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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

FLOW AROUND BENDS IN AN OPEN FLUME

By Ahmed Shukry,[1] Assoc. M. ASCE

Synopsis

In this paper are presented the results of an investigation of the spiral motion around bends by direct measurements taken with a specially designed pitot sphere, which records the velocity components in the three planes. The characteristics of this spiral motion, as well as other pertinent aspects of the bend action, were studied experimentally. The investigations show the effect of varying the Reynolds number, the depth-breadth ratio, the radius-breadth ratio, and the deflection angle of the bend.

Introduction

Notation.—The letter symbols in this paper are defined where they first appear, in the text or by illustrations, and are assembled for convenience of reference in the Appendix.

Research on flow around bends has engaged the attention of hydraulic engineers for more than a hundred and fifty years. Most of the investigations were on closed bends and were mainly concerned with the energy lost as the result of bend resistance. Although considerable data have been published,[2,3,4,5,6,7,8] the results have not been satisfactory because of their wide disagreement, which may be attributed to the absence of a standard basis for their correlation.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by November 1, 1949.

[1] Asst. Prof. of Irrig., Faculty of Eng., Farouk First Univ., Alexandria, Egypt.

[2] "Report Upon the Physics and Hydraulics of Mississippi River," by A. A. Humphreys and H. L. Abbot, Lippincott, Philadelphia, Pa., 1861, p. 315.

[3] "Loss of Pressure in Water Flowing through Straight and Curved Pipes," by A. W. Brightmore, *Minutes of Proceedings*, Inst. C. E., London, Vol. CLXIX, 1906–1907, p. 315.

[4] "Der Verlust in 90°-rohrkrümmern mit gleichbleibendem kreisquerschnitt," by A. Hofmann, München, Oldenbourg, 1929.

[5] "The Flow of Water in Flumes," by Fred C. Scobey, *Technical Bulletin No. 393*, U.S.D.A., Washington, D. C., 1933.

[6] "Resistance to Flow in Curved Open Channels," by S. P. Raju, in Abridged Translations of Hydraulic Papers, *Proceedings*, ASCE, November, 1937, p. 49 after p. 1834.

[7] "Effects of Channel Shape on Losses in a Canal Bend," by C. H. Yen and J. W. Howe, *Civil Engineering*, January, 1942, p. 28.

[8] "Theorestiche Grundlagen der Fluss-und Wildbachverbauungen," by R. Müller, Leeman & Co., Zürich, 1943.

If the energy lost due to bend resistance is obtained dimensionally, as in the case of problems dealing with pipe resistance, the application of the π-theorem[9] to any bend would show that its coefficient of resistance ξ $\left(\text{which expresses the head lost } h_b \text{ in terms of the mean velocity head } \frac{V^2}{2g}\right)$ is a function of:

(*a*) The Reynolds number **R**, expressed as

$$\mathbf{R} = \frac{V R}{\nu} \qquad (1)$$

in which V is the mean velocity; R is the hydraulic radius; and ν is the kinematic viscosity;

(*b*) The roughness parameter R/n_l in which n_l is a linear dimension of the wall roughness and the bed roughness; and

(*c*) The form parameter, which depends on the geometrical shape of the bend.

In a bend of an open, stable, rectangular channel the form parameter (item (*c*)) may be expressed in terms of the three ratios r_c/b, y/b, and $\theta/180°$ (r_c is the radius of the bend at the center of the stream; b is the width of the channel bed; y is the depth of flow; and θ is the angle of bend in degrees).

The general relationship between ξ and its parameters may be expressed as

$$\xi = C\,\phi\left(\mathbf{R}, \frac{R}{n_l}, \frac{r_c}{b}, \frac{y}{b}, \frac{\theta}{180°}\right) \qquad (2)$$

in which C is a constant and ϕ denotes a function.

Eq. 2 shows the importance of specifying the range of any experiment with respect to each of the five parameters. Furthermore, since the additional energy lost is but one of the phases in which the bend action manifests itself, other phenomena resulting from this action may reasonably be expected to vary with these parameters.

The lateral currents induced in bends were first observed by J. Thompson in 1876[10] in a series of experiments conducted in an open channel having an 180° bend, the spiral motion being observed by seeds and drops of anilin dyes. Later, many other investigators, following almost the same procedure as Professor Thompson, explored the spiral flow phenomena in closed and open bends.[11,12,13,14] In most of these researches only the forward-velocity components were recorded because of the limitation of the then existing hydraulic

[9] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York and London, 1938.

[10] "On the Origin and Winding of Rivers in Alluvial Plains, with Remarks on Flow Around Bends in Pipes," by James Thompson, *Proceedings*, Royal Soc. of London, March, 1876, p. 5.

[11] "Nebenströmungen in gekrümmten Kanälen," by A. Hinderks, *Zeitschrift*, Vereines Deutscher Ingenieure, Vol. 71, 1927, p. 1779.

[12] "Flow around a River Bend Investigated," by F. L. Blue, Jr., J. K. Herbert, and R. L. Lancefield, *Civil Engineering*, May, 1934, p. 258.

[13] "Flow of Water around 180°-Bend by D. L. Yarnell, *Technical Bulletin No. 526*, U.S.D.A., Washington, D. C., 1936.

[14] "Flow Around Bends in Stable Channels," by C. A. Mockmore, *Transactions*, ASCE, Vol. 109, 1944, p. 593.

instruments; thus, it was not possible to obtain a direct, clear image of the flow pattern. Moreover, the use of floats, threads, dyes, seeds, wet paints, and transparent models could show only the general features of the spiral flow.

Spiral flow cannot be studied properly unless direct measurements of the velocity vectors have been made. As the flow is distinctly three dimensional, a pitot sphere was built especially for the work described herein to measure the velocity components.

The Pitot Sphere

The pitot sphere (Fig. 1) has five brass tubes a, b, c, d, and e (Fig. 1(*b*)), enclosed in a casing stem, A, which is 9.0 mm in external diameter (see Fig. 1(*a*)).

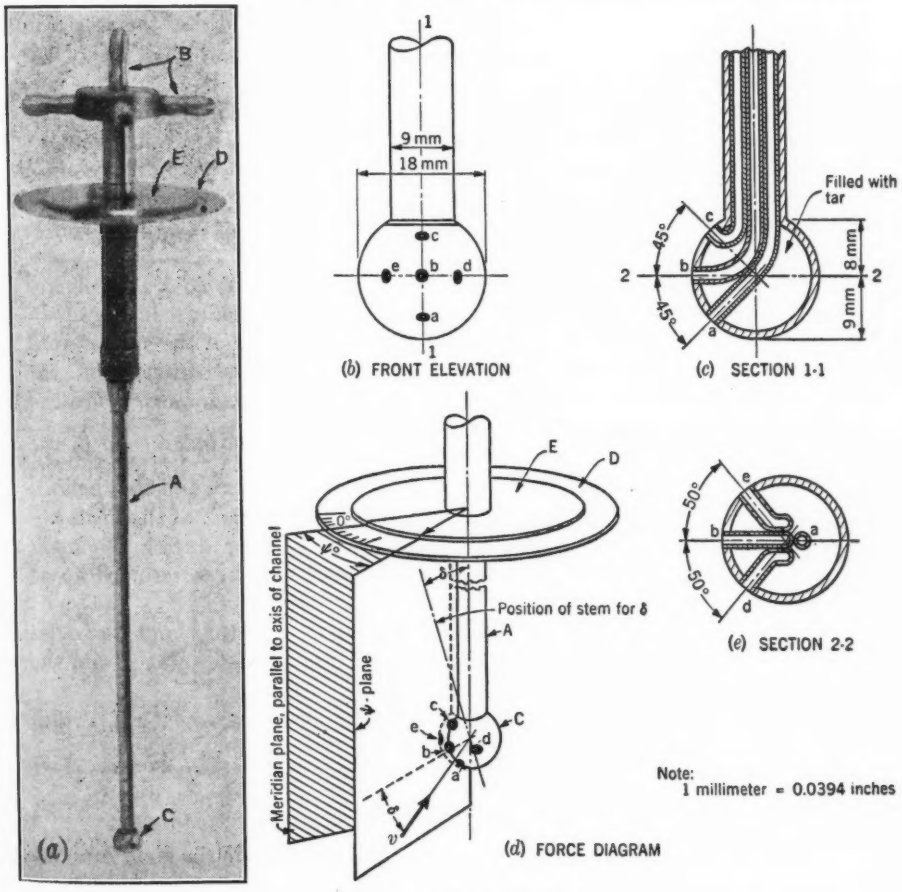


Fig. 1.—The Pitot Sphere

Each tube has a small brass tapping at the top end to permit a rubber tubing connection with a pressure manometer. At the bottom end the tube is bent inside a sphere, C, 18.0 mm in diameter (3/4 in.), and then terminates at a cer-

tain hole in the shell of the sphere which thus contains five holes. The center of hole b coincides with the horizontal axis of the sphere; holes a and c lie on its vertical great circle and have central angles of 45°, whereas holes e and d lie on the horizontal great circle and have central angles of 50°.

Two circular disks D and E (Fig. 1(*a*)) are mounted on the stem A. Disk D is graduated in degrees and disk E is fixed to the stem and contains a pointer that lies in the plane of holes a, b, and c. The stem A passes through a hole in disk D so that when the disk is fixed at a certain position and the stem is rotated, the angle of deviation in degrees can be readily obtained.

To use the pitot sphere, the procedure is as follows:

(1) The instrument is held with the stem A perpendicular to the direction of the longitudinal axis of the channel. It is then so adjusted that the meridian plane, containing holes a, b, and c and the zero of the graduated disk D, is parallel to that longitudinal axis.

(2) The stem is then rotated until the pressure manometers connected to holes d and e indicate the same reading. In this position, the velocity vector v lies in the vertical plane that passes through holes a, b, and c and the pointer of the disk E (Fig. 1(*d*)). Angle ψ between this plane and the meridian plane can be readily observed on disk D. The manometric heads h_a, h_b, h_c, and h_d are then recorded.

(3) To find, by direct measurement, the angle of inclination δ of the velocity vector with the direction of the longitudinal axis of the channel in Fig. 1(*d*), the stem should be rotated in the plane ψ about the center of the sphere until manometers a and c indicate the same reading. In this second position, the velocity vector would be directly opposite hole b, and the manometric head of this hole would give the velocity head $\frac{v^2}{2g}$ from which the magnitude of the vector v could be determined. The angle between the normal and the inclined positions of the stem would be equal to the inclination angle δ of the vector.

The angle, δ, however, is not easily measured directly during the main experiments and an indirect method, based on calibration, is resorted to as explained subsequently.

(4) Having obtained the magnitude of the velocity vector v and its angles ψ and δ, the velocity components in the three planes are determined from the relationships (see Fig. 2):

$$v_x = v \cos \delta \sin \psi \quad \ldots\ldots (3a)$$

$$v_y = v \sin \delta \quad \ldots\ldots (3b)$$

and

$$v_z = v \cos \delta \cos \psi \quad \ldots\ldots (3c)$$

During the main experiments it is rather difficult to rotate the stem A about the sphere whose center must remain stationary at the point of measurement. For this reason the pitot sphere has been calibrated beforehand in such a way that the manometric readings h_a, h_b, h_c, and h_d, taken during step (2), may be used directly to give values of the velocity head $\frac{v^2}{2g}$ of the vector and its inclination angle δ.

Calibration of the Pitot Sphere.—In this calibration, the instrument is given different angles of inclination δ, varying from $\mp 60°$ with a direction of a parallel flow whose velocity, v_o, and static pressure, p_o, are known (see Fig. 3). For any angle δ within the specified range, manometers e and d are first adjusted to give the same reading, and the manometric heads h'_a, h'_b, h'_c, and h'_d are then recorded.

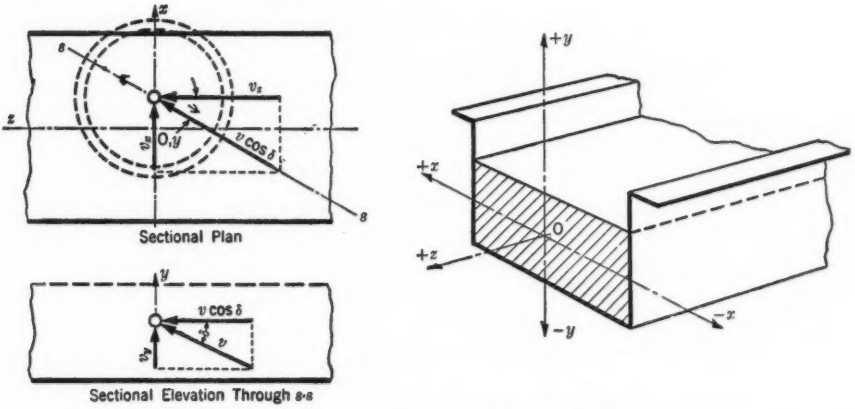


FIG. 2.—FLOW COORDINATES AND VELOCITY COMPONENTS

Consider the case of a potential flow around a frictionless sphere, as represented in Fig. 4. From the theory of fluid mechanics,[15] the pressure head at any hole m, lying at a central angle β_m in the direction of the parallel flow, is given by

$$h'_m = \frac{p_o}{\gamma} + \frac{v_o^2}{2g}\left(1 - \frac{9}{4}\sin^2 \beta_m\right) = \frac{p_o}{\gamma} + k_m \frac{v_o^2}{2g} \ldots\ldots\ldots\ldots (4)$$

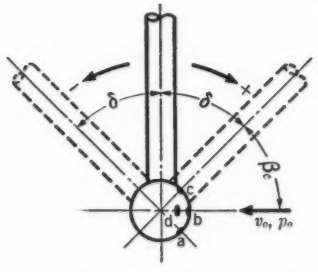


FIG. 3.—ANGLE OF INCLINATION δ

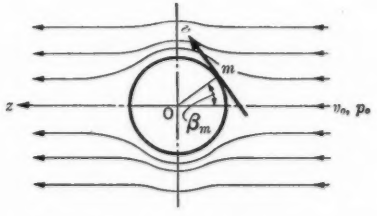


FIG. 4.—PATTERN OF STEADY FLOW AROUND A SPHERE

The symbol k_m is termed the "dynamic coefficient" of the hole m. As demonstrated by Eq. 4, it depends only on the central angle β_m. Substituting in Eq. 4 the manometric heads recorded in the calibration test (h'_a, h'_b, h'_c, and h'_d), the corresponding dynamic coefficients (k_a, k_b, k_c, and k_d), for any angle δ,

[15] "Aerodynamic Theory," by Julius Springer, Berlin, 1934, p. 213.

are determined. If different values for δ are taken, the functions of k_a, k_b, k_c, and k_d, with respect to angle δ, can now be obtained from Fig. 5.

Furthermore, there is a functional relationship between any angle δ and its corresponding manometric heads h'_a, h'_b, h'_c, and h'_d. Calling this function k_δ, an "inclination factor," the relationship may be expressed as follows:

$$k_\delta = \frac{h'_c - h'_a}{h'_b - h'_d} = \frac{k_c - k_a}{k_b - k_d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

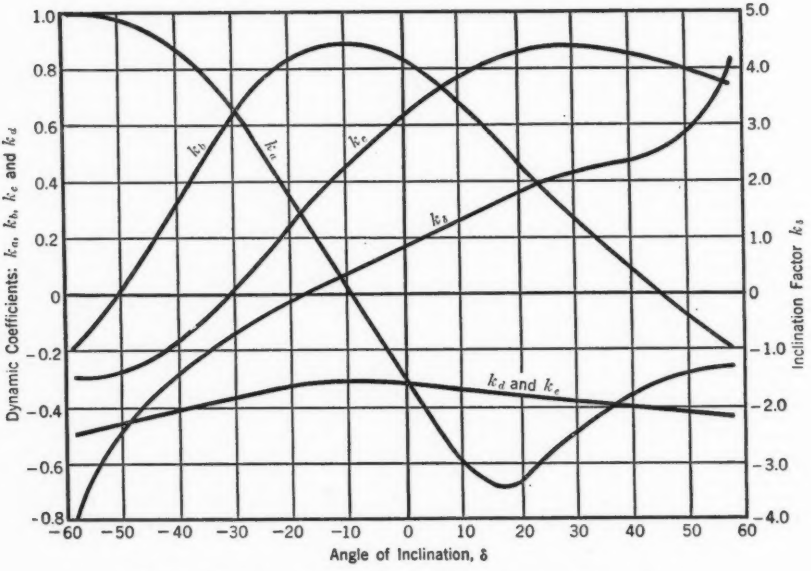


FIG. 5.—DYNAMIC COEFFICIENTS AND INCLINATION FACTORS FOR THE PITOT SPHERE, BY CALIBRATION

It is solely dependent on the manometric heads or the dynamic coefficients of the holes, which coefficients have been shown to depend entirely upon their central angles β_a, β_b, β_c, and β_d, with respect to the direction of the parallel flow. Since each one of these angles bears a direct relationship to the angle of inclination δ of the stem, as may be seen in Fig. 3, then k_δ depends only on angle δ. If k_δ is computed from Eq. 5 for the different values of δ lying between $\delta = \mp 60°$, then the relationship between k_δ and angle δ can be obtained from Fig. 5.

If the calibration curves of the pitot sphere have been predetermined, as in Fig. 5, step (3) of the foregoing measuring procedure may be replaced by the following step (3*a*), noting that v_o, p_o, and h', for a pitot sphere inclined in a parallel flow, are equal to v, p_s, and h, respectively, when the pitot sphere is held normal to the axis of the channel and the velocity vector is inclined at any angle δ.

(3*a*) Substituting the values of the manometric heads recorded in step (2) (namely, h_a, h_b, h_c, and h_d) in Eq. 4,

$$h_a = \frac{p_s}{\gamma} + k_a \frac{v^2}{2g} \quad \ldots\ldots (6a)$$

$$h_b = \frac{p_s}{\gamma} + k_b \frac{v^2}{2g} \quad \ldots\ldots (6b)$$

$$h_c = \frac{p_s}{\gamma} + k_c \frac{v^2}{2g} \quad \ldots\ldots (6c)$$

and

$$h_d = \frac{p_s}{\gamma} + k_d \frac{v^2}{2g} \quad \ldots\ldots (6d)$$

The velocity head of the vector may be computed from the following relationship:

$$\frac{v^2}{2g} = \frac{h_a - h_b}{k_a - k_b} = \frac{h_b - h_c}{k_b - k_c} = \text{(etc.)} \quad \ldots\ldots (7)$$

The static pressure head at the measuring point will be

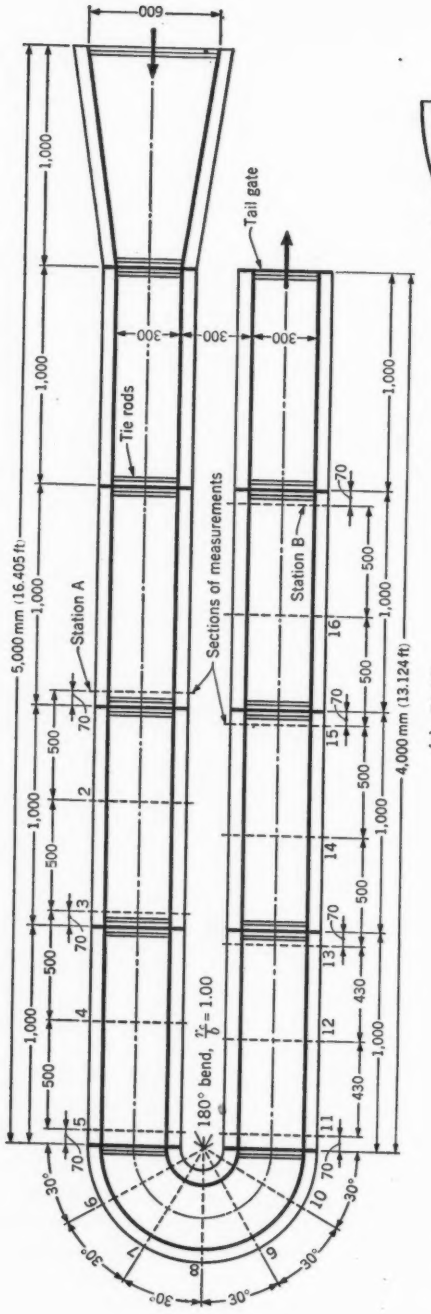
$$\frac{p_s}{\gamma} = h_a - k_a \frac{v^2}{2g} = h_b - k_b \frac{v^2}{2g} = \text{(etc.)} \quad \ldots\ldots (8)$$

To find the angle of inclination δ of the vector, the inclination factor k_δ is first computed from the manometric heads according to Eq. 5 and the corresponding angle δ is then obtained from Fig. 5.

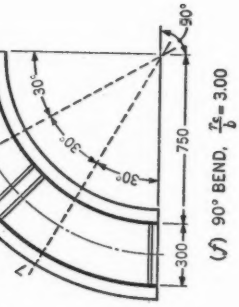
The pitot sphere used in this research work was calibrated in a still-water channel, 120 m long (393.70 ft). The instrument was given angles of inclination δ to the channel axis ranging between $\delta = \mp 60°$ and was then towed at various selected speeds ranging between 0.50 m per sec and 3.00 m per sec (9.8 ft per min to 59.0 ft per min). The dynamic heads were recorded during the uniform motion of the pitot sphere. In this calibration, the static pressure head in Eq. 4 was zero since h represented only the dynamic pressure head $k \frac{v_o^2}{2g}$. The results showed that the dynamic coefficients of the holes, for any angle δ, were almost constant in the range of speeds used. This is an important point in favor of the pitot sphere.

Laboratory Arrangement

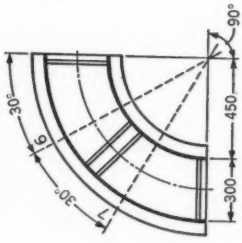
The experimental flume was constructed of steel sheets, and the inside surfaces were coated with smooth paint. It consisted of different separate straight and curved sections that could be assembled to give any desired alinement (Fig. 6). The joints between the sections were filled with rubber gaskets 5.0 mm thick. The flume had a uniform cross section 30 cm wide (11.8 in.) and 40 cm deep (15.75 in.). When tests were conducted on any given bend, the bend was preceded and followed by straight tangents 5.0 m and 4.0 m (16.40 ft and 13.12 ft) long, respectively; there were no other bends in the system. A stilling basin, floats, and a curved masonry entrance helped to provide a smooth flow in the flume; a tailgate was used for adjusting the depth



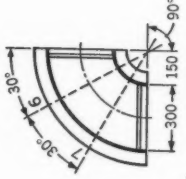
(a) PLAN



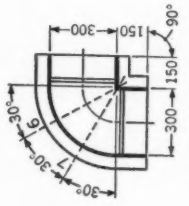
(f) 90° BEND, $\frac{r_c}{b}$ = 3.00



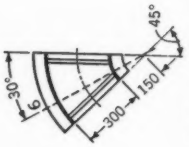
(e) 90° BEND, $\frac{r_c}{b}$ = 2.00



(d) 90° BEND, $\frac{r_c}{b}$ = 1.00



(c) 90° BEND, $\frac{r_c}{b}$ = 0.50



(b) 45° BEND, $\frac{r_c}{b}$ = 1.00

FIG. 6.—GENERAL PLAN OF FLUME AND BENDS

of flow. The broken lines in Fig. 6 show the positions of the sections at which measurements were taken. In the bed of both the first section (Station A, Fig. 6(*a*)), and the last section (Station B), three holes were drilled which were connected to a common tube so that the observer was able to measure the drop in the water surface between the two stations. This was done by two glass containers, mounted on a common board, each fitted with a point gage. The rate of discharge was measured by two previously calibrated sills.

In all the experiments the bed of the flume was kept horizontal and the flow approached the bend in a streaming state. The experimental study covered the effect of the partial variations of the following parameters: **R**, r_c/b, y/b, and $\theta/180°$. The mean value of Kutter's coefficient of roughness (as computed from the Manning formula) was found, for the range of experiments in the present research, to be 0.01012.

Velocity and Energy Distribution Around Bends

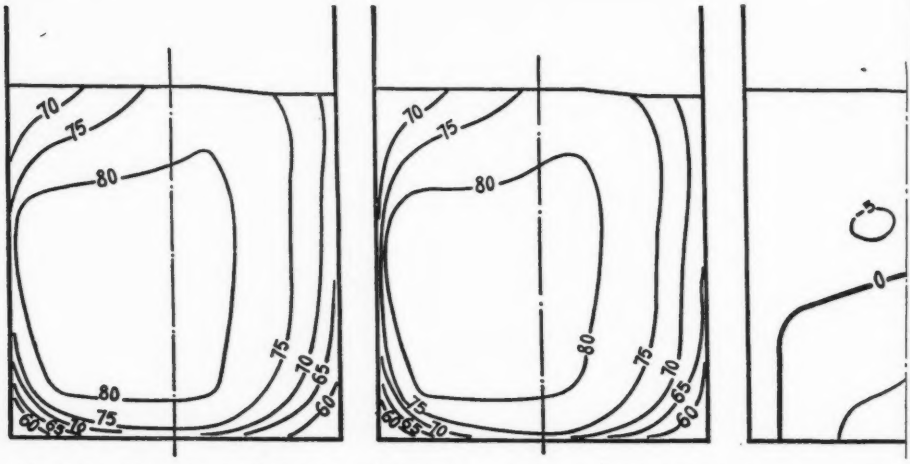
The Spiral Motion in a Straight Flume.—Fig. 7 presents the contour lines of the velocity components v_x, v_y, and v_z at the midsection of a straight flume 9.0 m long, the flow coordinates being as shown in Fig. 2. The magnitudes and directions of the lateral currents are represented by the dimensioned arrows at the measuring points. To visualize the lateral motion, "direction lines" were drawn by interpolation. These lines should not be confused with the "streamlines" since they do not obey the known equation of continuity. The zero contours of v_x and v_y intersect at point Z at which the flow is parallel to the longitudinal axis of the flume. Point Z may be considered as the intersection of the axis of the spiral motion with the plane of the cross section. Although the rotary motion about this axis is not uniform, a single spiral motion, nevertheless, is easily discernible. This differs from the spiral motion in straight channels, as discussed by A. H. Gibson,[16] M. Möller,[17] and others, in which the pattern included one spiral on each side of the center line where the maximum superelevation existed. It is quite probable that such a case of double spiral motion does not occur in a straight channel unless it has a long and uniform reach, sufficiently removed from the inlet, to permit the equalization of shear stresses on both sides. The highest water level would then be at the center of the stream. In short laboratory flumes, the small disturbance at the entrance, being usually an inevitable condition, is sufficient to cause the zone of highest water level to shift to one side, thus giving rise to a single spiral motion.

The Spiral Motion Around an 180° Bend.—Fig. 8 shows the contour lines of the velocity vector v and the lateral currents v_{xy} at eight sections around an 180° bend. The forward-velocity contours, v_z, are also given for some of these sections. The conditions of flow are the same as those for the straight flume discussed in the previous paragraph. Fig. 8 shows that the general distribution of the flow around the bend passes through three distinct stages.

In the first stage (sections 2 to 7, Fig. 6), the v-contours and the v_z-contours keep adjusting themselves until they become almost flattened out vertically

[16] "Hydraulics and Its Applications," by A. H. Gibson, Constable & Co., Ltd., London, 1934, 4th Ed., p. 332.

[17] "Hydraulic Laboratory Practice," by J. R. Freeman, A.S.M.E., 1929, p. 70.



(*a*) Contour Lines of Equal Vector (v) (*b*) Contour Lines of Equal Component (v_2) (*c*) Contour Lines of Equal

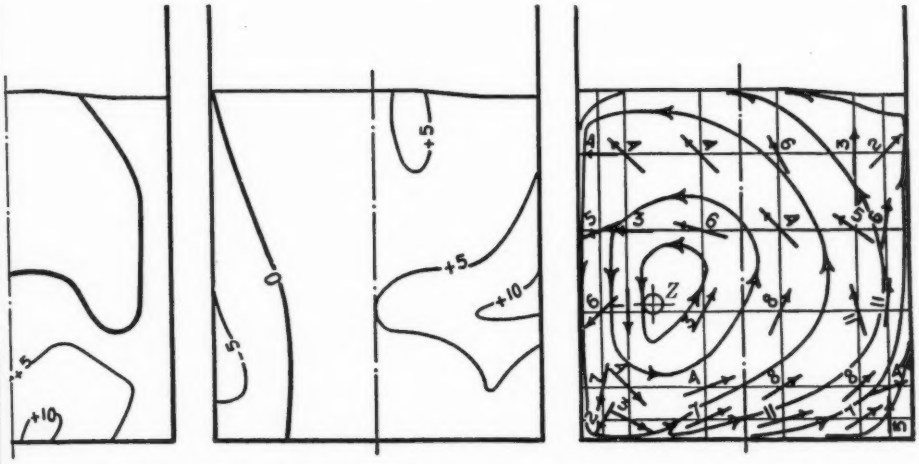
FIG. 7.—DISTRIBUTION OF THE VELOCITY COMPONENTS, FACING DOWN
(Velocities Are in Centimeters per Second (=1.9685 Ft per Min); y/b = 1.0; R = 7

and parallel to one another; and velocity contours of much higher values occur near the inside wall. This stage ends at section 7 (at 60°, Fig. 8(*c*)), where the superelevation of the water surface, the maximum velocity vectors, and the maximum forward-velocity components have values greater than those at any other section of the bend.

During the second stage (sections 7 to 11, Fig. 6), the contours of maximum velocity gradually move from the inside wall, causing such great deceleration there that a discontinuity region is formed at the exit of the bend adjacent to the inside wall (section 11). In spite of this discontinuity, however, the flow in the outer half of the section is fairly evenly distributed.

In the third stage (sections 11 to 16, Fig. 6), the distribution of the flow is readjusted until it reverts to the conditions in a straight flume.

By virtue of the centrifugal force, the lateral currents along the bend show a continuous tendency to deviate from the original pattern of the straight approach flume and to form a new one. The new pattern, with axis Z_2, will be called the "centrifugal spiral motion" to distinguish it from the "original spiral motion" of a straight flume whose pattern has an axis Z_1. The centrifugal motion becomes perceptible near the bed at section 7, and then expands gradually as the flow moves around the bend. As a result of this action, the original pattern is pushed to the upper outside corner of the section until it ultimately disappears near the exit of the bend, at section 11, Fig. 6. At this section the centrifugal motion almost occupies the full cross-sectional area, with the exception of the region close to the inside wall which is occupied by the wake of separation. As the flow leaves the bend and advances downstream, the original motion reappears in the upper regions (section 12). It then expands gradually,



Equal Component (v_x) (*d*) Contour Lines of Equal Component (v_y) (*e*) Direction Lines and Magnitudes of the Lateral Currents (v_{xy})

DOWNSTREAM AT THE MIDSECTION OF A STRAIGHT FLUME
R = 73,500; and Q = 70 Liters per Sec (=8,899.24 Cu Ft per Hr))

pushing down the centrifugal pattern, which, however, does not totally disappear until section 16 is reached, where the lateral motion becomes similar again to that of a straight flume.

It will be noted from the foregoing that the assumption that there is a single centrifugal spiral motion moving uniformly along the bend is erroneous. As a matter of fact, a complicated flow pattern is produced by the interference of the spiral motion (originally taking place in the straight approach flume) with that caused by the bend action. In the present work the centrifugal pattern was fully developed only for a short length of flume in the vicinity of section 11.

The effect of the spiral motion on the specific energy diagram of the curved flume is shown in Fig. 9, in which the five lines, 1, 2, 3, 4, and 5, may be defined as follows:

Curve	Definition
1.....	The line of mean depth of flow along the flume
2.....	The specific energy line obtained by adding the kinetic energy of the mean forward velocity, $\frac{V^2_x}{2g}$, to the mean depth of flow, y_m, at any section
3.....	The specific energy line obtained by adding to y_m the value of $\left(\frac{V^2_x}{2g}\right)_m$, the mean kinetic energy of the forward motion at any section, as given by

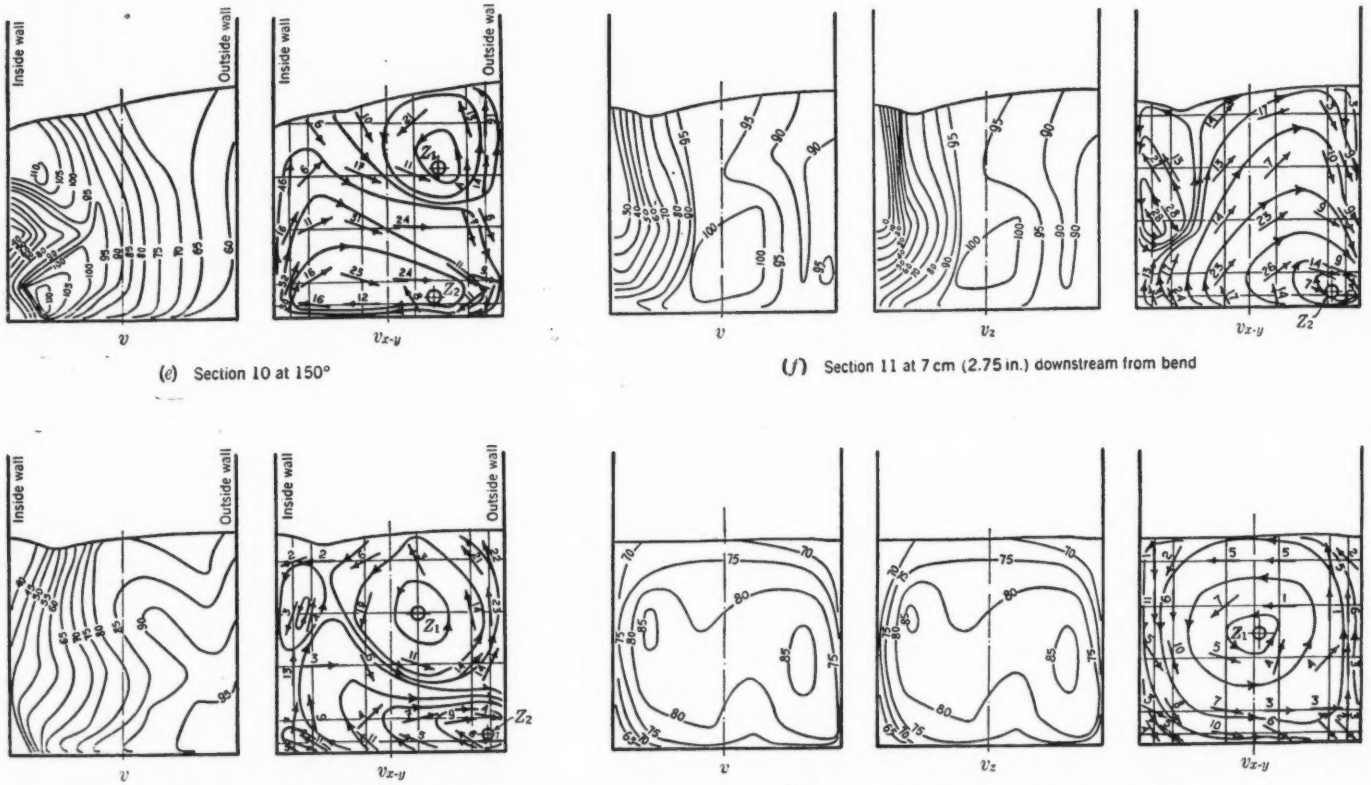
$$\left(\frac{V^2_x}{2g}\right)_m = \frac{1}{V_x A}\int_0^A \frac{v^3_x}{2g}\,dA \qquad (9a)$$

(*a*) Section 2 at 157 cm (5.2 ft) upstream from bend

(*b*) Section 5 at 7 cm (2.75 in.) upstream from bend

(*c*) Section 7 at 60°

(*d*) Section 8 at 90°



(e) Section 10 at 150°

(f) Section 11 at 7 cm (2.75 in.) downstream from bend

(g) Section 12 at 50 cm (1.6 ft) downstream from bend

(h) Section 16 at 247 cm (8.1 ft) downstream from bend

FIG. 8.—DISTRIBUTION OF THE VELOCITY VECTORS, v, THE FORWARD-VELOCITY COMPONENTS, v_z, AND THE LATERAL CURRENTS v_{x-y}

Curve Definition

in which V_s is the mean forward velocity; A is the area of the water cross section; and v_s is the forward-velocity component of an elemental area

4.....The true specific energy line of the flume obtained by adding to y_m the value of $\left(\frac{V^2}{2g}\right)_m$, the mean total kinetic energy at any section, as given by

$$\left(\frac{V^2}{2g}\right)_m = \frac{1}{V_s A}\int_0^A \frac{v^2}{2g} v_s \, dA \quad \text{.....(9b)}$$

5.....The friction line as obtained experimentally, representing the specific energy line for an equivalent length of a straight flume.

Eqs. 9, for any section, were solved graphically by a planimeter, from the v-contours and v_s-contours.

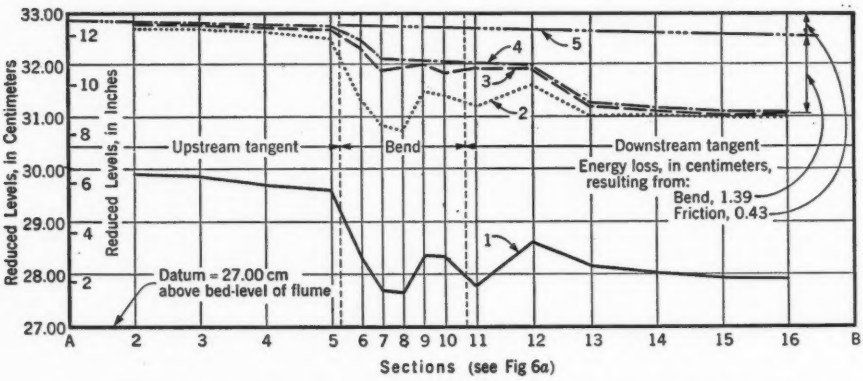
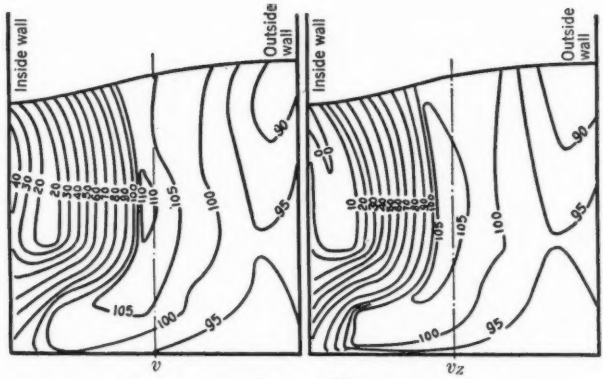


FIG. 9.—ENERGY DIAGRAMS FOR AN 180° BEND ($r_c/b = 1.00$; $y_A/b = 1.00$; and $R_A = 73{,}500$)

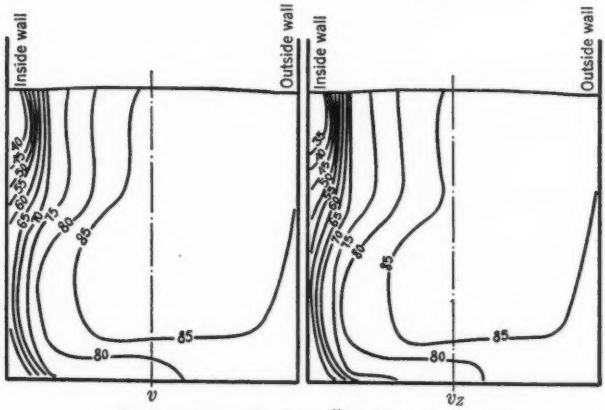
The ordinates between curves 4 and 3 at any section represent the mean kinetic energy of the lateral motion as given by

$$\left(\frac{V^2_{xy}}{2g}\right)_m = \left(\frac{V^2}{2g}\right)_m - \left(\frac{V^2_s}{2g}\right)_m \quad \text{.....(9c)}$$

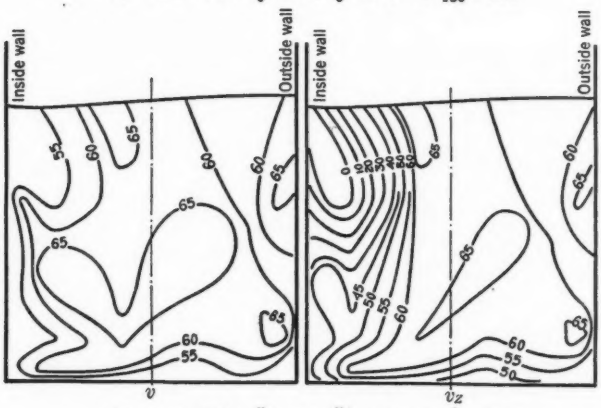
The ordinates between curves 4 and 5 represent the accumulated energy losses resulting from the bend resistance, whereas those between curve 5 and the static energy line, in Fig. 9, are the accumulated energy losses from friction. Although the values of $\left(\frac{V^2_{xy}}{2g}\right)_m$ at the different sections of the bend are relatively small, they are, nevertheless, the cause of the apparent inconsistency in the energy line, curve 3, Fig. 9, whose ordinates at some sections acquire greater values than at preceding ones. This shows the importance of using the pitot sphere for measuring the velocity components, if a true analysis of a pronounced three-dimensional flow is to be obtained.



(a) $R_A = 73{,}500$, $\frac{r_c}{b} = 1.00$, $\frac{y_A}{b} = 1.00$, AND $\frac{\theta}{180^\circ} = 0.50$



(b) $R_A = 73{,}500$, $\frac{r_c}{b} = 3.00$, $\frac{y_A}{b} = 1.00$, AND $\frac{\theta}{180^\circ} = 0.50$



(c) $R_A = 48{,}100$, $\frac{r_c}{b} = 1.00$, $\frac{y_A}{b} = 1.00$, AND $\frac{\theta}{180^\circ} = 0.50$

FIG. 10.—EFFECT OF VARYING THE REYNOLDS NUMBER, AND THE RADIUS-BREADTH RATIO, ON THE DISTRIBUTION OF THE VELOCITY VECTORS AND THE FORWARD-VELOCITY COMPONENTS, v_z, AT THE DOWNSTREAM SECTION OF A BEND; SEE SECTION 11, FIG. 6
(Velocities Are in Centimeters per Second (=1.9685 Ft per Min))

Strength of a Spiral Motion.—No definition of the strength of a three-dimensional flow has come to the writer's notice. Such a definition, however, is essential in order to be able to visualize the effect of the spiral flow in different

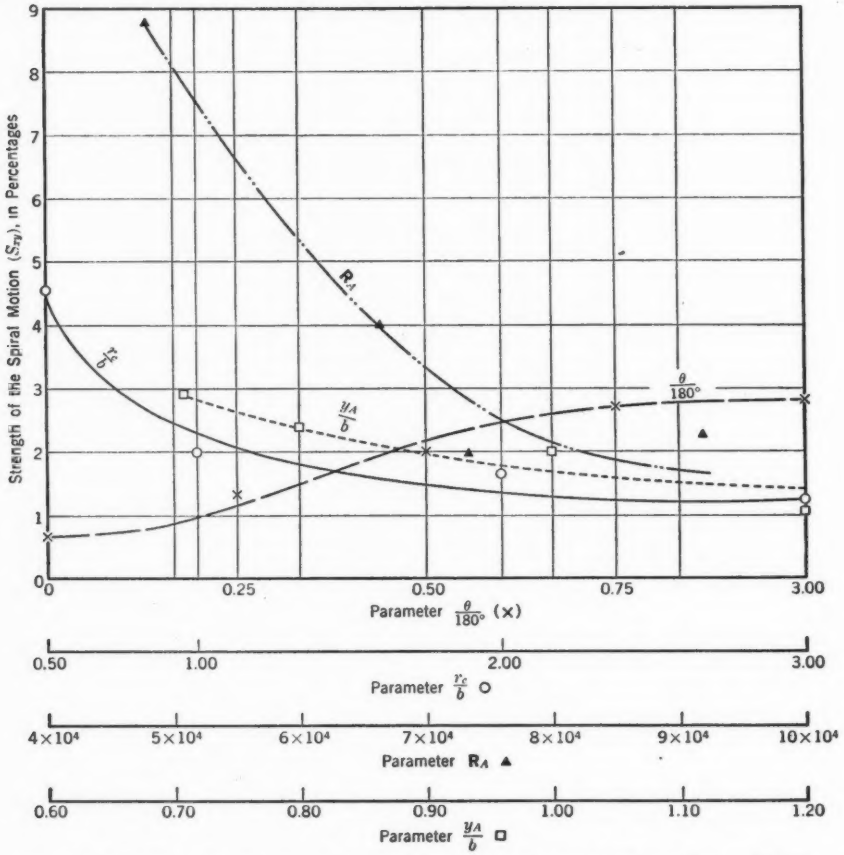


FIG. 11.—EFFECT OF THE DIFFERENT PARAMETERS ON THE STRENGTH OF THE SPIRAL MOTION

bends under variable conditions of flow. In this paper the strength of a spiral motion at any cross section is taken as the percentage ratio of the mean kinetic energy of the lateral motion to the total kinetic energy of flow, and is called S_{xy}:

$$S_{xy} = \frac{\left(\frac{V^2_{xy}}{2g}\right)_m}{\left(\frac{V^2}{2g}\right)_m} 100 \ldots . (10)$$

In the case of a flow with streamlines parallel to the axis of the conduit, $S_{xy} = 0$. The magnitude of S_{xy} is determined at any cross section from the contour lines of the velocity vector v and the forward-velocity components v_x, Eqs. 9.

TABLE 1.—MAGNITUDES OF CONSTANT PARAMETERS

Case	R_A	r_c/b	y_A/b	$\theta/180°$
R_A.....	..	1.0	1.0	0.5
r_c/b.....	73,500	..	1.0	0.5
y_A/b.....	68,500	1.0	..	0.5
$\theta/180°$....	73,500	1.0	1.0	..

The strength of the spiral motion was studied for each of the four parameters $\mathbf{R}_A$, r_c/b, y_A/b, and $\theta/180°$. (Subscript A in y_A and $\mathbf{R}_A$ refers to conditions at Station A, Fig. 6(a)). A particular section was chosen for comparing the results of all the bends, section 11, Fig. 6. Four groups of tests were run. In each

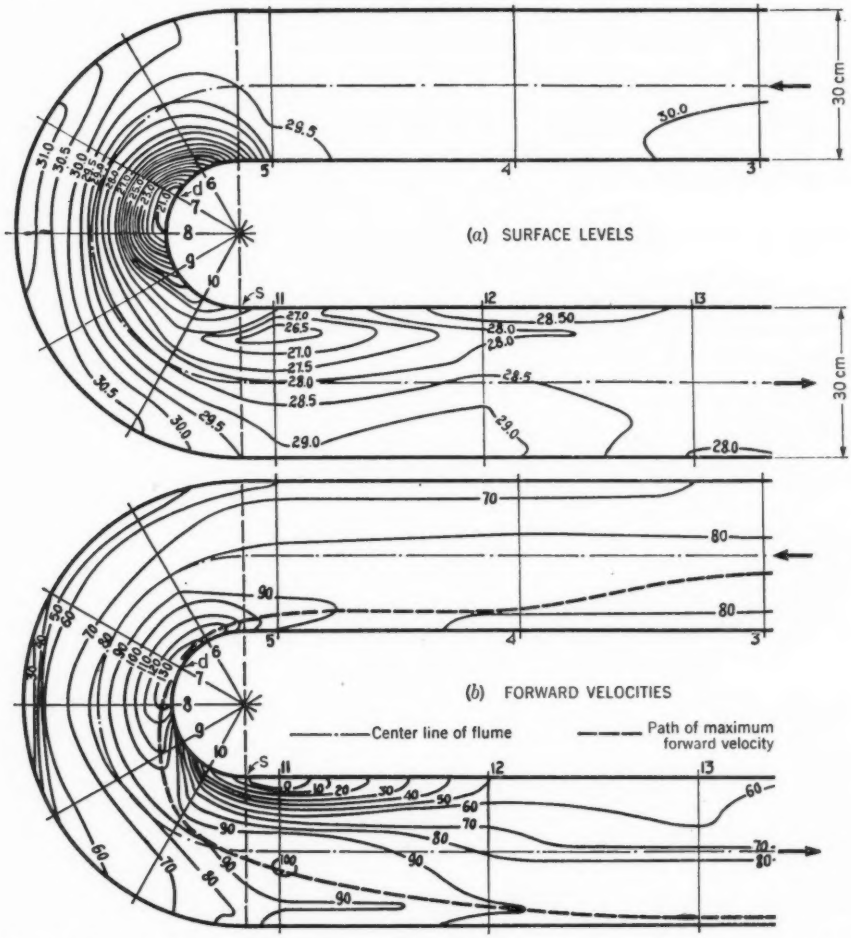


FIG. 12.—CONTOUR LINES OF EQUAL SURFACE LEVELS—IN CENTIMETERS (=0.3937 IN.) ABOVE THE HORIZONTAL BED OF THE FLUME—AND FORWARD-VELOCITY DISTRIBUTION IN AN 180° BEND—IN CENTIMETERS PER SECOND (=1.9685 FT PER MIN)

(r_c/b = 1.00; y_A/b = 1.00; and, with V_A = 77.8 Cm per Sec (=152.95 Ft per Min), R_A = 73,500)

group, three parameters were kept at constant values while the fourth parameter was given at least four different values. The v-contours and v_z-contours were plotted for each case. Fig. 10 shows the v-contours and the v_z-contours for only two values of each of the parameters $\mathbf{R}_A$ and r_c/b. Fig. 11 gives the four parametric functions of S_{xy} plotted on the same vertical axis but with a horizontal axis for each of the four parameters. The magnitudes of the three constant parameters for each case are shown in Table 1.

From Fig. 11 the following is noted:

(*a*) At low values of $\mathbf{R}_A$, the corresponding S_{xy}-values are comparatively high (by increasing $\mathbf{R}_A$, the gradient of the curve decreases);
(*b*) S_{xy} does not decrease appreciably in value beyond the ratio $r_c/b = 3.0$;
(*c*) S_{xy} decreases as the depth-breadth ratio y_A/b increases; and
(*d*) For the range $\theta/180° = 0$ (straight channel) to $\theta/180° = 0.5$, the increase in S_{xy} is nearly twice that for the range $\theta/180° = 0.5$ to 1.0.

Behavior of Water Surface and Forward-Velocity Components.—Fig. 12(*a*) shows the configuration of the water surface around an 180° bend. Fig. 12(*b*) gives the forward-velocity contours at a horizontal section taken at a height above the bed equal to four tenths of the mean depth of flow. The path of the thread of maximum velocity, Fig. 12(*b*), deviates from its normal course at a section upstream from the bend. This section (which is at a distance of three times the bed width from the inlet) marks the beginning of the "adjusting process" in which the flow is forced by the bend action to adapt itself to a certain distribution. This distribution occurs at the radial section passing through point d at which the water surface attains its minimum level and the path of maximum velocity almost touches the inside wall. Beyond point d, the path gradually moves outward until it crosses the center line of the flume at the section that passes through the separation point s. It then approaches the downstream outer wall and runs parallel to it for a distance 8 *b* in which the "readjusting process" takes place.

TABLE 2.—LOCATIONS OF POINTS OF MAXIMUM SURFACE DEPRESSION (POINT d, FIG. 12) AND POINTS OF SEPARATION (POINTS s, FIG. 12) FOR VARYING PARAMETERS (THE PARAMETER $y/b = 1.00$, A CONSTANT)

Parameter	R = 10,500		R = 21,000		R = 31,500		R = 52,500		R = 73,500	
	θ_d/θ	θ_s/θ	θ_d/θ	θ_s/θ	θ_d/θ	θ_s/θ	θ_d/θ	θ_s/θ	θ_d/θ	θ_s/θ
r_c/b	(*a*) $\theta/180° = 0.50$, A CONSTANT									
0.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.00	0.111	0.250	0.189	0.500	0.250	0.611	0.306	0.889	0.667	1.000
2.00	0.033	..[a]	0.067	..[a]	0.100	..[a]	0.133	..[a]	0.333	..[a]
3.00	0.000	..[a]	0.000	..[a]	0.044	..[a]	0.056	..[a]	0.167	..[a]
$\theta/180°$	(*b*) $r_c/b = 1.00$, A CONSTANT									
0.25	0.111	0.222	0.333	0.667	0.333	0.667	0.400	0.710	0.556	0.889
0.50	0.111	0.250	0.189	0.500	0.250	0.611	0.306	0.889	0.667	1.000
0.75	0.148	0.333	0.185	0.445	0.296	0.556	0.371	0.926	0.445	1.000
1.00	0.056	0.778	0.111	0.778	0.167	0.805	0.278	0.833	0.333	1.000

[a] No separation.

The point of maximum surface depression d and the point of separation s were located for different bends under widely different conditions of flow, as cited in Table 2. The positions of the two points are given in terms of the

ratios of their angles θ_d and θ_s, measured from the inlet of the bend to the total angle θ of the bend.

The ratios θ_d/θ and θ_s/θ were found to be only slightly affected by varying the parameter y/b. On the other hand, they decrease very quickly as the values of r_c/b increase. At $r_c/b = 3.0$, point d almost lies at the beginning of the bend and no separation in the flow occurs.

Fig. 12 shows that both the contour lines of the water surface and the forward velocity are more or less parallel to the walls of the bend in the vicinity of the radial section which passes through point d, indicating the existence of a functional relationship between the radius of curvature, the depth of flow, and the forward velocity. Investigations on different bends showed that the flow conditions at this radial section are very similar to those in a free or irrotational vortex, as long as the depth of flow at point d is greater than the critical depth of the streaming state. At radial sections on either side of point d, the similarity diminishes.

This characteristic of flow at section d may be applied to determine the forward-velocity distribution and surface profile curves. For example, in a free-vortex motion,

$$v_z = \frac{C}{r} \qquad (11a)$$

and

$$y = H - \frac{v_z^2}{2g} \qquad (11b)$$

also,

$$V_z = \frac{\int_{r_i}^{r_o} \frac{C}{r}\, dr}{r_o - r_i} = \frac{C}{r_o - r_i} \log_e \frac{r_o}{r_i} \qquad (12a)$$

$$y_m = \frac{\int_{r_i}^{r_o} y\, dr}{r_o - r_i} = \frac{\int_{r_i}^{r_o} \left(H - \frac{C^2}{2g r^2} \right)}{r_o - r_i} = H - \frac{C^2}{2g r_o r_i} \qquad (12b)$$

and

$$Q = V_z y_m (r_o - r_i) = C \log_e \frac{r_o}{r_i} \left(H - \frac{C^2}{2g r_o r_i} \right) \qquad (12c)$$

In Eqs. 12, r_i and r_o are the inner and outer radii of the bend, respectively; C is the circulation constant in a free-vortex motion; and H is the elevation of the specific energy line at any section.

If Q and H are known, the constant C can be determined from Eq. 12*c*. The velocity and the depth at any radius r are then obtained according to Eqs. 11. The superelevation Δh of the water surface will then be given by

$$\Delta h = \frac{C^2}{2g r^2_o r^2_i} (r^2_o - r^2_i) \qquad (13)$$

For the practical application of Eqs. 11*a*, 12*c*, and 13 to any bend, Table 2 is used to give the position of point d. The height of the specific energy line

is first determined for any section A in the straight approach channel (far removed from the inlet of the bend) from the formula:

$$H_A = y_A + \alpha \frac{V^2_z A}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which the velocity head corrective factor α may be assumed equal to unity without appreciable error. The elevation H is then obtained by subtracting from H_A the energy lost in normal friction and the additional energy lost from bend resistance up to the radial section passing through point d. If ρ represents the ratio of the latter loss to its total value in the bend, and L is the distance between the two sections, then,

$$H = H_A - L\,h'_f - \rho\,h_b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which h'_f is the energy lost in friction per unit length of a straight channel and may be either determined experimentally or calculated from the Manning

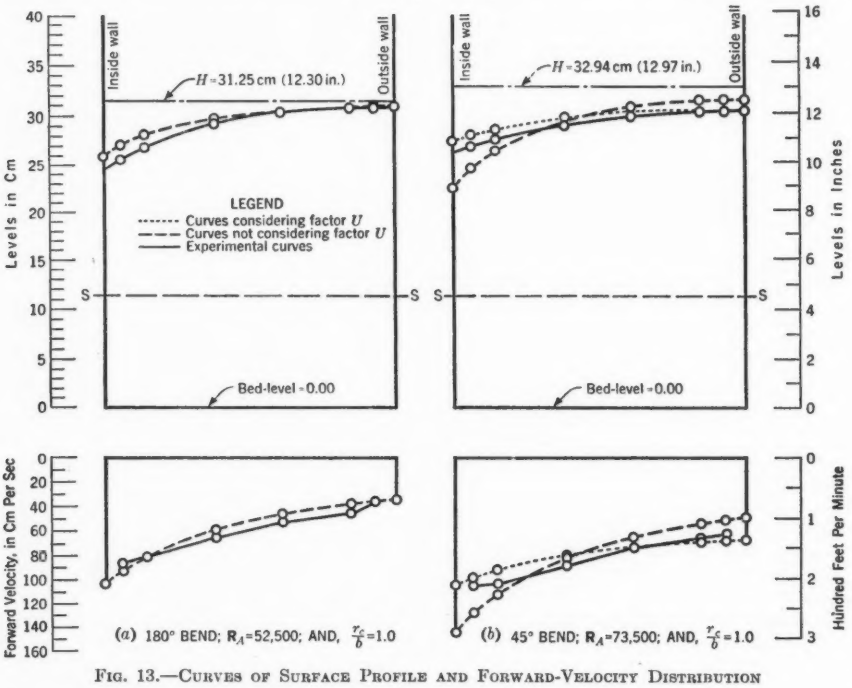


FIG. 13.—CURVES OF SURFACE PROFILE AND FORWARD-VELOCITY DISTRIBUTION

formula, and h_b is the energy lost due to bend resistance which may be obtained from Fig. 16 as explained subsequently. The coefficient ρ was found to be more or less constant for any bend and to have a value of 0.4.

The predetermined curves of the forward-velocity distribution and surface profile for any bend (obtained by assuming the existence of a theoretical free vortex) were found to be reasonably accurate as long as the angle of the bend was greater than 90°. The theoretical and experimental curves were obtained

for several bends under different conditions of flow. An 180° bend is taken as an example and the pertinent curves are shown in Fig. 13(*a*) in broken and full lines, respectively. The experimental curves of the forward-velocity distribution are for the horizontal section S–S.

When the angle of bend is less than 90°, the differences between the theoretical and the experimental values gradually increase. These differences, however, could be reduced by introducing in the free-vortex formula a coefficient U (which is a function of the angle of bend) so that the formula becomes

$$v_z = U \frac{C}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16a)$$

The coefficient U is obviously also a function of the radius r, since v_z must be independent of r as the angle of the bend becomes zero (straight channel). Assuming that U varies linearly between $\theta = 0$ and $\theta = 90°$, and that its values at these two extremities are $\frac{r}{C V_{zs}}$ and 1.0, respectively (V_{zs} being the mean forward velocity in a straight channel), then, for any angle θ, the coefficient U will be given by

$$U = \frac{r}{C} V_{zs} + \left(1.0 - \frac{r}{C} V_{zs} \right) \frac{\theta}{90°} \ldots\ldots\ldots\ldots (16b)$$

The experimental curves of the surface profile and forward-velocity distribution are compared in Fig. 13(*b*) with the theoretical curves as calculated with and without the introduction of the coefficient U.

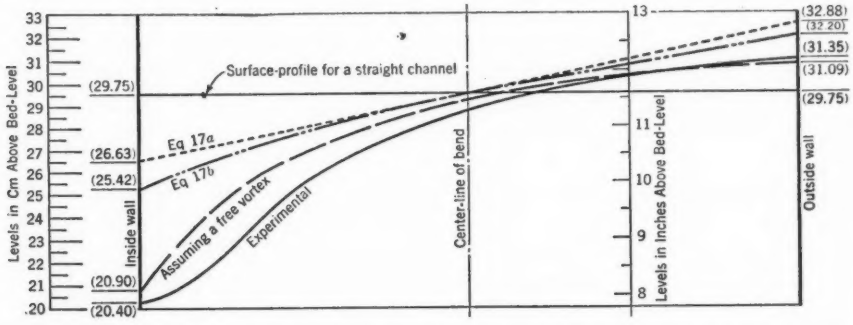


FIG. 14.—COMPARISON BETWEEN ACTUAL AND THEORETICAL SHAPES OF THE WATER-SURFACE PROFILE IN AN 180° BEND ($r_c/b = 100$; $y_A/b = 1.00$; AND $R_A = 73{,}500$)

Fig. 14 shows the actual water-surface profile in an 180° bend as compared with the theoretical profiles obtained by the foregoing assumption of a free vortex, and by the application of each of the following two formulas which are generally used in practice—

$$\Delta h = \frac{V^2_z b}{g r_c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17a)$$

and

$$\Delta h = \frac{V^2_z}{g} \log_e \frac{r}{r_i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17b)$$

In Eq. 17*a*, the surface profile is a straight line,[18] whereas in Eq. 17*b* (derived by F. Grashof[19]) it is a logarithmic curve. In both formulas the forward velocity is assumed constant throughout the entire cross section. In spite of the fact that the superelevation is attributed to the centrifugal force, the effect of this force on the velocity distribution, however, is not taken into consideration in either formula.

Effect of the Bend Action on Movable Beds.—When the bed of a curved channel is of an erodible material, a stable configuration in that bed is developed by virtue of the centrifugal spiral motion. It is noticeable in Fig. 8 that the strongest lateral currents occur near the outside wall of the midsection of the bend (section 8), with a downward movement. Beyond this section, the currents move diagonally along the bed toward the inside wall until they become close to it near the exit of the bend, section 11, with an upward inclination. These currents are not uniformly deflected toward the inside wall, however, but are grouped in separate zones which develop separate scour regions (pools), as shown in Fig. 15. The scoured material from these regions crosses the bend diagonally toward the inside wall near the exit of the bend where it settles because of the upward inclination of the bed currents. A sand bar is thus formed close to the inside wall at the downstream straight channel.

The different effects of the four parameters $\mathbf{R}_A$, r_c/b, y_A/b, and $\theta/180°$ were studied in four groups of tests. The bed level of the bend as well as that of an adjacent downstream section was lowered below the general bed level of the flume. The depression was then filled with sand of uniform grain, 1 mm to 2 mm in diameter. The surface of the sand was then given the same level as that of the remainder of the flume. The effect of the bed load which might be carried to the bend from the upstream channel was thus eliminated. Fig. 15(*a*) shows the results of tests on an 180° bend with three different depth-breadth ratios, and Fig. 15(*b*) gives the results on a 90° bend with three different radii. In both illustrations the full lines indicate the contours of the deformed bed, and the broken lines represent the main ridges. The original bed level before the flow was allowed in the flume was considered to be 0.0. The contour lines are given in terms of the final depths of deposit or scour to the original mean depth of flow y_A. Such representation produces dimensionless contours that help to illustrate the relative size of the deformation in the bed.

Fig. 15(*a*) shows that, as the depth-breadth ratio decreases, the volume of the sand bar at the inside wall of the downstream straight channel increases and the scour at the outside wall becomes deeper. In natural waterways, which generally have low depth-breadth ratios, this phenomenon might lead to a change in the watercourse downstream from the bend if the sides were also of an erodible material.

Fig. 15(*b*) demonstrates that an increase in r_c/b causes a considerable reduction in the size of bed deformation. For the present series of experiments no deformation occurred at $r_c/b = 3.0$.

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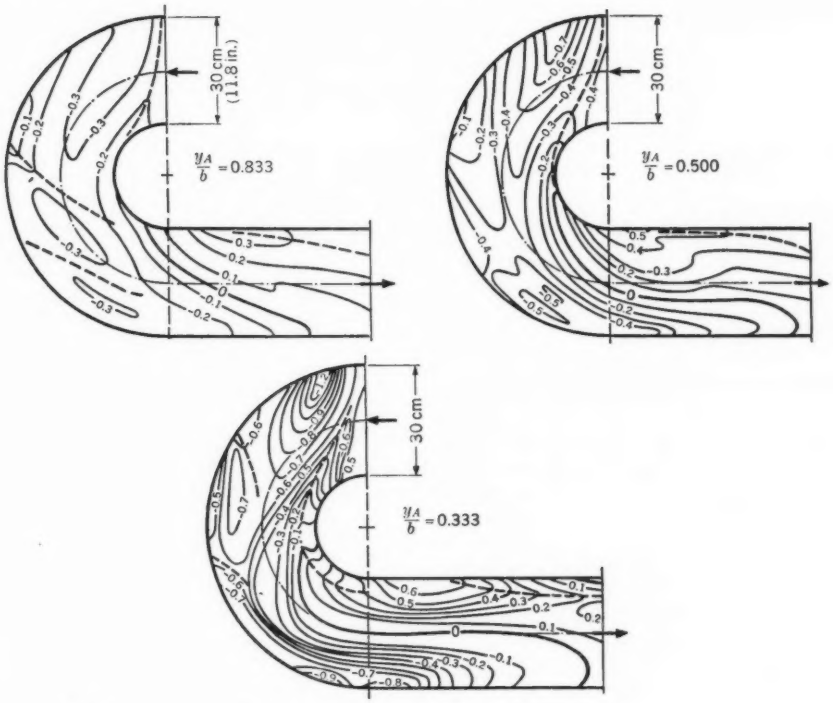
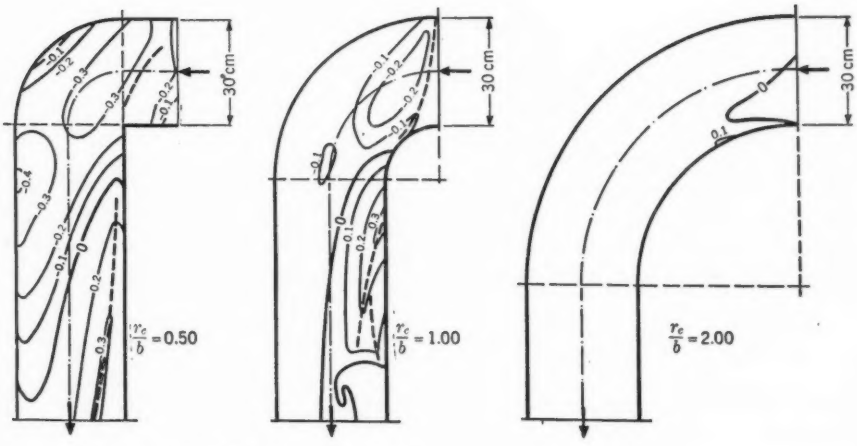
[18] "Hydraulics of Steady Flow in Open Channels," by S. M. Woodward and C. J. Posey, John Wiley & Sons, Inc., New York, N. Y., 1941, p. 112.

[19] "Hydraulic Structures," by A. Schoklitsch, A.S.M.E., 1937, Vol. I, p. 151.



(*a*) $\frac{y_A}{b}$ VARIABLE, WITH $\frac{r_c}{b}$ = 1.00 AND $\frac{\theta}{180°}$ = 1.00



(*b*) $\frac{r_c}{b}$ VARIABLE, WITH $\frac{y_A}{b}$ = 0.833 AND $\frac{\theta}{180°}$ = 0.50

FIG. 15.—EFFECT OF THE BEND ACTION ON THE DEFORMATION OF MOVABLE BEDS (CONTOUR LINES ARE INDICATED BY THE RATIOS OF THE FINAL DEPTH OF DEPOSIT OR SCOUR TO THE ORIGINAL MEAN DEPTH OF FLOW)
(V_A = 40 Cm per Sec (=78.74 Ft per Min)

Energy Lost as the Result of Bend Resistance

The energy lost from the resistance of a bend, h_b, was obtained by subtracting from the total loss, the energy lost in friction in an equivalent length of a straight flume under the same conditions of flow. Such frictional losses were determined experimentally beforehand. Energy losses were considered equal to the fall of the true specific energy line (not the water surface) between Stations A and B, Fig. 6(*a*). This was done because of the difference in the mean velocities at the two stations resulting from the horizontality of the bed.

The distribution of the energy lost along a flume from a bend can be studied from the true energy line, 4, and the friction line, 5, in Fig. 9. Since the main drops in line 4 occur at the regions of the adjustment and readjustment of the flow distribution, the energy loss chargeable to bend resistance may be attributed principally to the change of momentum at these two regions. In other parts of the curved flume, lines 4 and 5 are approximately parallel. This indicates that both the energy loss ascribed to the lateral currents of the spiral motion and the increase in shear stresses at the boundaries (the latter resulting from the increase in the lateral velocity gradient as the thread of maximum velocity approaches the side walls) are relatively small.

To determine the energy resulting from a given bend, let L_{A-B} be the distance between Stations A and B, measured along the center line of the flume; V_{zA} and V_{zB} be the mean forward velocities at Stations A and B; y_A and y_B be the mean depths of flow at Stations A and B; α_A and α_B be the velocity head corrective factors at Stations A and B; and h'_f be the energy lost in friction per unit length in a straight flume under the same conditions of flow.

The energy loss chargeable to the bend will then be given by

$$h_b = \left(y_A + \alpha_A \frac{V^2_{zA}}{2g} \right) - \left(y_B + \alpha_B \frac{V^2_{zB}}{2g} \right) - (L_{A-B}\, h'_f) \quad \ldots\ldots (18)$$

The velocity head corrective factor α for any cross section is given by

$$\alpha = \frac{1}{V^3_z\, A} \int_0^A v^2\, v_z\, dA \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Eq. 19 was solved graphically by the planimeter from the v-contours and the v_z-contours at both Stations A and B for different bends under widely different conditions of flow. In all cases α_A and α_B were each found to be nearly equal to unity, with a maximum value of 1.025. This showed that Stations A and B were not affected by the bend action. Eq. 18, therefore, becomes

$$h_b = (y_A - y_B) - \left(\frac{V^2_{zB} - V^2_{zA}}{2g} \right) - (L_{A-B}\, h'_f) \quad \ldots\ldots\ldots (20)$$

The results of the four parametric functions of the resistance coefficient ξ—in which

$$\xi = h_b \frac{2g}{V^2_{zm}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

—are presented in Fig. 16. For the parameter **R**, the three families of curves

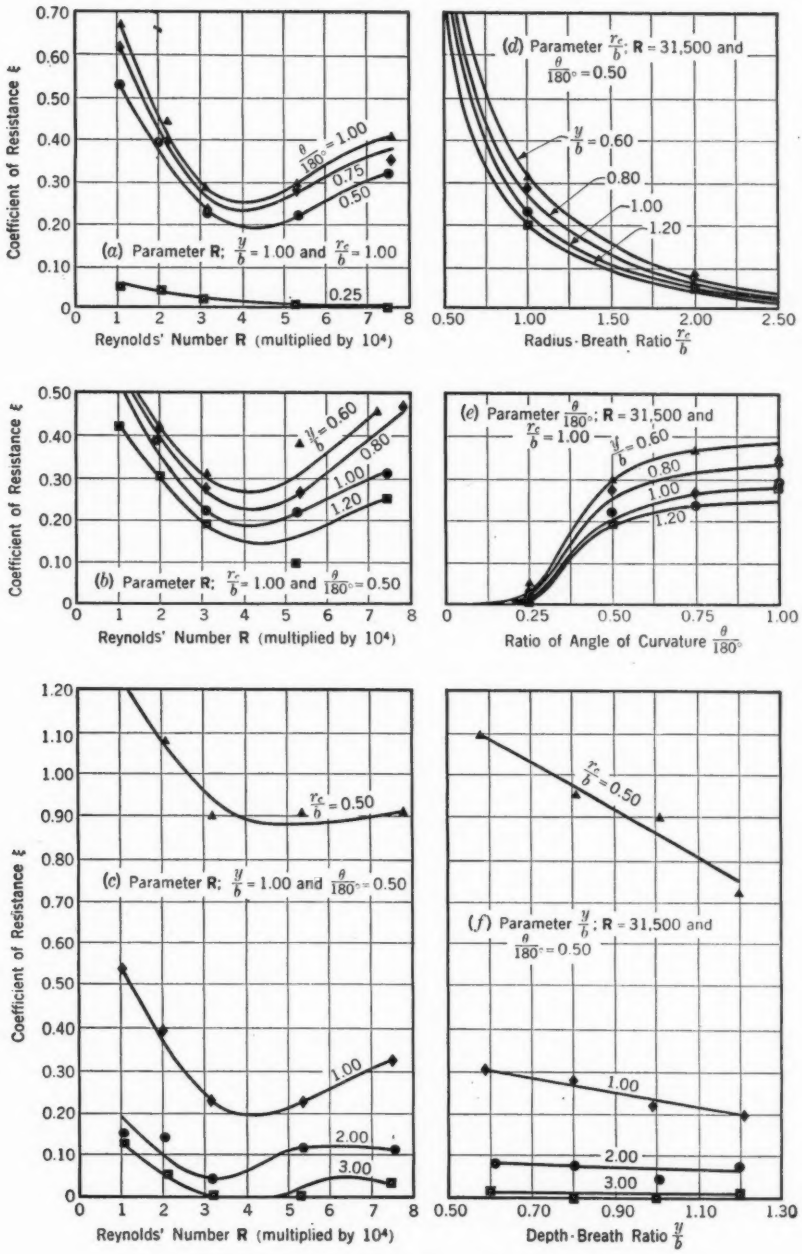

Coefficient of Resistance ξ
(a) Parameter R; y/b = 1.00 and rc/b = 1.00
θ/180° = 1.00
0.75
0.50
0.25
Reynolds' Number R (multiplied by 10⁴)
(d) Parameter rc/b; R = 31,500 and θ/180° = 0.50
y/b = 0.60
0.80
1.00
1.20
Radius-Breath Ratio rc/b
(b) Parameter R; rc/b = 1.00 and θ/180° = 0.50
(e) Parameter θ/180°; R = 31,500 and rc/b = 1.00
Ratio of Angle of Curvature θ/180°
(c) Parameter R; y/b = 1.00 and θ/180° = 0.50
rc/b = 0.50
2.00
3.00
(f) Parameter y/b; R = 31,500 and θ/180° = 0.50
Depth-Breath Ratio y/b

FIG. 16.—EXPERIMENTAL PARAMETRIC FUNCTIONS OF THE COEFFICIENT OF ENERGY RESULTING FROM BEND RESISTANCE (EQ. 19)

are presented in Figs. 16(*a*), 16(*b*), and 16(*c*) to indicate the behavior of ξ when two parameters are kept constant while the third parameter is given different constant values. For each of the parameters r_c/b, $\theta/180°$, and y/b, Figs. 16(*d*), 16(*e*), and 16(*f*), respectively, only one family of curves is given.

The general characteristics of the functions are, as follows:

1. The coefficient ξ decreases as **R** increases until a value of $\mathbf{R} = 3 \times 10^4$ is reached (Figs. 16(*a*), 16(*b*), and 16(*c*). It then shows smaller variations in the range between $\mathbf{R} = 3 \times 10^4$ and the maximum value of **R** which could be reached in the present set of experiments—namely, $\mathbf{R} = 8 \times 10^4$. In this range, the function has a tendency to rise temporarily in a manner similar to that of the drag coefficients of immersed circular cylinders where the temporary rise occurs between $\mathbf{R} = 1{,}800$ and $\mathbf{R} = 32{,}000$.[20]

2. Coefficient ξ decreases as y/b increases (Fig. 16(*b*)). The slopes of the curves, however, become smaller at higher values of r_c/b. At $r_c/b = 3.0$, ξ becomes independent of y/b.

3. Coefficient ξ varies with the parameter $\theta/180°$ in a reversed curve whose inflection point and maximum gradient lie between $\theta = 45°$ and $\theta = 90°$ (Fig. 16(*e*)). In this range, ξ increases approximately from 10% to 80% of its maximum value at $\theta = 180°$.

4. The ξ-curves in Fig. 16(*f*) have very steep downward slopes between $r_c/b = 0.5$ and $r_c/b = 2.0$, ξ falling to about 7% of its maximum value at $r_c/b = 0.5$. Coefficient ξ then continues to diminish until its value becomes negligible at $r_c/b = 3.0$, and the curves become almost parallel to the $\frac{r_c}{b}$-axis. The ratio $r_c/b = 3.0$ is outstandingly significant in all phenomena resulting from the bend action, since, at this ratio, the bend action effects attain practically their minimum values. The curves in Figs. 16(*a*), 16(*b*), and 16(*c*) may be applied in practice, within the foregoing specified ranges, to determine ξ for bends of smooth open channels. For any given case, ξ is obtained by fixing its value with respect to two variables. It is then adjusted with respect to the third and fourth variables, consecutively.

As an example, assume a bend having the following data: $\mathbf{R} = 60{,}000$, $y/b = 0.7$, $\theta/180° = 0.667$ (120° bend), and $r_c/b = 1.50$. Let it be required to find the value of ξ. From the family of curves in Fig. 16(*c*), for $\mathbf{R} = 60{,}000$ and $r_c/b = 1.50$, $\xi'' = 0.190$. This value is for $y/b = 1.00$ and $\theta/180° = 0.500$. From Fig. 16(*b*), for $\mathbf{R} = 60{,}000$, the values of ξ corresponding to ratios $y/b = 0.7$ and $y/b = 1.0$ are 0.34 and 0.26, respectively. Therefore, to correct ξ'' for $y/b = 0.7$, multiply its previously obtained value by 0.34/0.26. Thus, $\xi' = 0.19 \times 0.34/0.26 = 0.249$. Similarly, to correct ξ' for $\theta/180° = 0.667$, multiply it by 0.30/0.26, as obtained from Fig. 16(*a*). The required value of ξ, thus, becomes $0.249 \times 0.30/0.26 = 0.287$.

[20] "Applied Fluid Mechanics," by M. P. O'Brien and G. H. Hickox, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, p. 186.

General Conclusions

(1) Spiral motion exists in straight flumes as well as in curved flumes. This fact is proved definitely by direct measurements taken by the pitot sphere. In a curved flume, the spiral motion induced by the centrifugal force is not uniform along the bend. There is a complicated pattern caused by the interference of the spiral motion (originating in the straight approach flume) with that produced by the bend. The strongest lateral currents appear close to the outer wall at the midsection of the bend, where they have a general direction toward the bed. Their direction and position gradually change through the second half of the bend until the exit is reached, where they approach the inside wall with upward inclination.

(2) The strength of the spiral motion S_{xy} in a bend (defined as the ratio of the kinetic energy of the lateral currents to the total kinetic energy of the flow at a particular section) decreases considerably by increasing either the Reynolds number **R** or the radius-breadth ratio r_c/b. At $r_c/b = 3.0$, S_{xy} attains practically its minimum value. The spiral motion S_{xy} also decreases (but in a lesser degree) as the depth-breadth ratio y/b increases. On the other hand, it increases as the deviation angle θ of the bend becomes greater. From $\theta = 0$ to $\theta = 90°$, the increase in S_{xy} is nearly double that of the range from $\theta = 90°$ to $\theta = 180°$. The kinetic energy of the lateral currents in a bend is relatively small, however, and, consequently, plays a minor part in producing the additional energy loss which is mainly the result of the change of momentum upstream and downstream from the bend.

(3) The coefficient ξ (of the energy lost due to bend resistance) varies considerably with each of the parameters **R**, r_c/b, y/b, and $\theta/180°$. This variation explains the enormous differences in the results if they are not properly correlated. Coefficient ξ varies with the parameter $\theta/180°$ in a reversed curve whose inflection point and maximum gradient lie between 45° and 90°. In this range, ξ increases approximately from 10% to 80% of its maximum value at 180°. Between ratios $r_c/b = 0.5$ and $r_c/b = 2.0$, ξ falls very rapidly. At $r_c/b = 3.0$, ξ attains practically its minimum value. Parametric families of curves are given here for practical use in determining ξ in smooth open bends.

(4) When the channel bed is formed of an erodible material, the bend action causes a stable configuration which diminishes considerably by increasing r_c/b or y/b. The practical minimum size of this configuration seems to be at $r_c/b = 3.0$.

(5) The forward-velocity distribution and the water-surface profile at the section passing through the point of maximum surface depression can be predetermined, with a reasonably good degree of accuracy, by the assumption of a theoretical free-vortex distribution. This assumption holds good as long as the depth of flow does not fall below the critical depth of the streaming state.

(6) For the proper proportioning of bends the ratio $r_c/b = 3.0$ is recommended as it gives the minimum radius at which all the bend action effects are negligible.

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APPENDIX. NOTATION

The following letter symbols adopted for use in this paper, and in its discussion, conform essentially with American Standard Letter Symbols for Hydraulics (ASA–ZI0.2–1942) prepared by a Committee of the American Standards Association, with Society participation and approved by the Association in 1942.

A = area of the water cross section;
b = breadth of a water stream in a rectangular flume; width of the channel bed;
C = a constant; circulation constant in a free-vortex motion;
g = acceleration of gravity;
H = specific energy; elevation of the specific energy line at any section;
h = head, with subscripts generally denoting the point of measurement:
  h_a, h_b, etc. = manometric heads;
  h_b = energy head loss in bend from resistance only;
  h' = calibrated head;
  h'_f = energy head lost by friction per unit length of straight channel;
  Δh = difference in water levels at the outer and inner walls of the flume; superelevation of the water surface;
k = a dynamic coefficient, with subscript such as m denoting point of reference; k_i = inclination factor;
L = length; distance between two sections;
n = roughness coefficient; n_l = a linear dimension of wall and bed roughness;
p = unit pressure:
  p_o = static pressure;
  p_s = pressure when pitot sphere is normal to the axis of the channel;

Q = quantity of flow; rate of discharge;
R = hydraulic radius;
$\mathbf{R}$ = Reynolds' number;
r = radius of bend; radius of curvature at any point:
  r_c = radius at the center of the stream;
  r_i = radius of the inner wall of the flume;
  r_o = radius of the outer wall of the flume;
S = strength of a spiral motion at any cross section, as S_{xy};
U = a coefficient that is a function of the angle of bend (Eq. 16*a*);
V = mean velocity in a given section; V_z = mean forward velocity;
v = velocity vector at any point (Fig. 1(*d*)):
  v_x = velocity component in the lateral direction;
  v_y = velocity component in the vertical direction;
  v_z = velocity component in the forward direction;
  v_{xy} = velocity vector projected on the plane of the cross section;
y = depth of flow; y_m = mean depth of flow;
α = velocity head correction factor;
β = a central vertical angle (Fig. 3) between the direction of the velocity vector and a given manometer, as indicated by subscript;
γ = weight of 1 cu ft of water; specific weight;
δ = angle of inclination of the velocity vector to the longitudinal axis of the channel;
θ = angle of deviation in a bend;
ν = kinematic viscosity;
ξ = bend coefficient of resistance; coefficient of energy loss from bend resistance;
ρ = ratio of energy lost from bend resistance to the total loss in the bends;
ϕ = a function; and
ψ = a horizontal angle of axial rotation (Fig. 1(*d*) between the meridian plane and the plane of the velocity vector.

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PAPERS

LIGHT-WEIGHT PUMICE CONCRETE

By A. E. Niederhoff,[1] M. ASCE

Synopsis

Pumice has been used as an aggregate in concrete to give it light weight, thermal insulating value, and certain other characteristics not usually associated with hard rock concrete. The experience of the United States Navy at Inyokern, Calif., has confirmed the fact that pumice concrete can be mixed and placed like any other concrete except that presaturation of the pumice is required and care must be exercised not to overvibrate the mixture. The strength of pumice concrete is controlled by the volume of cement used per cubic yard. Generally, one additional sack of cement is required per cubic yard to obtain the same strength as hard rock concrete.

On the west coast where pumice deposits are prevalent, the cost of light-weight concrete is no more than the cost of hard rock concrete. Because of its light weight and insulating qualities it has been found feasible and economical to ship pumice as far as 300 miles for use in building construction.

Pumice concrete has a low modulus of elasticity, which influences the design of reinforced members. Considerable saving in materials results when correctly designed structures of pumice concrete are used in earthquake-proof structures. Field tests of buildings made of pumice concrete have verified laboratory findings.

This material weighs about one half as much as conventional concrete, can be designed for strengths up to 3,500 lb per sq in., has low heat conductivity, and is nailable in about the same degree as certain woods. It will not spall under influence of direct flames and has sound absorption values which can be controlled by the type of surface finish that is used. Caution is indicated when using this concrete in structures subject to abrasion or erosion.

Introduction

The use of pumice concrete for building construction in California and Mexico is not new; but, because of its recent acceptance and widespread use by the United States Army, the United States Navy, and private structural

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **November 1, 1949.**

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engineers on important multistory buildings, it is well to review the characteristics of this material. A rare opportunity is afforded by experience on the $6,500,000 Michelson Research Laboratory built for the U. S. Navy at Inyokern. During the construction of the laboratory[2] it was found that coarse aggregate of local pumice and fine aggregate of sand would produce workable concrete having the required strength of 2,800 lb per sq in. and would weigh only 110 lb per cu ft. The thermal qualities of this material as an insulator against desert heat were superior to those of hard rock concrete and there appeared to be a reduction in noise reverberations in rooms whose walls and ceilings were composed of pumice concrete because of the numerous, minute air holes in the surface.

Pumice is an igneous rock, white or gray in color, formed in prehistoric time by the extravasation of water vapor from molten lava. This highly vesicular glass, produced by the inclusion of air bubbles in the magma during consolidation, will float on fresh water. The specific gravity ranges from 1.12 to 1.60 but flotation occurs because the many channels are sealed, preventing the water from entering. From one half to two thirds of the volume of pumice rock is pore space. The hardness ranges from 4.0 to 5.0 on the Moh scale of hardness, making pumice about as hard as fluorspar of feldspar.

Anywhere in the vicinity of volcanoes one can expect to find pumice. Many western states have large deposits but in the midwest and east there are no volcanoes and there is no pumice. Pure, large deposits that are free from clays, volcanic ash, and dust, and have the strength, particle size, shape, and proper strength-weight ratio for concrete aggregates are relatively hard to find. Such deposits occur in the Coso Mountain Range about 30 miles north of Inyokern and about 50 miles from the borderline of Death Valley National Monument in California. These deposits are ideally situated so that daily truck shipments of hundreds of yards are made to Los Angeles, Calif., and its vicinity, each load identical with the preceding load, and each load meeting the standards set up by the purchaser.

The U. S. Navy has specified pumice aggregate for concrete poured in many of the buildings constructed at the Ordnance Testing Station at Inyokern. The requirements are that: Pumice must have a minimum hardness of 4.0 on the Moh scale; it must have a specific gravity of 1.17 or less; it must be free of clay or other deleterious matter; it must not disintegrate in water; and it must have the following gradation (percentage passing, by weight):

Sieve size	Fines	Coarse
1 in.	100	95–100
¾ in.	100	50–85
⅜ in.	95–100	20–50
No. 4	90–95	5–10
No. 8	70–80	0–7
No. 16	50–60	....
No. 28	35–45	....
No. 50	25–35	....
No. 100	15–25	....
No. 200	5–15	....

[2] "Naval Testing Lab in Desert," by A. E. Niederhoff, *Western Construction News*, September, 1946, p. 90.

Materials for concrete houses have been taken from pits at Bishop, Calif. Tests on combined coarse and fine aggregate showed a specific gravity of 1.62 and a weight, as it came from the mine, of 53.2 lb per cu ft. The dry, loose weight per cubic foot was 8 lb less. Chemical analysis showed the following constituents (in percentages):

Silica (SiO_2)	70.22
Aluminum oxide (Al_2O_3)	22.56
Iron oxide (Fe_2O_3)	0.48
Titanium oxide (TiO_2)	trace
Calcium oxide (CaO)	0.54
Magnesium oxide (MgO)	0.24
Sodium oxide (Na_2O)	2.70
Potassium oxide (K_2O)	0.11
Loss on ignition	3.10
Total	99.95

Sieve analysis on this mine run pumice showed a large percentage of fines:

Sieve size	Percentage passing
¾ in.	100
⅜ in.	88.8
No. 4	67.1
No. 8	53.3
No. 14	46.4
No. 28	40.5
No. 50	32.7
No. 100	19.0
No. 200	11.3

Proportioning Pumice Concrete

Using pumice aggregates or pumice coarse aggregate and clean sand, it is possible to make light-weight structural concrete having 28-day crushing strengths ranging from 1,000 lb per sq in. to 3,500 lb per sq in. by varying the proportion of cement added to the mix. As an illustration, the mixtures shown in Table 1 were used for three characteristic strengths of concrete. The weights are based on saturated and surface dry materials. Pumice coarse aggregate is usually sprayed with water for 48 hours prior to mixing or until it has absorbed 33% of its dry weight of water.

Fig. 1 shows the age-strength relationship of several pumice concrete mixes including the three shown in Table 1. Tests were made at the U. S. Naval Testing Laboratory in Inyokern. In common with hard rock concrete the ultimate strength of pumice concrete increases with age. Test results of 1-year-old cylinders are not available; but, judging from the difficulty in driving nails in old pumice concrete walls as compared with the ease with which this same operation can be performed at the age of 28 days, it is assumed that the gain in strength is equal to that of conventional concrete.

Table 2 gives recommended mixes for pumice concrete based on desired strength. It is assumed that the pumice conforms generally to the standards adopted at the U. S. Naval Ordnance Test Station at Inyokern.

Experience with "Pozzolith" as an admixture has shown that, although it increases the workability of a harsh mix, it does not have much effect on the ultimate strength. If the Pozzolith is added in a dry form, it is difficult to control, and it has been known to retard the setting of concrete to such an extent

TABLE 1.—TYPICAL MIXES OF PUMICE CONCRETE FOR CHARACTERISTIC STRENGTHS

(Quantities Are in Pounds per Cubic Yard of Concrete)

Material	Specific gravity	Strengths (Pounds per Square Inch)		
		1,700	2,800	3,200
Pumice fines	1.57	868	....	532
Sand	2.60	....	1,471	482
Pumice coarse	1.12	670	682	690
Portland cement	3.1	564	658	658
Admixture (Pozzolith)	....	....	4.2	4.2
Water (in gallons)	....	(42)	(36)	(44)

that formwork could not safely be removed even 7 days after pouring. If the Pozzolith is added to a solution of water, a much greater dispersion is obtained throughout the mix; and, if the addition of the Pozzolith is accompanied by a reduction in the total water, thereby decreasing the water-cement ratio, the strength of 28-day-old cylinders will be increased about 20%. However, the same workability and increased strength can be obtained for the same cost by increasing the cement content and presaturating the coarse aggregate.

TABLE 2.—SPECIFICATION FOR PUMICE CONCRETE CONSTRUCTION

Weight (pounds per cubic foot)	Estimated 28-day compressive strength (pounds per square inch)	Cement factor (sacks per cubic yard[a])	Maximum water (gallons per sack of cement[b])	Approximate Weights of Saturated Surface Dry Aggregates (Pounds per Cubic Yard)		
				Sand	Pumice fine aggregate	Pumice coarse aggregate[c]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
75	1,500	5.5	6.0	....	898	697
83	2,000	6.5	6.0	....	875	650
92	2,500	7.0	7.0	463	512	663
110	3,000	7.2	5¼	1,471	....	682
100	3,500	8.0	6¼	482	532	690

[a] Number of 94-lb sacks of cement per cubic yard of concrete freshly mixed. [b] Maximum gallons of water per 94-lb sack of cement. [c] Saturated to 30% moisture content.

A commercial laboratory (Smith-Emery Company) made a series of tests on pumice concrete with and without an admixture. Trial mixes were first studied to decide the proportion of water required to produce a 4¼-in. slump. The same total volume of mixing water was then used for samples with and without admixtures, thus keeping the yield and the cement factor the same for

each design. In the field, the same mix design is used in many comparative compression tests, without the use of an admixture. An admixture that reduces the total water to produce the same slump reduces the yield per sack of cement or increases the cement content per cubic yard, thereby showing a higher percentage increase in compressive strength with the use of admixture. The laboratory control of the design avoided this error.

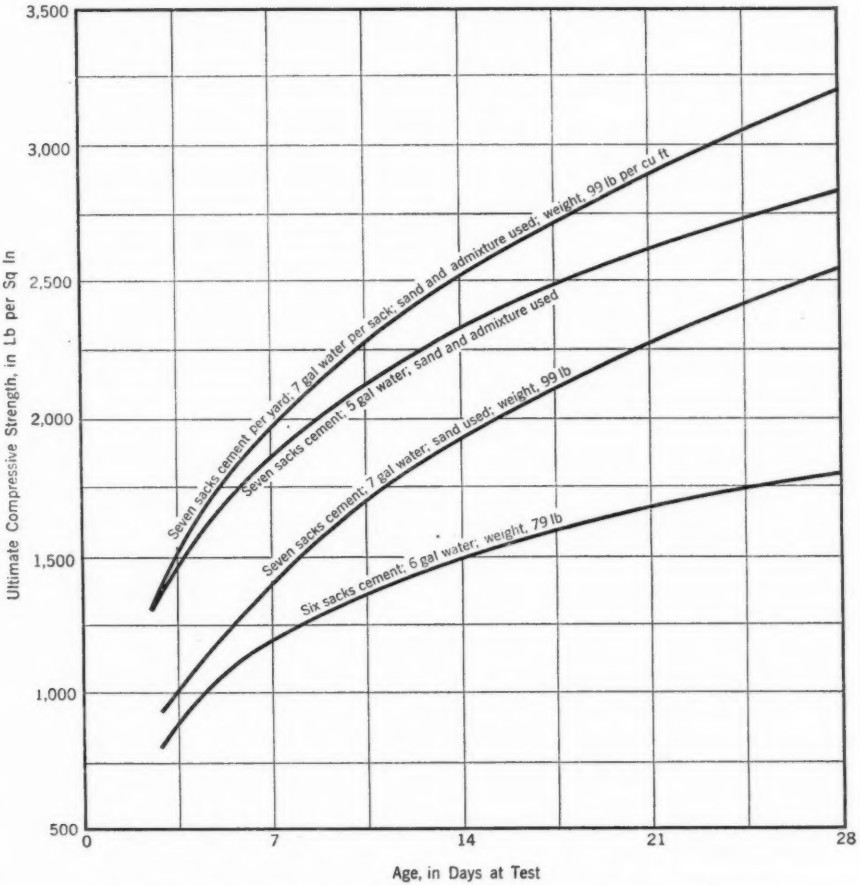
Ultimate Compressive Strength, in Lb per Sq In
3,500
3,000
2,500
2,000
1,500
1,000
500
0
7
14
21
28
Age, in Days at Test
Seven sacks cement per yard; 7 gal water per sack; sand and admixture used; weight, 99 lb per cu ft
Seven sacks cement; 5 gal water; sand and admixture used
Seven sacks cement; 7 gal water; sand used; weight, 99 lb
Six sacks cement; 6 gal water; weight, 79 lb

FIG. 1.—AGE-STRENGTH RELATIONSHIP FOR SOME PUMICE CONCRETE MIXES

In making the test specimens the natural sand and pumice aggregate were saturated fully and correction was made for free water. Thus, all mix designs are based on saturated surface dry material, which experience has shown to be the only proper procedure for using light-weight aggregate without trouble during placing. All materials were weighed and all batches were one-sack batches, mixed in a power mixer for 4 min.

To accelerate the initial set of pumice concrete or to prevent freezing before concrete has cured sufficiently, some contractors have successfully used a small

percentage of calcium chloride in the mix. In a mix for a 28-day specified strength of 1,500 lb per sq in., contractors have used 9½ lb of calcium chloride per yard of concrete. This concrete was poured in the wintertime when desert temperatures go below freezing nearly every night. The addition of calcium chloride had no appreciable effect on the final strength or on the durability of the concrete.

Mixing and Placing

There are some peculiarities about mixing and placing pumice concrete which were learned through experience in placing thousands of cubic yards of various consistencies and strengths of the material. Best results are obtained when the pumice is soaked with water for 48 hours before use—or until it absorbs water up to 30% of its dry weight. It was also found advantageous to increase the mixing time to 4 min, particularly if the coarse aggregate had not been presaturated adequately. Mixing in a conventional, revolving drum-type mixer is not as satisfactory as in a rotating paddle-type mixer. The latter machine produces a homogeneous concrete in less time and causes less wear and breakage of the coarse aggregate than does the conventional mixer.

External vibrating of pumice concrete is necessary to force the material to flow under openings in wall forms. On flat slabs, particularly in pre-cast work, external vibration is desirable to produce consolidation, but care must be exercised not to overvibrate. In pouring concrete houses on the same project[3] both internal vibration and external vibration were required. More internal vibrators were needed for wall concrete placed below the window sills, but above these openings external vibration (and very little of it) was used exclusively. Concrete was placed and vibrated with discretion, and in a certain sequence, painfully learned through experience, to prevent aggregate from floating to the top and to eliminate honeycomb and voids in the pour.

Curing

Pumice concrete should be cured in the same manner as hard rock concrete. The coal-tar pitch cutback sealing process for curing has been widely used on the desert where water is scarce[4] or where surrounding work would be damaged by ponding or spraying. On some pre-cast work[2] initial curing was accomplished in the forms by covering with wet burlap bags. After 12 hours the bags were removed from the forms and the concrete was sprayed with water. In pre-casting slabs for concrete houses, a contractor in California cured the slabs in a steam tunnel for from 6 hours to 8 hours and obtained an equivalent of the usual 7-day strength of the concrete.[5] A combination of a high temperature of 170° F and a saturated atmosphere accelerated the hardening process. Even after removal from the steam tunnels the slabs continued to gain strength beyond that of unsteamed concrete.

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[3] "Machine-Built Desert Houses," by A. E. Niederhoff, *Western Construction News*, September, 1947, p. 89.

[4] "The Metropolitan Water District of Southern California, History and First Annual Report," Los Angeles, Calif., 1939, p. 180.

[5] "Prefabricated Pumice Concrete Houses," by H. L. Mathews, *Journal*, A. C. I., May, 1948, p. 797.

Cost of Pumice Concrete

The cost of pumice concrete is dependent on the distance between the deposits and the point of ultimate use of the aggregate. For the favorable situation that exists at the Naval Ordnance Test Station at Inyokern, the comparison between hard rock concrete and pumice concrete of equal strength shows a small saving in cost of materials in favor of the light-weight aggregate.

Based on 1946 job prices of aggregate delivered to one contractor's mixer at Inyokern, for light-weight and conventional concrete having 28-day strengths of 2,800 lb per sq in., costs were as shown in Table 3(*a*). Because of the extra

TABLE 3.—Comparison of Costs, Pumice Concrete and Hard Rock Concrete

Material	Pumice Concrete			Hard Rock Concrete		
	Quantity	Unit price	Sum	Quantity	Unit price	Sum
(*a*) Light-weight and Conventional Concrete; Inyokern, Calif.; 28-Day Strength, 2,800 Pounds per Square Inch; 1946 Job Prices						
Sand, in cubic feet	9.039	$0.104[a]	$0.94	8.7	$0.104[a]	$0.905
Rock, in tons	....	....	....	0.965	3.31[b]	3.195
Portland cement, in sacks	7	0.80	5.60	5.2	0.80	4.16
Pumice, in cubic yards	0.352	3.59[c]	1.30	....	....	....
Admixture, in pounds	4.2	0.081[d]	0.34	....	....	....
Total, per cubic yard	....	....	$8.18[e]	....	....	$8.26[e]
(*b*) Prefabricated Concrete House; Specified Crushing Strength; 1,500 Pounds per Square Inch; 1947 Prices						
Pumice fines, in cubic yards	0.87	$3.00	$2.61	....	....	....
Pumice coarse, in cubic yards	0.574	3.00	1.72	....	....	....
Portland cement, in sacks	6	0.80	4.80	4.2	$0.80	$3.36
Sand, in cubic feet	....	....	....	14.5	0.12	1.73
Rock, in tons	....	....	....	0.97	3.75	3.64
Total, per cubic yard	....	....	$9.13[e]	....	....	$8.73[e]

[a] $\frac{\$2.75}{yd^3} + 2.5\%$ California state sales tax $= \frac{\$2.81}{27\ ft^3} = \$0.104$ per cu ft. [b] Coarse hard aggregate, classified as $\frac{3}{4}$-in. rock, cost $0.81 per ton f.o.b. (free on board) Los Angeles, Calif., plus $1.94 transportation plus $0.06 federal tax on rail hauls, plus $0.50 per ton for stock piling at the batcher. [c] Pumice aggregate cost $3.50 per cu yd, plus 2.5% sales tax delivered at Inyokern, or $3.59 per cu yd. [d] Pozzolith admixture, bought in quantities, cost $0.081 per lb. [e] Total cost of dry ingredients at the mixer.

care in presaturating pumice, the extra mixing time, and the pains with which the batch must be vibrated, the unit cost, in the forms, probably amounts to 10% more than that of hard rock concrete of the same strength. On a prefabricated concrete house job made with pumice aggregate having a specified crushing strength of 1,500 lb per sq in., the cost in dollars per cubic yard of dry materials at the batcher, in 1947, is given in Table 3(*b*).

Merely comparing unit costs for conventional and for pumice concrete does not give a true picture of the relative economy of the two materials when made into structural slabs, beams, and girders. Comparisons have been made on the sizes of slabs and beams to support a given live load. Because of the smaller

dead weight of pumice concrete, and the lower modulus of elasticity, it was cheaper to build the ten-story building in Los Angeles for the Prudential Insurance Company of this material. Foundation conditions and cost dictated the use of pumice concrete on the new telephone building also in Los Angeles. The General Petroleum Building, a ten-story structure in Los Angeles, showed appreciable savings in reinforcing and structural steel used in and under pumice concrete floors. The sizes of the foundations were reduced because the dead weight of pumice concrete is only two thirds that of conventional concrete.

It has been shown elsewhere[6] that a pumice concrete beam can carry a greater live load than can a hard rock concrete beam designed for the same bending moment because of the smaller dead load. Additional advantages of the pumice concrete beam were: Increased headroom because the beam was shallower, and a saving of 21% in concrete.

In the design of pumice concrete slabs there is also a possibility of saving materials.[7] For the same bending moment and the same allowable unit stresses the pumice concrete slab requires an effective depth to steel only 80% of that which was necessary for hard rock concrete. Slightly more steel was necessary for the shallower slab, but the live-load carrying capacity was greater because the dead weight of pumice concrete was only 75% of the dead weight of conventional concrete. Considering the prevailing market price of materials in Los Angeles, multistory buildings with relatively poor foundation material can be built of pumice concrete for less than can buildings of conventional concrete.

Thermal Qualities

The thermal qualities of pumice concrete have been adequately proved on the Mojave Desert in California. On this subject Capt. H. L. Mathews has written:[8]

> "Because of extremes in desert temperatures and the necessity for air cooling in all buildings during 8 months of the year, the thermal characteristics of the roof and walls of the houses are of utmost importance. Operational expense in keeping a poorly insulated building cool in summer and warm in winter could be prohibitive and would continue unabated during the life of the building. Accordingly, it was specified that the conductivity or *k* factor for concrete should not be greater than 1.95 BTU [British thermal units] per sq ft per in. thickness per hour per degree. This is one-sixth that of hard rock concrete, four-tenths the conductivity of cinder concrete, and half that of Haydite concrete. Reference is made to Fig. 2[9] which gives the insulating values of various building materials. Similarly the thermal transmission coefficient or *U* factor for the 5-in. thick pumice concrete wall was specified as not greater than 0.30. * * *"

The Pittsburgh (Pa.) Testing Laboratory made a thermal conductivity test to determine the *k*-value for a sample of 2,500-lb concrete, with the following results:

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[6] "Chart for Pumice-Concrete Beams," by A. E. Niederhoff, *Engineering News-Record*, September 18, 1947, p. 89.

[7] "Nomograph Chart Aids Pumice-Concrete Slab Design," by A. E. Niederhoff, *Civil Engineering*, October, 1947, p. 47.

[8] "Prefabricated Pumice Concrete Houses," by H. L. Mathews, *Journal*, A. C. I., May, 1948, p. 800.

[9] *Ibid.*, p. 801.

Thickness tested, average in inches	2.049
Density as tested, in pounds per cubic foot	93
Hot plate temperature, in degrees Fahrenheit	71.3
Cold plate temperature, in degrees Fahrenheit	29.5
Mean temperature, in degrees Fahrenheit	50.4
k-value	2.69

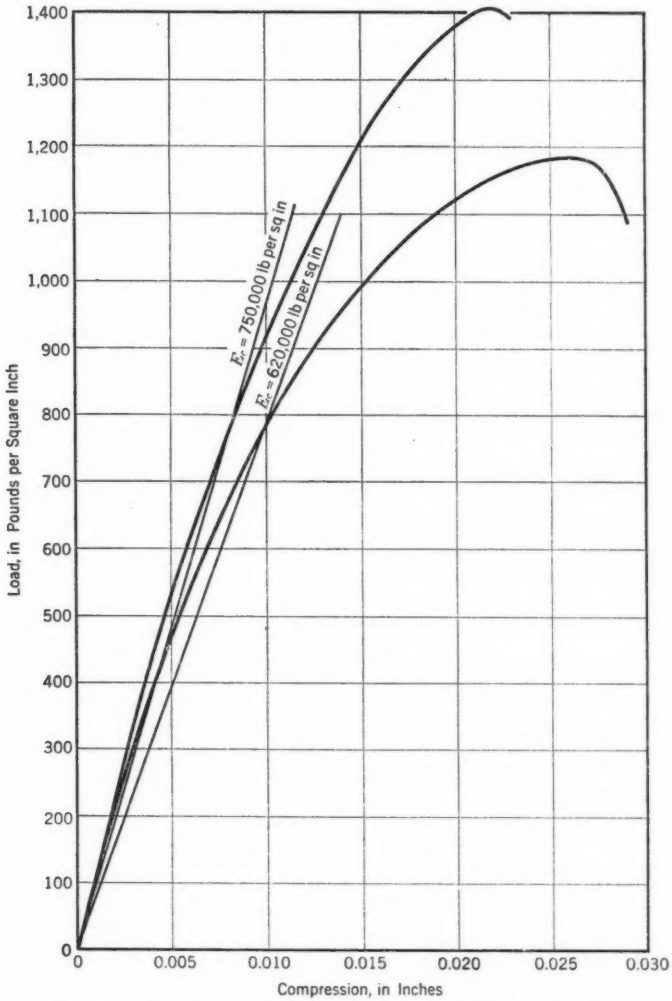

FIG. 2.—STRESS-STRAIN CURVES FOR PUMICE CONCRETE CYLINDERS, 6 IN. BY 12 IN., 28 DAYS OLD

The thermal conductivity k is expressed in British thermal units per hour, per square foot, per 1-in. thickness, per 1° F temperature.

Light-weight concrete having much less strength, and made of one part of cement to eight parts of pumice, had a k-value of 1.61. Apparently the more

pumice used and the less sand and cement, the better the k-value. A k-value of 1.92 has frequently been specified in connection with 2,000-lb concrete.

Modulus of Elasticity

The stress-strain diagrams obtained from measurements taken on thirty-two cylinders, 6 in. by 12 in., of pumice concrete were curves throughout their length (see Fig. 2), indicating greater increase in rate of strain for increased stress. Tests were made at the U. S. Naval Laboratory at Inyokern during the first quarter of 1948. Loadings were applied relatively fast to avoid plastic flow and to eliminate permanent set; but, since pumice concrete is no more elastic than conventional concrete, there was considerable variation in the observed modulus.

TABLE 4.—Modulus of Elasticity (Secant), in Pounds per Square Inch, to Be Used in Design

Type of concrete	Range of Ultimate Compressive Strengths (Pounds per Square Inch)				
	1,000 to 1,499	1,500 to 1,999	2,000 to 2,499	2,500 to 2,999	3,000 to 3,499
Conventional	....	....	2,000,000	2,500,000	3,000,000
Pumice	750,000	830,000	1,000,000	1,250,000	1,500,000

The secant modulus of elasticity was used and the slope of a straight line was obtained drawn from the origin through a point on the curve at a compressive stress of 800 lb per sq in. It was generally observed that the stronger the pumice concrete the higher would be the modulus. Also, it was found that the

TABLE 5.—Summary of Data

Test No.	Sacks of cement per cubic yard	Water-cement ratio; $\frac{w}{c}$ (gallons per sack)	Admixture	Slump (inches)	Workability	Segregation	Weight (Pounds per Cubic Foot)		Modulus of elasticity (pounds per square inch)
							As tested	Dry	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	7.0	7.0	None	4.25	Good	None	99.5	91.75	1,296,440
2	7.0	6.25	Pozzolith	4.25	Excellent	None	99.0	99.4	1,529,799

modulus of elasticity increases with age. For purposes of comparison, the recommendations of the 1940 Joint Committee on Reinforced Concrete Design[10] are included together with the findings of the Smith-Emery and U. S. Navy laboratories on the moduli of elasticity of conventional and pumice con-

[10] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, ASCE, June, 1940, Pt. 2.

crete. The values in Table 4 are based on a pumice aggregate having a hardness of from 4.0 to 5.0 on the Moh scale and a specific gravity ranging from 1.12 to 1.57. For harder or more dense pumice aggregate used in concrete it is expected that the modulus of elasticity would increase.

Ordinary tables and design charts for the design of reinforced concrete are not usable for design in pumice concrete. If the modulus of elasticity of reinforcing steel is 30,000,000 lb per sq in., then the ratio of $\frac{E_c}{E_s}$ (called n in reinforced concrete design parlance) ranges from 40 for 1,000-lb pumice concrete to 20 for 3,000-lb pumice concrete. Table 4 indicates n-values twice those of conventional concrete of equivalent ultimate strength.

Other Test Results

The Smith-Emery Company made many tests on pumice concrete, some of which are of interest to the builder and others which are primarily of interest to pumice concrete designers. The test summary given in Table 5 is based on several specimens made with sand, pumice fines, and pumice coarse aggregate. Test 2 had concrete of the same mixture and consistency as test 1 except that Pozzolith was added and other ingredients were varied to obtain a 4.25-in. slump.

Enlightening field tests on pumice concrete structures have been made from time to time. It is well known that a fire in a conventional concrete building will cause aggregate to pop, and then the concrete will spall and crack. Extensive spalling may expose reinforcing steel, which, when damaged by fire, makes the entire structure unfit for occupancy. Pumice concrete, on the other hand, will not spall. It can be heated to fusion and will not crack or explode. Early

ON TESTS OF PUMICE CONCRETE

Pounds per Square Inch									
Modulus of rupture	Ultimate Compressive Strength (Average, Three Cylinders)		Bond Strengths (Pull Out)			Shrinkage (total inches per inch)	Absorption (percentages)	Bleeding (pounds of water per cubic yard of concrete)	Test No.
			Vertical	Horizontal					
	7 days	28 days		Bottom	Top				
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	
418.9	1,345	2,565	403.9	418.7	297.0	0.00061	11.1	2.87	1
425.2	1,953	3,278	461.6	537.8	350.9	0.00060	7.10	1.21	2

in 1940 a pumice concrete building was soaked with gasoline and waste, ignited, and allowed to burn. When the fire had burned out, the building was inspected and the only evidence of the fire was smoke stain on the walls and ceiling. The experiment was repeated except that after the fire was well started the second time it was put out by water from fire hoses. Inspection of the building again showed no structural damage.

In rainy climates where buildings have pumice concrete exterior walls and roofs, they can be waterproofed by ordinary methods with any one of a dozen or more commercial products on the market. Smooth exterior walls have been pleasingly waterproofed by three coats of colored, unthinned paint consisting of vinyl resins, pigments, and solvents. Pumice concrete roof decks coated with fiberized asphalt emulsions are inexpensive and effectively waterproofed.

The capability of pumice concrete members to accept internal strain without structural damage is demonstrated every time carpenters drive nails into pumice concrete. It is common practice to scab 2-in. by 4-in. pieces of lumber onto pumice concrete walls by nailing with 16-penny nails. Many concrete buildings have the baseboard and moldings nailed on with finishing nails. At least one motel, recently constructed with pre-cast pumice concrete roof slabs 3 in. thick, had the roof nailed through to the supporting rafters below. By actual test with pumice concrete slabs 1.5 in. thick it was found that 16-penny nails could be driven through them into wood supports below without cracking the unreinforced concrete. Nails had to be kept at least 6 in. from the edge of the thin slab.

The holding power of nails in pumice concrete is almost as good as that of nails driven in wood. In fastening pipes or air ducts by hangers to a ceiling or wall there is a large saving in time by merely nailing the hangers in place.

Conclusions

Laboratory and field tests and job experience involving thousands of cubic yards of reinforced pumice concrete warrant the following conclusions with respect to this material:

1. Pumice concrete varies in weight from 75 lb to 110 lb per cu ft, dependent, to some extent, on the specified strength. It is about one half to two thirds the weight of sand and gravel concrete. The light weight reduces the dead weight of structures and there is a marked saving in reinforcing steel Extra stories can be added to existing buildings where footings are designed for specified dead loads. During construction, the formwork for light-weight concrete is not required to be as strong as that used for conventional concrete, resulting in a lower initial cost for the building. Where earthquake considerations are vital, the low dead weight of the structure and, consequently, the reduced horizontal forces to be taken care of in design, are exceedingly advantageous. In placing pre-cast units by hand, the light-weight pumice concrete saves time, energy, equipment, and money.

2. Strength of pumice concrete can be varied to suit design conditions within practical limits of from 1,000 lb per sq in. to 3,500 lb per sq in. Many inexpensive mixes have been designed to specified strengths and many data are available to aid the engineer in a predetermination of pumice concrete mix and cost.

3. Pumice concrete has a low heat conductivity that puts it in the same category with some of the best insulating materials. This property is inherent and, when combined with light weight and good structural strength, makes wall construction with pumice concrete less expensive than that with other materials that must have applied insulation. Because of the poor heat conductivity of

pumice concrete there is little condensation, allowing large savings in building construction because furred walls are not necessary. Air conditioning of pumice concrete buildings is facilitated as proved in houses constructed on the Mojave Desert. For walk-in refrigerators, pumice concrete floors, walls, and roofs have been specified without further applied insulation, thus lessening the initial cost of construction

4. Nails can be driven into pumice concrete so that they have a holding power almost equal to that of wood. This fact combined with experience in sawing thin slabs of lean pumice concrete presents definite advantages over ordinary concrete construction. The low modulus of elasticity allows this internal strain without catastrophic structural damage. In concrete masonry and pre-cast units the low modulus of elasticity decreases breakage during production, transportation, stock piling, and final placement. Expansion and contraction, vibration, concussion, and earthquake strains are ameliorated by this relatively inelastic material.

5. Tests have shown that pumice concrete can be heated to fusion (about 2,400° F) without spalling or cracking (see Fig. 3). Sudden quenching of hot concrete in cold water produces no cracks and the structural integrity of pumice concrete buildings remains practically unchanged even after a 4-hour fire (see Fig. 4). Welding steel reinforcement together right on top of a pumice concrete slab has not spalled the concrete. In fact, pumice concrete has been used successfully as a backup for boiler and firebox installations.

6. The quality of the mix and surface texture of pumice concrete determines the absorption of sound waves. In this respect it differs from conventional concrete building construction where other methods are necessary to reduce noise reverberations. A lean pumice concrete with a "popcorn" finish will absorb up to 50% of sound. This advantage is useful not only in small homes, but in auditoriums, theaters, and factories, where sound is controlled only by expensive processes for conventional concrete structures. Values of sound transmission are decreased from eight decibels to twelve dicebels.

7. All the equipment used for mixing, hauling, and placing light-weight pumice concrete is less heavily loaded than when the same operations are performed with sand and gravel concrete. Monetary savings are immediately reflected in more efficient labor, less time involved in doing a job, and less wear and tear.

8. Pumice concrete offers a wide selection of textures or finishes ranging from a rough "popcorn" to a smooth "sack" finish. The labor involved in obtaining any particular finish costs no more than does that required for conventional concrete finishing.

9. Except for presaturation of coarse aggregate, longer mixing time, and discretion in the use of a vibrator when placing, pumice concrete is handled the same as conventional concrete.

10. Wearing surfaces should not be made of pumice concrete. Light-weight concrete floor slabs in multistory office buildings are usually covered with asphalt tile or other wearing surfaces. In this case full advantage can be taken of the light-weight material.

FIG. 3.—IN A FIRE TEST, THE TRANSMISSION OF HEAT WAS RETARDED: NO SPALLING OF PUMICE CONCRETE

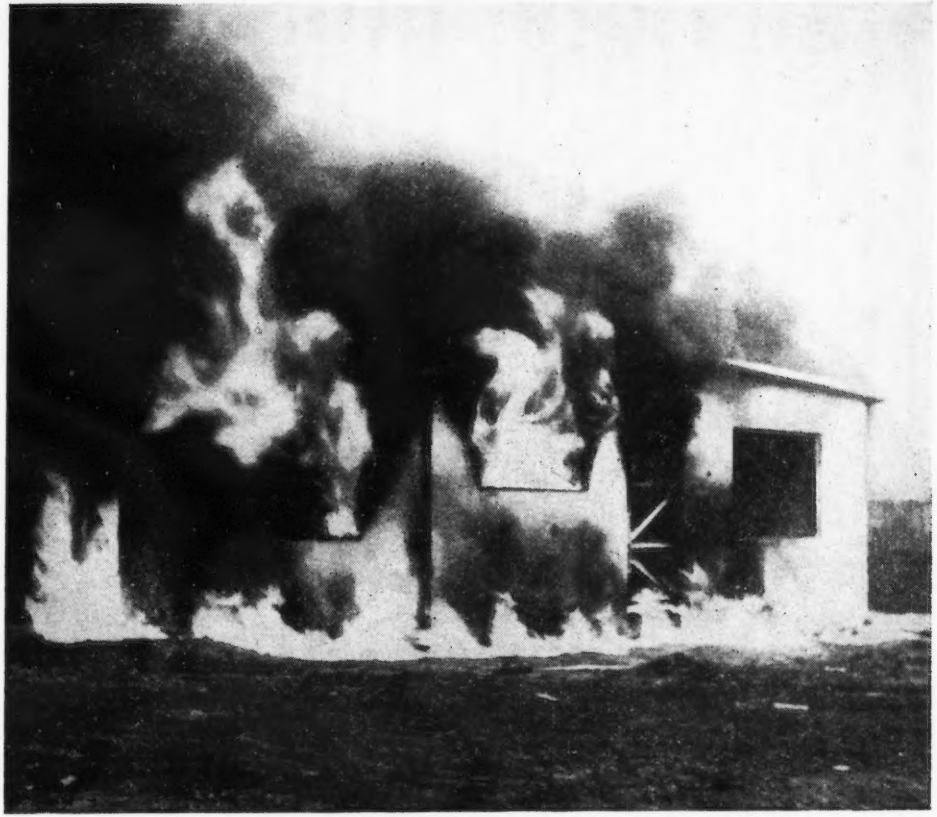

FIG. 4.—THERE WAS NO DAMAGE IN THIS PUMICE PREFABRICATED HOUSE, AFTER FIRE WAS QUENCHED BY WATER

Acknowledgments

Grateful acknowledgment is given to the Bureau of Yards and Docks, United States Navy, and in particular to Captain Mathews, officer in charge of construction at the U. S. Naval Ordnance Test Station at Inyokern, for the privilege of collecting data and preparing this paper. C. S. McKenzie, design superintendent at Inyokern and the writer's immediate superior, lent encouragement and assistance on many of the phases of design and construction.

Paul R. Splane contributed photographs, test data, and many job records available to him as a former pumice producer in Los Angeles.

Many others have assisted indirectly or directly and to these unnamed persons the writer offers grateful thanks.

After Fire Was Quenched by Water

Was Retarded: No Spalling of Pumice Concrete

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

APPLIED COLUMN THEORY

By F. R. Shanley [1]

Synopsis

The generalized Euler equation (involving the tangent modulus) is used as a basis for column strength analysis. The influence of the shape of the stress-strain diagram is shown by nondimensional column curves and an envelope formula is derived. The effects of initial eccentricity are investigated by interaction curves for combined bending and compression. Correction of test data, end fixity effects, and optimum design of columns are discussed briefly.

1. The Generalized Euler Theory

The failure of members under compressive loading has long been a subject of great interest to the structural engineer. In aircraft structures the phenomenon of instability is the basis for many strength calculations, because nearly all the structure is critical under some design condition that causes compressive loads. The demand for minimum structural weight has had two important effects—it has forced the aircraft designer to use special alloys and it has raised the level of design stresses well beyond the proportional limit, into the inelastic range. This combination of circumstances has made it necessary to abandon many of the formulas and methods that were satisfactory for heavier types of construction.

The search for more rational methods leads the investigator backward, chronologically, to the year 1757, when Leonhard Euler published his famous treatise on the strength of columns. The following quotation is taken from a translation[2] of the Euler paper by J. A. Van den Broek, M. ASCE:

> "To begin with I should indicate that this moment is not limited to elastic bodies, among which we have good reason to doubt that columns could be included. It concerns in essence a force by which any body resists a change in curvature, and it is totally immaterial whether such a body after flexure is endowed with a force to re-establish its original shape or not. For

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **November 1, 1949.**

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[2] "On the Strength of Columns," by Leonhard Euler, translation by J. A. Van den Broek, *American Journal of Physics*, July–August, 1947, p. 315.

this reason we might preferably designate this moment as stiffness moment (*moment de roideur*) because it occurs in all bodies that resist flexure, whether they are elastic or not."

The "moment" to which Euler referred was designated by him as $E\,k\,k$, the product of the modulus of elasticity and the square of the shape parameter. Today the product of E and the moment of inertia I replaces $E\,k\,k$ in the Euler formula.

The foregoing quotation shows that Euler did not intend to restrict his formula to the elastic range. Because Young's modulus and the tangent modulus were unknown in Euler's time, he could not have been more specific; but he left the door wide open for a general interpretation of the term E.

Since E is usually considered to be a constant (up to a certain stress), Euler's formula is restricted to the elastic range. In 1889, F. Engesser published a paper[3] in which he suggested that E be considered as the slope of the compressive stress-strain diagram, commonly called the "tangent modulus." (In a letter to the writer, Theodor von Kármán, M. ASCE, noted that he had independently made a similar suggestion in an early paper written in Hungarian). Several investigators later pointed out that this theory would not account for the fact that, when the strains are decreasing on one side of the column, during bending, the stresses will decrease at a rate given by the elastic modulus, E. In 1909, Mr. von Kármán[4] made a thorough analysis of column action and performed some very careful experiments, by which the double-modulus (or reduced modulus) theory was apparently substantiated.

The writer has shown[5,6] that the tangent-modulus theory is actually correct for predicting the load at which buckling of a perfect column will begin and that the double-modulus theory gives the upper limit for the load as the bending increases. Mr. von Kármán's own interpretation is clearly indicated by the following excerpts from his discussion of one of the writer's papers:[7]

> "* * * the first equilibrium bifurcation from the straight equilibrium configuration occurs at a load given by the Euler formula when the Young's modulus is replaced by the tangent modulus. * * * The definition of the stability limit must be revised for nonreversible processes. * * * Although the Euler formula with the tangent modulus does not, in general, give the maximum axial load to which the column can be subjected without large deflection, it is conservative and therefore advisable to use this formula for practical computation of column loads. As I have shown in my paper of 1909, also the load deflection curve that starts from the upper limiting load in general soon assumes a negative slope. Consequently, it is difficult to determine the actual peak of the axial load. It will certainly be between the two values that correspond to the tangent and the reduced moduli. These two values can be respectively designated as the lower and upper limits of the critical load."

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[3] *Zeitschrift fur Architektur und Ingenieurwesen*, 1889.

[4] "Untersuchungen über Knickfestigkeit," by Theodor von Kármán, *Forschungsarbeiten No. 81*, Berlin, 1910.

[5] "The Column Paradox," by F. R. Shanley, *Journal of the Aeronautical Sciences*, December, 1946, p. 678.

[6] "Inelastic Column Theory," by F. R. Shanley, *ibid.*, May, 1947, p. 261.

[7] *Journal of the Aeronautical Sciences*, May, 1947, p. 267.

The generalized Euler formula (which should perhaps be designated the Euler-Engesser formula) may now be written as

$$P = \frac{\pi^2 E_t I}{L^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which P is an axial load on the column; E_t is the tangent modulus; and L is the length for a column with pinned ends. (The effects of end restraint are discussed in Section 8.)

To obtain the average axial stress f corresponding to the load P, Eq. 1 is divided by the cross-sectional area:

$$f = \frac{P}{A} = \frac{\pi^2 E_t I}{L^2 A} = \frac{\pi^2 E_t}{(L/\rho)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which $\rho^2 = I/A$ defines the least radius of gyration.

The following statement may be used as a basis for further discussion: The generalized Euler equation gives the load at which a perfect column will begin to bend.

The word "perfect" is used to describe a hypothetical centrally loaded straight column of uniform cross section and homogeneous material properties. It should be noted, too, that only primary bending failure is involved, not the bending that would result from a local instability failure. The statement takes care of another point that has sometimes caused confusion. It is well known that, in the elastic range, the buckling load is not a constant but increases with increasing deflection. In fact, for very large deflections the load required to maintain equilibrium may be considerably greater than the Euler load. In the inelastic range, too, it is possible for the column load to exceed the load given by the tangent-modulus formula, as previously noted. In both these cases, however, the generalized Euler formula is correct for predicting the load at which bending (or buckling) begins.

There are now two questions to answer:

(*a*) Shall the theoretical load at which bending of a perfect column begins be taken as the upper limit for structural design?

(*b*) How is this load further reduced by various imperfections that may exist in the actual structure?

It should not be difficult to agree on the answer to the first question. In the elastic range the column load will not exceed the Euler value by any appreciable amount until the bending deflection becomes quite large.[8] In the inelastic range the increase beyond the Euler value will depend on the variation of the tangent modulus with increasing stress. In the highly curved region of the stress-strain diagram (sometimes termed the knee) the tangent modulus drops very rapidly with increasing stress (shown subsequently in Fig. 5); consequently very little increase in load would be obtained. In the so-called plastic range, beyond the knee of the diagram, the tangent modulus usually has a relatively low value and does not change rapidly with increasing strain. The

[8] "An Introduction to the Theory of Elasticity," by R. V. Southwell, Oxford Univ. Press, Oxford, England, 2d. Ed., p. 434, Fig. 114.

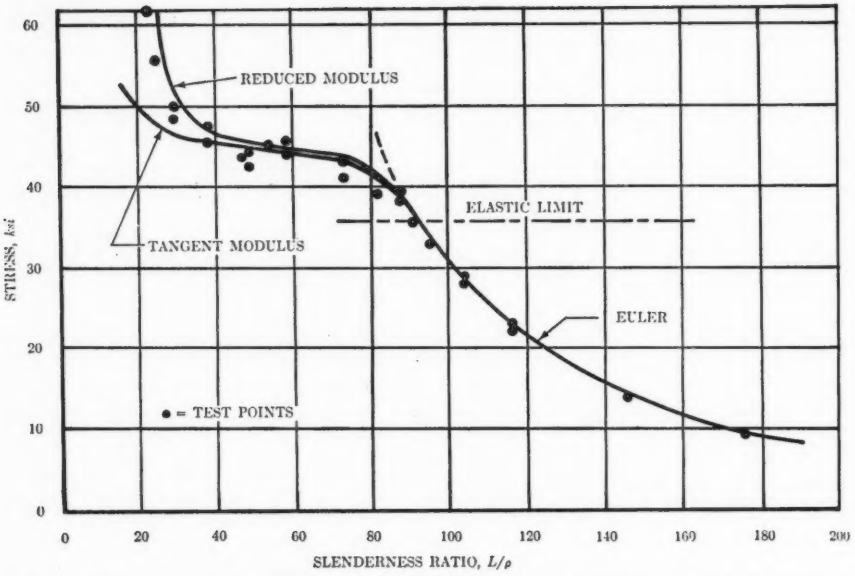

FIG. 1.—RECTANGULAR STEEL BAR WITH KNIFE-EDGE ENDS (THE ABBREVIATION "KSI" DENOTES KIPS PER SQUARE INCH—THAT IS, THOUSAND POUNDS PER SQUARE INCH)

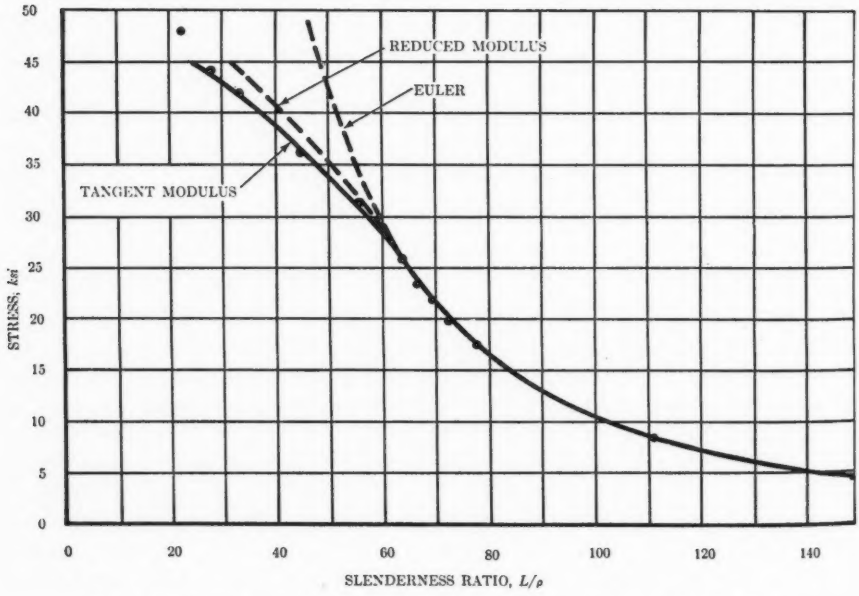

FIG. 2.—SQUARE ALUMINUM ALLOY COLUMN WITH SPHERICAL ENDS (24ST ⅛ IN.)

maximum column load will therefore approach that predicted by the double-modulus theory,[9] which permits a relatively large increase for low values of $\frac{E_t}{E}$. It is doubtful whether any structural engineer would wish to take advantage of this factor in actual design, because in this region an increase in stress will involve a large amount of permanent set. It seems logical, therefore, to regard the beginning of the bending in a perfect column as the upper limit for structural purposes.

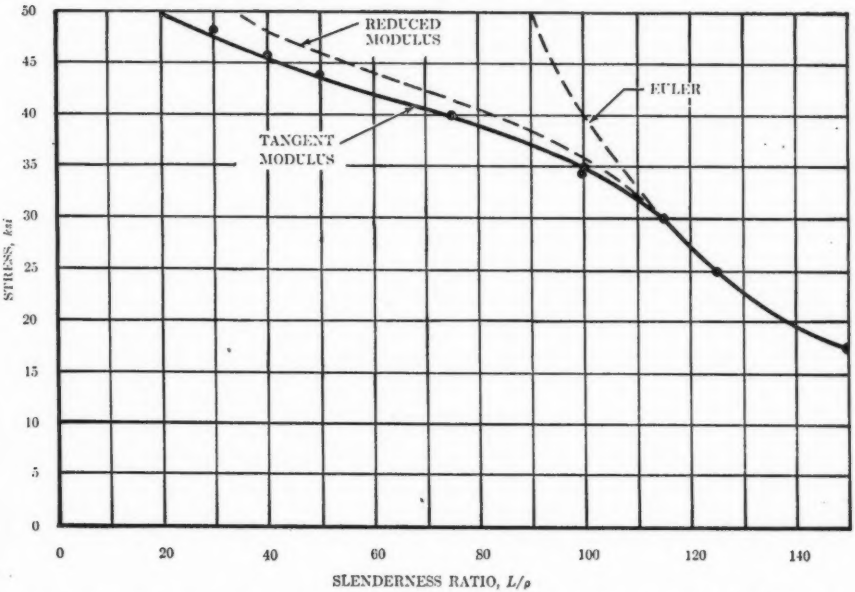

FIG. 3.—SOLID ROUND ALUMINUM (17ST) ALLOY ROD, WITH FLAT ENDS; DIAMETER 0.282 IN.

Figs. 1,[4] 2,[10] and 3[11] show how test data compare with the generalized Euler equation. As denoted by the caption, the specimens for Fig. 1[4] were steel, of rectangular cross section, and were loaded through knife-edges. (The tangent-modulus column curve has been added.) The test points approach the double-modulus curves more closely in the region that lies well above the proportional limit. Mr. Van den Broek's tests[10] were made with square bars of 24ST aluminum alloy (see Fig. 2), using spherical steel loading blocks. (The symbol 24ST denotes a trade designation by which the alloy is commonly recognized. In the drawings, stresses are abbreviated ksi for kips—or thousands of pounds—per square inch, and psi for pounds per square inch.) Fig. 3[11] shows the results of tests made by the Research Laboratories of the Aluminum

[9] "The Double-Modulus Theory of Column Action," by William R. Osgood, *Civil Engineering*, March, 1935, p. 174, Fig. 2.

[10] "Column Formula for Materials of Variable Modulus," by J. A. Van den Broek, *The Engineering Journal*, December, 1945, p. 772.

[11] "Column Strength of Various Aluminum Alloys," by R. L. Templin, R. G. Sturm, E. C. Hartmann, and M. Holt, *Technical Paper No. 1*, Aluminum Research Laboratories, Pittsburgh, Pa., 1938.

Company of America, in Pittsburgh, Pa. The specimens were 17ST aluminum alloy of round cross section and were tested with flat ends (in these tests a fixity coefficient of 4 was developed).

It would be possible to devote the remainder of this paper to the reproduction of test data that show excellent agreement with the generalized Euler theory. Figs. 1, 2, and 3, however, should be sufficient evidence that the theory can be substantiated experimentally. In fact, it is doubtful whether any other structural theory could be verified so accurately by tests covering such a wide range of variables (materials, shape, method of loading, and end fixity). Even the departure of the test points from the tangent-modulus curve is predicted by the theory, as explained previously.

Although it may be agreed that the generalized Euler equation represents the upper limit for column strength, the engineer will very properly raise the second question, previously noted, regarding the reduction in strength caused by imperfections. The major imperfections may be classified as follows:

(*a*) Variations in material properties;
(*b*) Eccentricity of load application;
(*c*) Initial crookedness of the column; and
(*d*) Effects of end conditions:

Items (*b*) and (*c*) are usually classed together, since their effects are very similar. The effect of end conditions should probably be regarded as a separate problem, not as a type of imperfection. The difficulty in this case is the calculation of the degree of end restraint in actual structures.

Before these subjects are considered, it will be helpful to review some of the methods that are used in the engineering application of the generalized Euler theory.

2. Engineering Application of the Column Theory

To one who has not actually tried to use the tangent modulus in the Euler equation the lack of a formula that can be solved on a slide rule may appear to be a disadvantage. Actually, however, the entire column curve for a given material can be obtained in a few minutes, directly from the stress-strain diagram. The slope at various stress values is obtained by a straightedge.[12] These values of the tangent modulus may then be used, together with the corresponding stresses, to solve Eq. 3 (presented subsequently) for the slenderness ratio. It is interesting to plot the values of the tangent modulus against stress. This method will aid in obtaining a smooth curve and will also reveal how the stiffness of the material decreases as the stress is increased. Mr. von Kármán followed this procedure[4] in obtaining the column curve shown in Fig. 1. The curve presented by Mr. von Kármán is reproduced as Fig. 4. The reduced modulus is also shown in Fig. 4. It is important to note that the difference between the tangent modulus and the reduced modulus does not cause a very large reduction in stress on the column curve (Fig. 1). The reason for this

[12] "Compressive Stress-Strain Properties of Some Aircraft Materials," by P. E. Sandorff and R. K. Dillon, *Proceedings*, A.S.T.M., Vol. 46, 1946, pp. 1039–1052.

can be noted by writing Eq. 2 as follows:

$$\frac{L}{\rho} = \pi \sqrt{\frac{E_t}{f}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

At a given stress, a change in the value of the modulus will simply displace the column curve to the left or right, and then only by the square root of the ratio involved. For stress-strain diagrams with a relatively sharp knee, the difference between the two theories is almost negligible, over the range of slenderness ratios used in design.

Fig. 5 shows the tangent-modulus curves for several materials. The column curves derived from these curves are shown in Fig. 6. It is quite evident that the shape of the column curve may vary widely between different

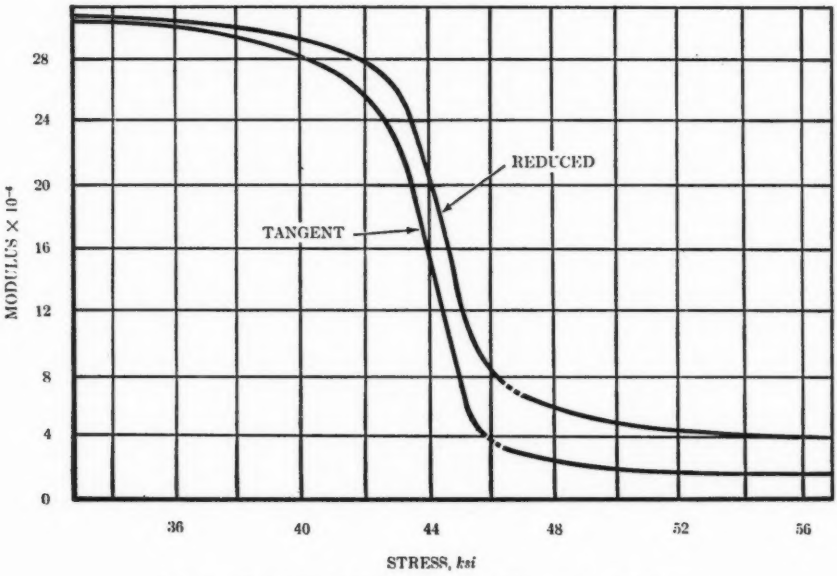

FIG. 4.—RELATION BETWEEN TANGENT MODULUS AND STRESS

materials. With such an array of curves to be fitted, it is no wonder that engineers have used so many different types of empirical formulas for the "short-column" range.

One of the very practical advantages of the tangent-modulus method is that there is no need to define the "short column." One formula covers the entire range and it is impossible to make the mistake of using the wrong formula. It would be well to abandon the term "short column."

Some of the other engineering advantages of the generalized Euler equation will become apparent in the discussion of material variations and fixity effects. Before considering these questions, however, two very useful engineering tools will be discussed: The mathematical stress-strain diagram and the nondimensional column curve.

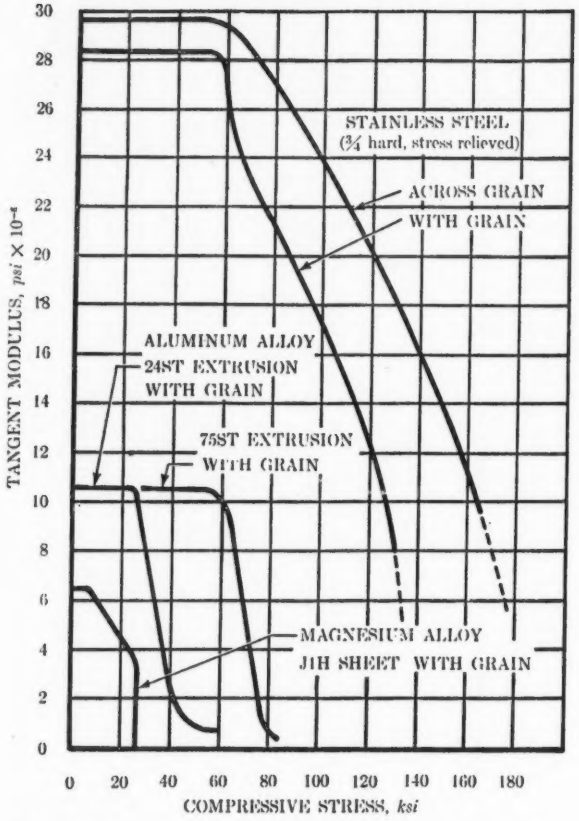

FIG. 5.—TANGENT-MODULUS CURVES FOR VARIOUS MATERIALS

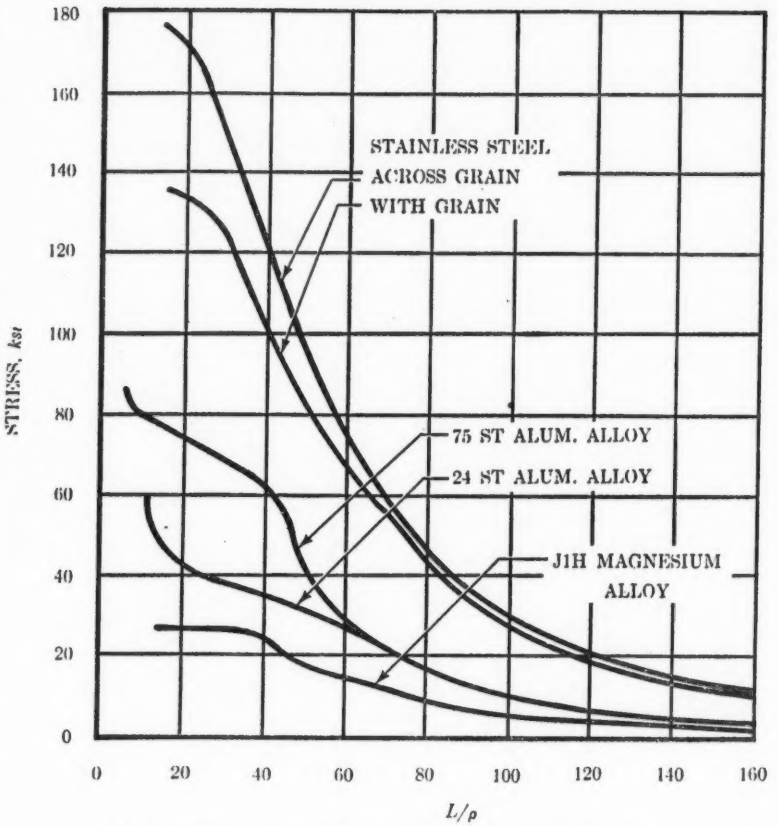

FIG. 6.—GENERALIZED EULER CURVES FOR VARIOUS MATERIALS

3. Mathematical Determination of the Tangent Modulus

W. Ramberg and William R. Osgood,[13] M. ASCE, have provided a method of expressing the stress-strain diagram by an equation which has the general form:

$$\epsilon = \frac{f}{E} + K\left(\frac{f}{E}\right)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which ϵ is a strain; E is the modulus of elasticity (slope at origin); n is an exponent which controls the shape of the diagram; and K is a constant (see note under Eq. 5).

If the constants are known, the tangent modulus may be found by differentiation. A more general solution may be obtained by writing Eq. 4 in nondimensional form—which may be accomplished by dividing every value of stress, on the stress-strain diagram, by a base stress and every value of strain by a base strain. The base stress, f_o, is approximately equal to the yield stress (0.002) but is determined by drawing a line from the origin at a slope equal to 0.7 E. The resultant stress is designated the "secant yield stress." (The secant method is used because it is nondimensional.) The base strain, ϵ_o, is taken as the strain corresponding to E and f_o.

The stress-strain equation, in nondimensional form, may be written as

$$\frac{\epsilon}{\epsilon_o} = \frac{f}{f_o} + \frac{3}{7}\left(\frac{f}{f_o}\right)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which $\epsilon_o = f_o/E$; and f_o is the secant yield stress for a secant modulus equal to 0.7 E. The constant 3/7 in Eq. 5 results from the selection of 0.7 E for the secant modulus.

Fig. 7 shows[13] the nondimensional stress-strain diagrams for various values of n. It is possible to determine n for an actual diagram by plotting it in nondimensional form on such a chart. Messrs. Ramberg and Osgood also present[13] a formula for determining n by a second secant yield stress.

The equation for the tangent modulus, in nondimensional form, may be obtained from Eq. 5:

$$\frac{E_t}{E} = \frac{1}{1 + \frac{3}{7}n\left(\frac{f}{f_o}\right)^{n-1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Curves representing this equation are reproduced in Fig. 8.[13]

4. Nondimensional Column Curves

The nondimensional column curve was first presented (to the writer's knowledge) in 1929.[14] Sometime later it was included in a handbook[15] printed by the federal government which establishes the allowable stresses used by

[13] "Description of Stress-Strain Curves by Three Parameters," by W. Ramberg and William R. Osgood, *Technical Note No. 902*, National Advisory Committee for Aeronautics, Washington, D. C., July, 1943.

[14] "Strength of Tubing Under Combined Axial and Transverse Loading," by L. B. Tuckerman, S. N. Petrenko, and C. D. Johnson, *Technical Note No. 307*, National Advisory Committee for Aeronautics, Washington, D. C., June, 1929.

[15] "Strength of Metal Aircraft Elements," *ANC-5*, Army-Naval-Civil Committee on Aircraft Design Criteria, U. S. Govt. Printing Office, Washington, D. C., 1942.

Fig. 6.—Generalized Euler Curves for Various Materials

Fig. 5.—Tangent-Modulus Curves for Various Materials

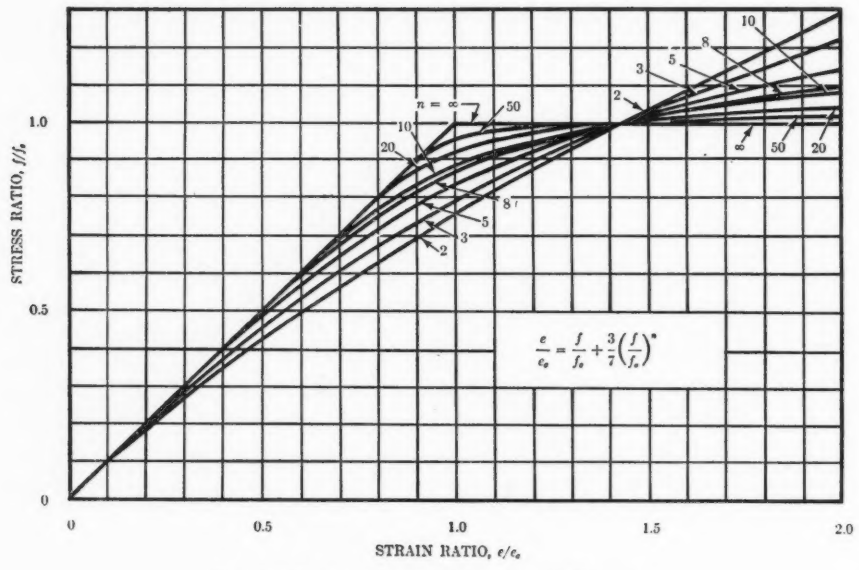

FIG. 7.—NONDIMENSIONAL STRESS-STRAIN DIAGRAMS

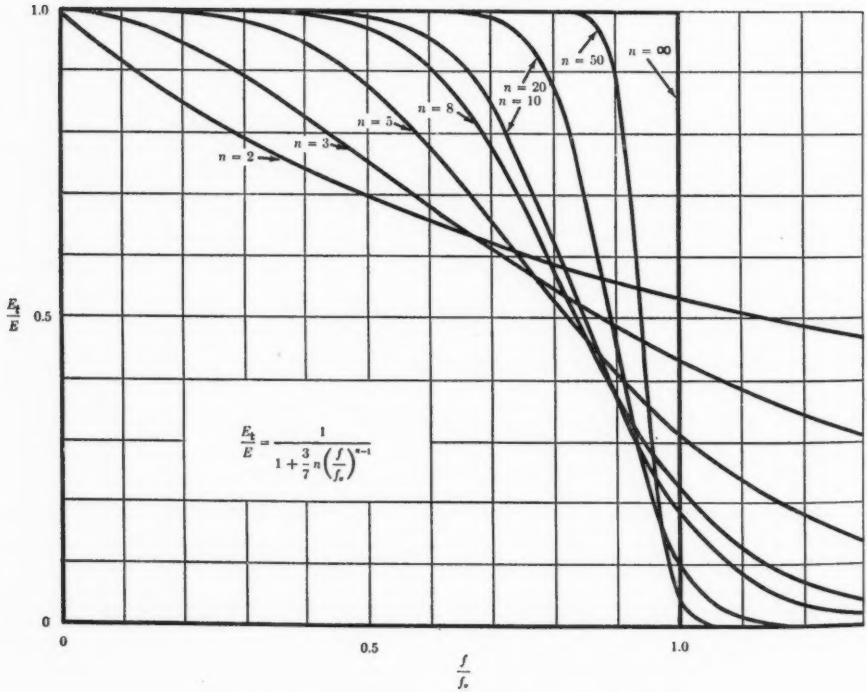

FIG. 8.—NONDIMENSIONAL TANGENT-MODULUS CURVES

airplane designers. The use of the nondimensional column curve in connection with aircraft stress analysis was described by F. P. Cozzone and M. A. Melcon in 1946.[16]

To reduce the column stress curve to nondimensional form, every point on the curve must be divided by a base value of stress, f_o, and also by a base value of slenderness ratio, $(L/\rho)_o$. The stress value originally chosen by L. B. Tuckerman, S. N. Petrenko, and C. D. Johnson[14] corresponded to the "column intercept" and was approximately equal to the 0.002 yield stress. If the nondimensional stress-strain diagram is used as a basis, the value of f_o will be the same as that used for Eq. 5.

To make the Euler (elastic) curves all coincide on the nondimensional diagram, the base value of $(L/\rho)_o$ is determined as the slenderness ratio at which the Euler stress equals the base stress. The equation for this point is

$$f_o = \pi^2 \frac{E}{(L/\rho)^2_o} \quad \ldots\ldots (7)$$

The nondimensional column equation is determined by dividing Eq. 2 by Eq. 7:

$$\frac{f}{f_o} = \frac{E_t/E}{\left[\dfrac{L/\rho}{(L/\rho)_o}\right]^2} \quad \ldots\ldots (8)$$

Let

$$R_f = f/f_o \quad \ldots\ldots (9)$$

and

$$B = \frac{L/\rho}{(L/\rho)_o} \quad \ldots\ldots (10)$$

The terms in Eqs. 9 and 10 may be called the column stress ratio and the reduced slenderness ratio. The column equation may now be written as

$$R_f = \frac{E_t/E}{B^2} \quad \ldots\ldots (11)$$

For the Euler (elastic) curve, $E_t = E$, from which the Euler equation is

$$R_f = \frac{1}{B^2} \quad \ldots\ldots (12)$$

The value of $(L/\rho)_o$ is a constant for a given material and is found by solving Eq. 7:

$$\left(\frac{L}{\rho}\right)_o = \pi \sqrt{\frac{E}{f_o}} \quad \ldots\ldots (13)$$

The expression for B is sometimes written as

$$B = \frac{1}{\pi} \sqrt{\frac{f_o}{E}} \frac{L}{\rho} \quad \ldots\ldots (14)$$

[16] "Nondimensional Buckling Curves—Their Development and Application," by F. P. Cozzone and M. A. Melcon, *Journal of the Aeronautical Sciences*, October, 1946, p. 511.

This form, however, conceals the physical significance of B. It is probably best to determine the constant $(L/\rho)_o$ for the material being investigated and then to use that value in Eq. 10 to determine B for any given slenderness ratio.

Messrs. Cozzone and Melcon[16] show that the curves of Fig. 8 may also be used for any type of buckling in which the equation has the form:

$$f = K \frac{E_t}{(a/b)^2} \quad \ldots\ldots (15)$$

in which K is a constant; and a and b are the two dimensions that affect the buckling stress. In this more general form,

$$B = \frac{a/b}{(a/b)_o} \quad \ldots\ldots (16)$$

and

$$\left(\frac{a}{b}\right)_o = \sqrt{K \frac{E}{f_o}} \quad \ldots\ldots (17)$$

The nondimensional column curve may be derived equally well from the load-length curves. Thus, test data can be plotted directly in nondimensional form, if desired. The column load, P, is divided by the base load, P_o, in which

$$P_o = f_o A \quad \ldots\ldots (18)$$

The effective column length, L, is divided by the base length, L_o:

$$L_o = \pi \sqrt{\frac{E I}{P_o}} \quad \ldots\ldots (19)$$

Then

$$R_f = \frac{P}{P_o} \quad \ldots\ldots (20)$$

and

$$B = \frac{L}{L_o} \quad \ldots\ldots (21)$$

Using Eqs. 18 to 21, any nondimensional column curve may be transformed directly into a design curve of load versus length, for a member of given material and cross section.

To express the nondimensional column curves in mathematical form it is necessary only to insert in Eq. 11 the expression for E_t/E from Eq. 6. The generalized column equation then becomes:

$$R_f + \frac{3}{7} n (R_f)^n = \frac{1}{B^2} \quad \ldots\ldots (22)$$

Eq. 22 is plotted in Fig. 9,[16] revealing a number of interesting points. The effects of varying the shape of the stress-strain diagram (by varying n) are greatest in the region where $B = 1.0$. Thus, it is apparent that the shape of the stress-strain diagram near the knee will have a relatively large influence on the column stress in this region.

5. The Envelope Equation

Fig. 9 shows that there is an envelope curve representing the minimum values of column stress. The equation for this envelope may be obtained from Eq. 22:

$$R_f = e^{-1/n} \quad \ldots\ldots (23)$$

from which

$$n = -\frac{1}{\log_e R_f} \quad \ldots\ldots (24)$$

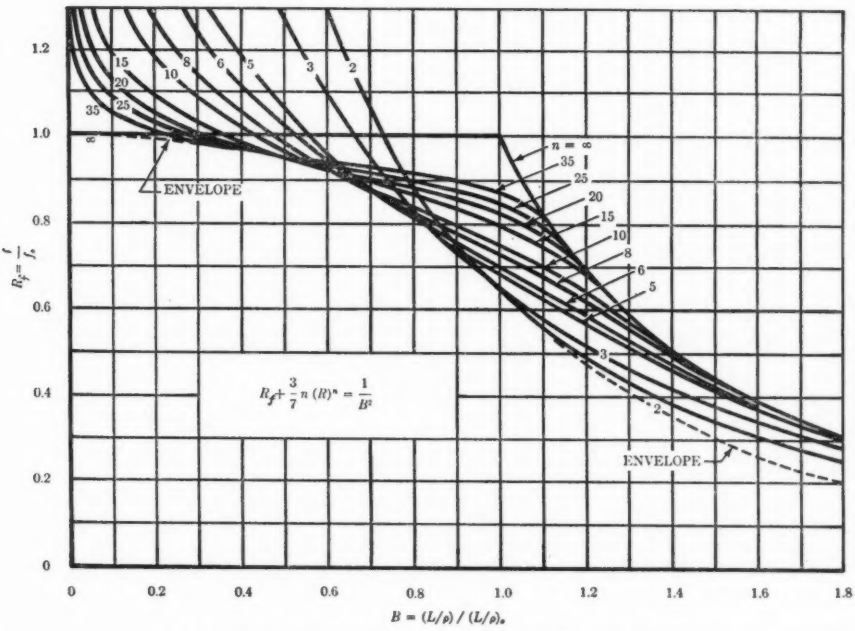

Fig. 9.—Nondimensional Tangent-Modulus Column Curves

Substituting the value of n in Eq. 24 into Eq. 22 gives the equation of the envelope curve:

$$R_f - \frac{3}{7} \frac{1}{e \log_e R_f} = \frac{1}{B^2} \quad \ldots\ldots (25)$$

Eq. 25 is based on the tangent-modulus theory, but it does not involve the tangent modulus; in fact, the only two material properties that enter into Eq. 25 are E (Young's modulus) and f_c (the secant yield stress). The physical significance of Eq. 25 is evident from Fig. 10. Together with the Euler curve, Eq. 25 bounds a region which encloses the column curves for all mathematical stress-strain diagrams (that is, all diagrams expressed by Eq. 22) that can be passed through the shaded area of Fig. 10(*a*); and it covers all positive values of n in Eq. 22.

Eq. 25 would be too conservative for most design purposes (at least for aircraft) since it does not permit the designer to take full advantage of available knowledge on the shape of the stress-strain diagram. Where weight saving is not too important, however, Eq. 25 could be used as a kind of over-all column curve for any material. It would then only be necessary to know the secant yield stress (the stress at which a line having a slope of 0.7 E cuts the stress-strain diagram) and the modulus of elasticity E.

The resemblance of the envelope curve to a parabola shows why parabolic curves have been found useful in representing the "short column" range. The relationship of various other column formulas is shown in Fig. 11. The equations of these formulas, in nondimensional form, are: Parabolic formula (exponent = 2):

$$R_f = 1 - \frac{B^2}{4} \quad \ldots\ldots (26)$$

straight line formula (intercept at $R_f = 1.0$):

$$R_f = 1 - 0.385\,B \quad \ldots\ldots (27)$$

Gordon-Rankine formula (with Ritter's constant):

$$R_f = \frac{1}{1 + B^2} \quad \ldots\ldots (28)$$

and Van den Broek formula:[17]

$$R_f = \sqrt{1 + \left(\frac{B^2}{2}\right)^2} - \frac{B^2}{2} \quad \ldots\ldots (29)$$

As a whole, the various empirical formulas form a "scatter band," as shown in Fig. 11.

6. Effects of Material Variations

From the discussion of nondimensional column formulas, it is obvious that the three primary material variables are the initial modulus of elasticity E, the secant yield stress f_o, and the shape of the knee of the stress-strain diagram (indicated by n).

It is unnecessary to consider E, as far as variations between lots of material are concerned. From an engineering point of view E can usually be regarded as constant for a given material or alloy and at a given temperature.

The secant yield stress based on 0.7 E is approximately equal to the conventional (0.002) yield stress.[13] For material-correction purposes either type of yield stress may be used. In general terms it can be stated that the compressive yield stress controls the height of the left end of the column curve. In the nondimensional form this stress controls the value by which all the ordinates of the diagram are multiplied. Therefore, it is the most important property, as far as material variations are concerned.

The specifications for structural materials to be used in compression should contain a minimum value for the compressive yield stress. The tensile yield

[17] "Theory of Limit Design," by J. A. Van den Broek, John Wiley & Sons, Inc., New York, N. Y., 1948, p. 103, formula IX.

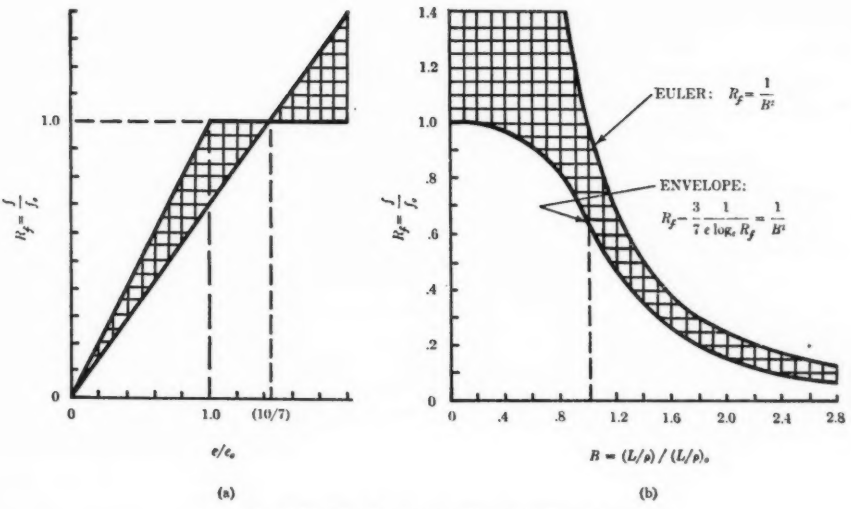

FIG. 10.—THE ENVELOPE COLUMN CURVE

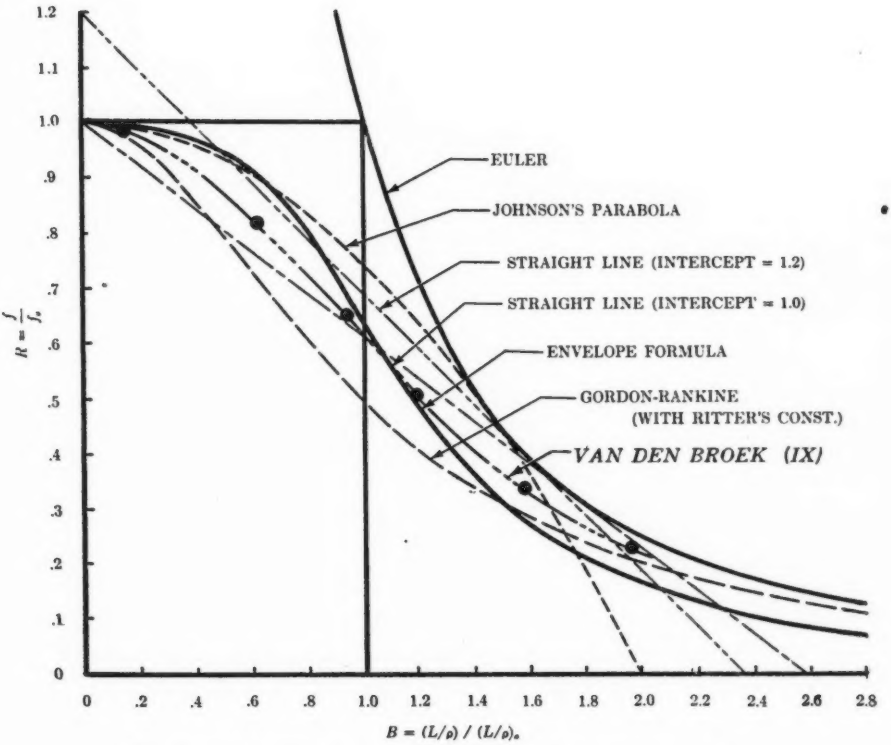

FIG. 11.—COMPARISON OF COLUMN FORMULAS IN NONDIMENSIONAL FORM

stress is sometimes a reasonably good substitute since the yield stresses in tension and compression are nearly the same, but this assumption can be misleading. Certain types of magnesium alloy show considerably lower yield stresses in compression than in tension. The "grain" (direction of rolling) may also have a pronounced influence (see Fig. 6). Most aluminum alloy sheet is straightened by stretching, which raises the tensile yield stress in one direction and raises the compressive yield stress in the other (cross grain).

To be entirely safe, it would be necessary to select, as a basis for the design column curve, the lowest probable value of compressive yield stress and the "flattest" type of diagram likely to be obtained (minimum values for f_o and n, when the stress-strain diagram is expressed mathematically). This procedure would take care of all material variations. (Theoretically the curve should be obtained from the two limiting values of n, with the envelope curve in between.)

In aircraft work many compression tests are performed to obtain allowable stresses for complicated types of construction. The results of such tests must be "reduced to standard," or to minimum specified values. Several methods are available, but the most logical is that using the nondimensional column curve. All test data may be divided by the constants corresponding to the test material, to obtain the nondimensional diagram. To convert back to "standard," the nondimensional values are multiplied by the constants for the standard material.

Fig. 12 shows a family of column curves for a high-strength aluminum alloy. The effects of varying the yield stress have been determined by the nondimensional system, assuming that the stress-strain diagrams remain affine (that is, that n does not vary).

Curves of this type may be used directly for "correcting" from one yield stress to another, as indicated in Fig. 12. Such curves may also be used directly for any type of buckling in which failure occurs by bending in one plane only, even though L/ρ is not the proper geometrical ratio. (This procedure is similar to the use of B for various types of buckling, as discussed in Section 4.)

7. The Effects of Eccentricity

The theories and formulas that have been discussed are all based on the conception of a perfectly straight, centrally loaded column. Since no actual column can fully meet this requirement, engineers have long been interested in the effects of eccentric loading. There is a definite need, also, for a sound method of design for columns that have design eccentricities of considerable magnitude. This problem therefore has two distinct phases: (*a*) The effects of small "incidental" eccentricities, and (*b*) the design of columns with known eccentricities.

For the design of columns that are nominally straight, it is customary and logical to consider small degrees of crookedness as equivalent to a certain initial eccentricity. There has been much discussion of the proper value to use for the equivalent eccentricity in actual design work, but obviously this problem comes under the heading of "tolerances" and must be controlled by the same processes that are used to control other tolerances in manufacture. It will be

assumed, therefore, that the designer either controls this variable by suitable specifications, shop practices, etc., or estimates a maximum value that is compatible with the kind of structures with which he is dealing. The question then becomes, is it necessary to make a further reduction in the column strength to account for the maximum probable equivalent eccentricity?

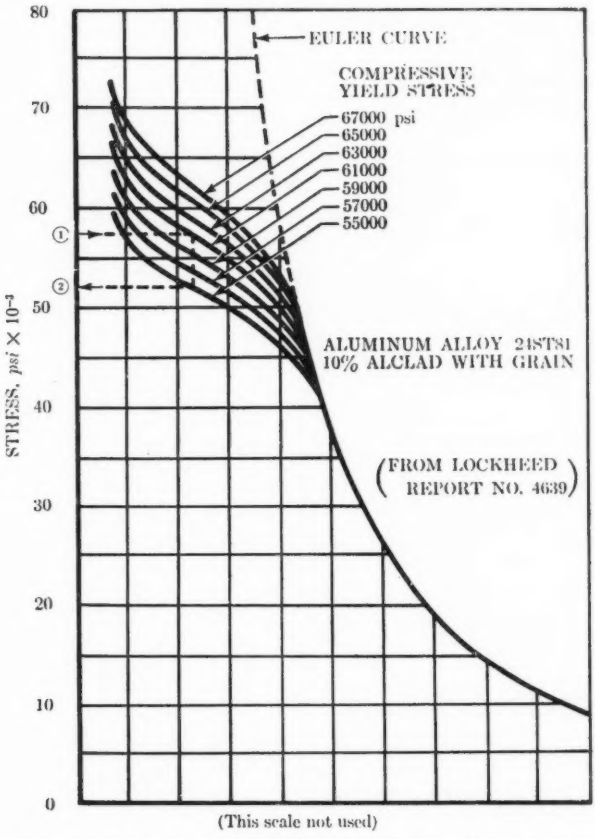

FIG. 12.—METHOD OF CORRECTING TEST DATA FOR YIELD STRESS

Before this question can be answered it is necessary to have a method of determining the effects of eccentricity. For the design of elastic members the answer is simple—use the secant formula. This well-known formula gives the maximum fiber stress that will be produced by a definite load (or average stress) in the column, provided that the stresses stay within the elastic range. If certain values are chosen for the eccentricity and the maximum permissible stress, the equation becomes a column formula.

For a material which has a very pronounced yield point, and a "flat-topped" stress-strain diagram, the secant formula can be used by substituting the yield stress for the maximum permissible stress. (It is still necessary to specify the

eccentricity, however.) For materials that do not exhibit this characteristic it is impossible to select a single value of maximum stress that will make the formula fit the test data. Furthermore, the shape of the secant column curve will have no relation to the shape determined by the tangent-modulus theory for zero eccentricity; yet it is obvious that the eccentric column curve must approach the basic column curve as the eccentricity approaches zero. Because of these shortcomings, the secant column formula cannot be considered as generally satisfactory.

The eccentric column was treated very thoroughly by Mr. von Kármán, in 1910.[4] He showed that, in the elastic range, initial eccentricity simply causes a constantly increasing lateral deflection as the axial load is increased. The curves are asymptotic to the Euler load, which therefore represents the maximum load for the eccentric elastic column. (This theory neglects the further increase in load at large deflections, previously discussed.)

For the inelastic case, Mr. von Kármán showed that increased eccentricity causes greater bending strains (for the same axial load), which force some of the material farther into the inelastic region of the stress-strain diagram. Thus, a lower value of effective modulus of elasticity is obtained. During loading, a point is finally reached where the internal resisting moment does not increase as rapidly as does the external moment. The external load must then decrease to maintain equilibrium. The maximum load reached in this process will depend on the slenderness ratio, the degree of initial eccentricity, the shape of the stress-strain curve, and the shape of the cross section.

This problem is extremely complex. Mr. von Kármán solved it for a particular material (steel) and cross section (rectangular) using the double-modulus theory to determine the internal resisting moment. Graphical integration methods were used. The resulting curves have been reproduced in many textbooks, but the exact solution has never been improved on, to the writer's knowledge. In fact, the recent discovery that the double-modulus theory (which Mr. von Kármán used) is not strictly accurate for small deflections further complicates the situation.

Mr. Van den Broek[10] has developed a formula for predicting the maximum load of the eccentric column. He assumes, in effect, that the stress-strain diagram becomes substantially flat at some known value of stress and that the column has bent to a point where opposite edges are working in these flat regions, one in tension and one in compression. By making some simplified assumptions as to the shape of the deflection curve, he obtains a formula for the maximum load, for a column of rectangular cross section. For materials having a curved stress-strain diagram, the Van den Broek method has the same limitation as the secant formula—there is no good definition for the "upper yield stress."

K. Ježek[18] has developed an algebraic method based on a simplified stress-strain diagram composed of two straight lines, one horizontal. He obtains factors by which the concentric column values are divided. Thus, the errors caused by the original assumptions are reduced, but for materials that have a

[18] "Approximate Theory of Eccentrically Loaded Steel Columns," by K. Ježek, *Der Stahlbau*, Vol. 8, 1935.

gradually curving stress-strain diagram the error may still be considerable. (The writer is indebted to Friedrich Bleich, M. ASCE, for the information on the Ježek method.)

These examples indicate that the eccentric column, in the inelastic range, cannot be treated adequately by any simple engineering formula or theory. The reason for this may be that there are in reality two problems—(1) stability under axial loading and (2) failure in bending. The stability problem is complicated even for zero eccentricity, but it can be handled by the tangent-modulus theory. Failure in bending is in itself a large subject, involving both the shape of the cross section and the shape of the stress-strain diagram.

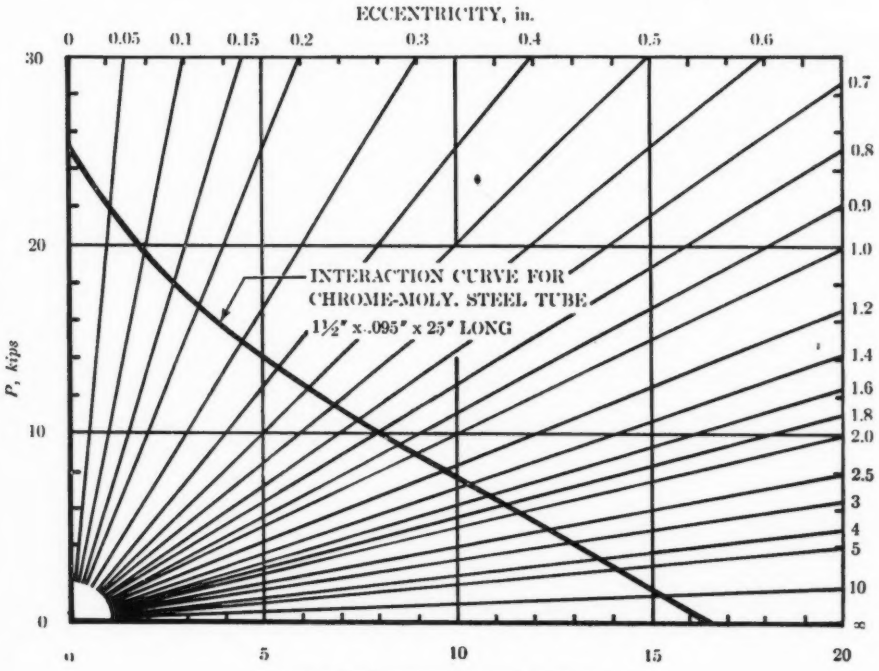

FIG. 13.—INTERACTION CURVE FOR COMBINED BENDING AND COMPRESSION

This situation is similar to one that existed about 1935, in aircraft structural analysis. There was no suitable formula for the strength of members subjected to combined bending and torsion. The solution was found in the interaction curve. This method has since become standard for handling many kinds of combined loading.[19]

Why not use the interaction curve to treat the eccentric column problem? The member can be regarded as being subjected to combined compression and bending, with the bending moment equal to $P\,e$ (e denoting eccentricity).

[19] "Basic Structures," by F. R. Shanley, John Wiley & Sons, Inc., New York, N. Y., 1944, Chapter 20, p. 340.

Interaction curves for combined bending and compression were obtained from tests of steel and aluminum alloy tubing by Messrs. Tuckerman, Petrenko, and Johnson.[14] In these tests the bending moment was applied by transverse loading at the third points, whereas in the eccentric column the bending moment must be considered as being applied at the ends of the member. Since the

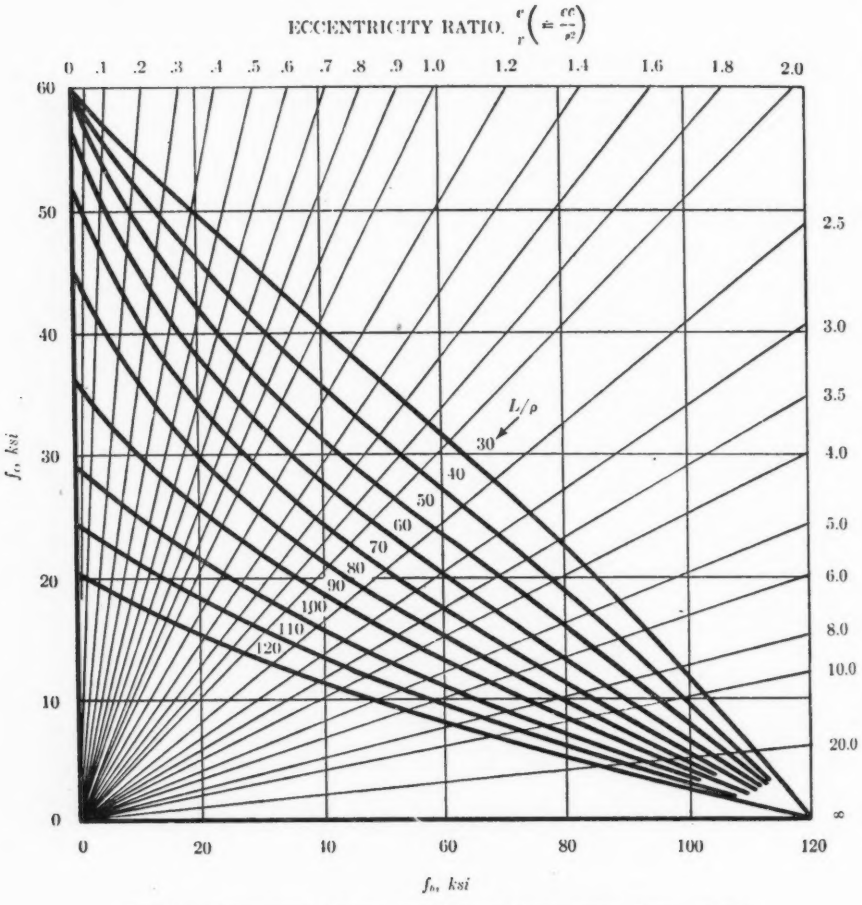

FIG. 14.—INTERACTION CURVES FOR CHROME-MOLYBDENUM STEEL TUBING

interaction curves for the two cases should be quite similar, the curves developed by Messrs. Tuckerman, Petrenko, and Johnson[14] will be used to illustrate the proposed method.

Fig. 13 shows a typical interaction curve for a single member under all possible combinations of axial compression and bending.[14] The two end points of the curve correspond to the column strength and to the ultimate bending moment, respectively. If it is assumed that the bending moment was applied at the ends of the member, as in an eccentric column test, the eccentricity (e)

is given by the ratio M/P. Straight lines of constant eccentricity therefore radiate from the origin, as shown in Fig. 13. The interaction curve may thus be used directly to determine the effects of eccentricity in axial loading.

Tests of different kinds and sizes of round tubing were made[14] and the results were plotted as interaction curves, both in terms of stress and in terms of stress ratios. Fig. 14 shows[14] that there is a family of curves for different values of the slenderness ratio. At low values of the slenderness ratio the curves tend to be straight, but at higher values there is a considerable sag (the result of the secondary bending moment caused by the deflection of the column). At zero bending stress the curves give a series of values that represent the concentric column curve; at zero axial stress the curves converge to a single value of f_b, which is the modulus of rupture for pure bending.

When the interaction curves are plotted in terms of stress, as in Fig. 14, the eccentricity lines denote values of the eccentricity ratio e/r, in which the core radius $r = \frac{\rho^2}{c}$; and c is the distance from neutral axis to outer fiber.

Fig. 14 may be converted to nondimensional form by either of two methods. Messrs. Tuckerman, Petrenko, and Johnson[14] divided the values of f_c by f_{co}, which was taken as the allowable stress for short columns (intercept value) and f_b was divided by f_{bo}, the modulus of rupture in pure bending. The resulting curves are like those of Fig. 14, with $\frac{L}{\rho}$-values replaced by values of B, the reduced slenderness ratio. For such a chart the eccentricity lines will define values of reduced eccentricity ratio, determined by

$$R_e = \frac{e}{e_o} \qquad (30)$$

in which $e_o = r\frac{f_{bo}}{f_{co}} = \frac{\rho^2}{c}\frac{f_{bo}}{f_{co}}$. To eliminate everything but the sag, divide all axial stresses by the concentric column stress for the member in question, instead of by the value of f_o. The following terms will then apply: f_{cr} is the allowable column stress for $e = 0$ and f_{bo} is the modulus of rupture in pure bending, and

$$R'_e = \frac{f_c}{f_{cr}}\left(= \frac{P}{P_{cr}}\right) \qquad (31)$$

$$R_b = \frac{f_b}{f_{bo}}\left(= \frac{M}{M_o}\right) \qquad (32)$$

$$R'_e = \frac{e}{e'_o} \qquad (33)$$

and

$$e'_o = \frac{\rho^2}{c}\frac{f_{bo}}{f_{cr}}\left(= \frac{M_o}{P_{cr}}\right) \qquad (34)$$

The terms in parentheses show that the ratios in Eqs. 31, 32, and 34 may all be expressed in terms of axial load and bending moment. Thus, test data may be converted directly into the nondimensional form. Conversely, the

nondimensional design charts may be used directly to find the allowable eccentric column load, if the load for the concentric column, and its ultimate bending moment, are known. The curves of Fig. 14 have been replotted in Fig. 15, in this reduced form. The actual slenderness ratio (L/ρ) has been retained, instead of the reduced slenderness ratio B. If B is used, the curves will be similar in shape; the eccentricity lines will remain unchanged.

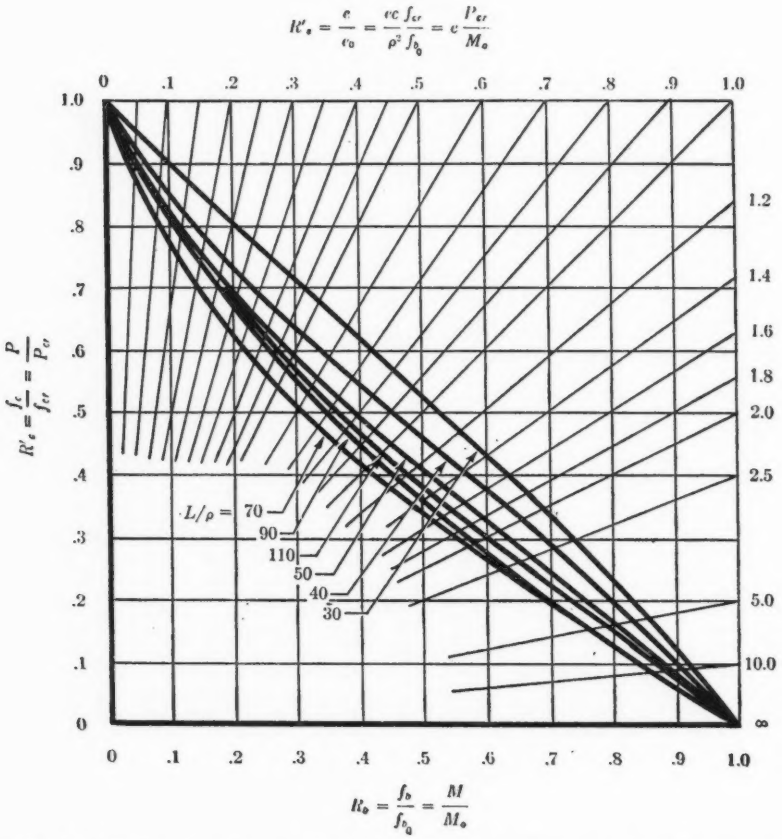
$R'_e = \frac{e}{c_0} = \frac{ec}{\rho^2} \frac{f_{cr}}{f_{b_0}} = e \frac{P_{cr}}{M_o}$
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1 0
$R'_e = \frac{f_c}{f_{cr}} = \frac{P}{P_{cr}}$
1.2 1.4 1.6 1.8 2.0 2.5 5.0 10.0 ∞
$L/\rho =$ 70 90 110 50 40 30
$R_b = \frac{f_b}{f_{b_0}} = \frac{M}{M_o}$

FIG. 15.—REDUCED INTERACTION CURVES FOR COMBINED BENDING AND AXIAL LOADING

Fig. 15 indicates that the nondimensional interaction curves tend to lie within a definite band and that the amount of sag is a maximum for slenderness ratios in the region corresponding roughly to $L/\rho = 70$ (or to $B = 1.0$ to 1.2). At higher values of slenderness ratio, the interaction curves tend to become straighter again. If the bending moment had been applied at the ends of the member, the amount of sag would no doubt be greater.

Since the tests on which Figs. 14 and 15 are based did not truly represent the eccentric column, it would be unsafe to use them for design purposes, but they can be used to draw some interesting conclusions. One of the most valu-

able features of interaction curves is that their shape seems to be primarily a function of the kind of loading conditions involved and is little affected by other variables. Further research may indicate that a few sets of interaction curves, such as those shown in Fig. 15, could be used for a wide variety of columns. By using the reduced slenderness ratio B, it may be possible to show that the shape of the curves depends on the value of the exponent n for the nondimensional stress-strain diagram.

For small values of eccentricity the curves of Fig. 15 are converging and therefore must lie relatively close together. If the lowest curve is considered to apply for all values of slenderness ratio, the effect of eccentricity would be a reduction of all the column stresses by a constant factor. The nondimensional column curve would not be changed in this case; it would only be necessary to multiply R_c by a reduced value of f_o (or P_o) in converting to stress (or load). (The original value of f_o would still be used in the term B, however.) Thus, small eccentricities are more closely related to a reduction in the compressive yield stress than to a reduction in the effective column length.

It can be noted that the shape of the eccentric column curves derived by the interaction method must approach that of the concentric column, as the eccentricity approaches zero. This assumption agrees with theory and tests, as shown by the eccentric column curves presented by Mr. von Kármán.[4] One of the objectionable features of the secant formula is thereby eliminated (since it has no relation to the shape of the stress-strain diagram). It should be noted that the interaction curves need not be based on any particular column formula—they may be used with any formula that may be acceptable for the concentric column. Once the interaction curves are determined, they may be converted into eccentric column curves, or plotted as curves of reduction factors versus slenderness ratio.

Interaction curves are equally applicable to the case of large initial eccentricities. The convergence of the curves at a point representing pure bending permits the designer to take advantage of all that is known about the modulus of rupture. Even failure by local buckling may be included, although it is not yet possible to foresee what change in the shape of the interaction curves would be necessary. The case of combined bending and local buckling (no column effects) is a simple interaction problem and could very likely be handled by using the interaction curve for low slenderness ratios and substituting the crushing strength for the column strength in determining f_o or P_o.

The effect of small eccentricities may now be determined approximately. Assume, for example, that the equivalent eccentricity, e, for a thin-walled tube of mean radius R lies within the tolerance $\pm 0.025\,R$ (for a column 20 in. in diameter this would be $\pm$ 0.25 in.). The value of the core radius r ($= \rho^2/c$) for such a tube is $R/2$. The eccentricity ratio e/r is therefore 0.05. The corresponding reduction in the column stress may be determined from Fig. 14 or Fig. 15. (If Fig. 15 is used, the eccentricity must be further reduced by using Eqs. 33 and 34.) The maximum reduction so determined is in the order of 6%, at $L/\rho = 70$. Since the curves are probably only slightly on the unsafe side at such low eccentricities, this result should be a fairly accurate indication of the reduction involved for the assumed case. The magnitude of the reduction

appears to be no greater than the usual variation in material properties and could be provided for (at least in part) in selecting a conservative value for the minimum compressive yield stress.

Mr. von Kármán[4] recommended that columns be designed on the basis of their concentric buckling strength and that the effects of initial eccentricities be taken care of in the safety factor. This recommendation is still sound and is followed in modern aircraft practice in the United States. (Although it is not specifically stated in the airworthiness requirements, the matter is provided for by a factor of safety to increase the applied loads, not to reduce the allowable stresses.) With the use of the tangent modulus in the column formula, together with proper control of manufacturing tolerances, the specification of a minimum compressive yield stress, and the reduction of all test data to minimum guaranteed properties, it would seem that the unknown factors in compressive loading are certainly no greater than those for any other type of loading and that no increase in the usual factor of safety is required.

8. Effects of End Restraint

In theory, end restraint has a very simple effect: It merely changes the effective column length. In practice, however, the accurate determination of the effects of end restraint for any column except a pin-ended column is a difficult problem. The situation that exists in aircraft structural analysis has been very well described by Paul E. Sandorff,[20] and there is little more that can be added at this time. Experience has shown that a fixity coefficient of 1.5 has been generally satisfactory for the design of the continuous compression panel forming the top of the wing structure. Mr. Sandorff shows that very little of this fixity is supplied by the supporting ribs. Edge restraint, chordwise curvature, and other effects usually provide the increase corresponding to the assumed fixity coefficient.

The long-standing conception of a fixity coefficient as a multiplying factor has perhaps given rise to an exaggerated idea of the effects of end restraint. Mr. von Kármán has shown[4] that the fixity coefficient can be used as a multiplying factor, but only for the elastic case (Euler formula). In the inelastic case the coefficient must be used indirectly, to obtain the effective length:

$$L_{\text{eff}} = L/\sqrt{C} \qquad (35)$$

It is easy to overlook this important fact in dealing with the generalized Euler equation. (The writer made this mistake in his own textbook!) It would have been much better if the effects of fixity had been introduced by a length coefficient in the first place, because length is really what is directly affected, not the buckling load nor the stress.

It is important to realize that efficiently designed structures cannot be made proportionately stronger by increasing the amount of end restraint. This fact, too, was pointed out by Mr. von Kármán[4] when he declared, in effect, that end restraint causes a marked increase in buckling strength only for long slender columns, not for shorter ones. In aircraft structures, for example, the com-

[20] "Notes on Columns," by Paul E. Sandorff, *Journal of the Aeronautical Sciences*, January, 1944, p. 1.

pression members are designed to develop ultimate stresses that approach the compressive yield stress.

Fig. 16 illustrates the effects of the fixity coefficient by column curves (24ST aluminum alloy) plotted for different degrees of end fixity C. Assume, for example, that a designer has used $C = 1$ to design a structure for the column stress f_1. If he now goes to considerable trouble to restrain the ends he may be able to develop a fixity coefficient C of 3.0, but the increase in stress thus obtained is almost negligible, as shown by case A, Fig. 16. On the other hand, assume that he has selected too high a value for the fixity coefficient (say, $C = 3.0$) and that in the actual structure it proves to be only $C = 1.0$. The

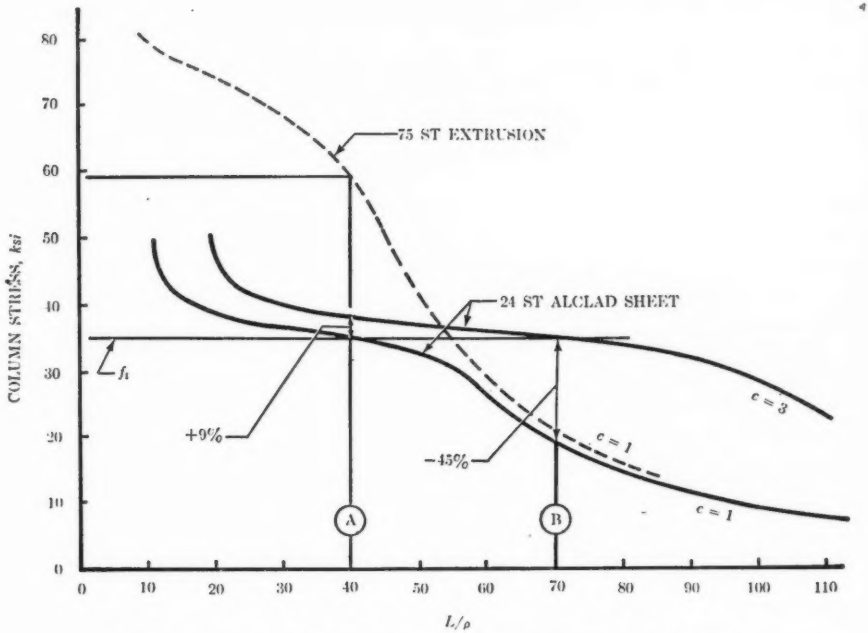

FIG. 16.—EFFECTS OF END RESTRAINT (CURVES OBTAINED FROM TANGENT MODULUS FOR TYPICAL MATERIAL)

decrease in allowable stress is now quite large, as shown by case B. This situation is rather disturbing. An excess of actual fixity (over the assumed value) can do little good, but a lack of fixity (under the assumed value) can do much harm. Thus, conservatism should be used in selecting the fixity coefficient for use in design. Research on this subject cannot be expected to result in substantial weight savings for efficiently designed structures.

Fixity effects should be thought of as "length" effects. In aircraft wing structures, for example, changes in end restraint primarily affect the rib or bulkhead spacing. A few extra ribs represent the price that would be paid for being quite conservative in selecting the fixity coefficient for design. Fig. 16 also shows how the use of a material having a higher yield stress pays divi-

dends in the lower range of slenderness ratios, but such material would give little benefit at high slenderness ratios.

End restraint in an eccentric column reduces the effective eccentricity. An examination of the elastic case will show that the reduction increases as the column load is increased. In the inelastic case this effect should be even more pronounced. This condition is one more reason why, in actual structures, the effects of small eccentricities may prove to be relatively unimportant. The combined effect of large eccentricities and end restraint is an attractive subject for further research; the interaction curves previously described might be useful in this connection.

9. Design for Minimum Weight

The principles of structural design for minimum weight have been treated in some detail elsewhere.[21] In this paper the discussion will be confined to the procedure for determining the optimum design as applied to simple pin-ended columns of tubular cross section, using the generalized Euler theory. Since the weight of the column (for a given material) will be a minimum when the allowable stress is a maximum, the object is to find the maximum value of allowable stress in terms of the load P, the distance through which it must be transmitted L, and a "shape parameter" that controls the proportions of the cross section. If this is done for various materials, a direct comparison of weights may be made.

The generalized Euler stress formula, Eq. 2, contains one of the design variables, L, but it does not include the load P directly. The radius of gyration, being dimensional, cannot be used as a shape parameter. Since ρ^2 has the dimensions of an area, a ratio can be obtained by dividing it by A, giving the shape parameter:[22]

$$k = \frac{\rho^2}{A} \quad \ldots\ldots (36)$$

Eq. 2 may now be written

$$f = \frac{\pi^2 E_t}{L^2} \left(\frac{\rho^2}{A} \right) A \quad \ldots\ldots (37)$$

Substituting k for (ρ^2/A), (P/f) for A, and solving for f or P/L^2 gives the desired equations:

$$f = \pi \sqrt{E_t} \sqrt{k} \sqrt{\frac{P}{L^2}} \quad \ldots\ldots (38)$$

or

$$\frac{P}{L^2} = \frac{f^2}{\pi^2 E_t k} \quad \ldots\ldots (39)$$

The term P/L^2 is called the "structural index" or the "structural coefficient." (A nondimensional coefficient may be obtained by dividing by the elastic modulus, E.) Eq. 39 is more convenient to use than Eq. 38 since the value of

[21] "Principles of Structural Design for Minimum Weight," by F. R. Shanley. *Journal of the Aeronautical Sciences*, March, 1949, p. 133.

[22] "Structures of Minimum Weight," by H. Cox and H. Smith, *Reports and Memoranda No. 1923*, H. M. Stationery Office, London, 1943.

E_t depends on f. From a curve of tangent modulus versus stress the value of $\frac{f^2}{E_t}$ may be determined. Eq. 39 can then be plotted as allowable stress f against P/L^2, for different values of k.

For a thin-walled tube of average diameter D, the value of the shape parameter is

$$k = \frac{\rho^2}{A} = \frac{1}{8\pi}\left(\frac{D}{t}\right) \quad \ldots\ldots (40)$$

For the round tube, Eq. 39 then becomes

$$\frac{P}{L^2} = \frac{8f^2}{\pi E_t \frac{D}{t}} \quad \ldots\ldots (41)$$

From Eq. 41, it can be noted that high values of the stress (f) for a given material will be obtained at high values of the structural index (P/L^2) for large values of D/t. There is a limit to the value of D/t that may be used, however, because the local buckling or crushing stress decreases as D/t is increased.

The optimum stress and the corresponding $\frac{D}{t}$-ratio may be obtained graphically by plotting Eq. 41 as a family of curves for different values of D/t, after which another curve of local buckling stress against D/t is drawn, by which the curves are "cut off." If an equation for the crushing stress can be written in terms of D/t and E_t, the problem may be solved mathematically.

Mr. Timoshenko[23] has suggested an equation of the following type for local buckling:

$$f_{cr} = K\frac{\sqrt{E\,E_t}}{D/t} \quad \ldots\ldots (42)$$

(Recent developments in the theory of plasticity indicate that this may be considerably on the safe side.) If this value is substituted for f in Eq. 41, the following equation will be obtained:

$$\left(\frac{D}{t}\right)_{\text{opt}} = 2\left[\frac{K^2 E}{\pi (P/L^2)}\right]^{\frac{1}{3}} \quad \ldots\ldots (43)$$

It is interesting to note that the tangent modulus does not appear in Eq. 43 even though the equation applies to the inelastic range. Since E remains practically the same for any one class of alloys (steel, aluminum, etc.), the optimum value of D/t will be unaffected by changes in the shape or height of the stress-strain diagram: It depends only on the initial slope of the diagram.

By substituting the value from Eq. 43 in either Eq. 41 or Eq. 42, the optimum stress is obtained as a function of P/L^2:

$$\frac{P}{L^2} = \frac{8f^3}{\pi K E^{\frac{1}{2}} E_t^{\frac{1}{2}}} \quad \ldots\ldots (44)$$

Mr. Timoshenko[23] has shown that the theoretical value of K is about 1.2, but that much lower values are usually obtained in tests. For practical use

[23] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936, p. 442.

in design, Eq. 44 can be plotted for various materials[21] as shown in Fig. 17, where $K = 0.40$. The procedure for optimum design consists in determining

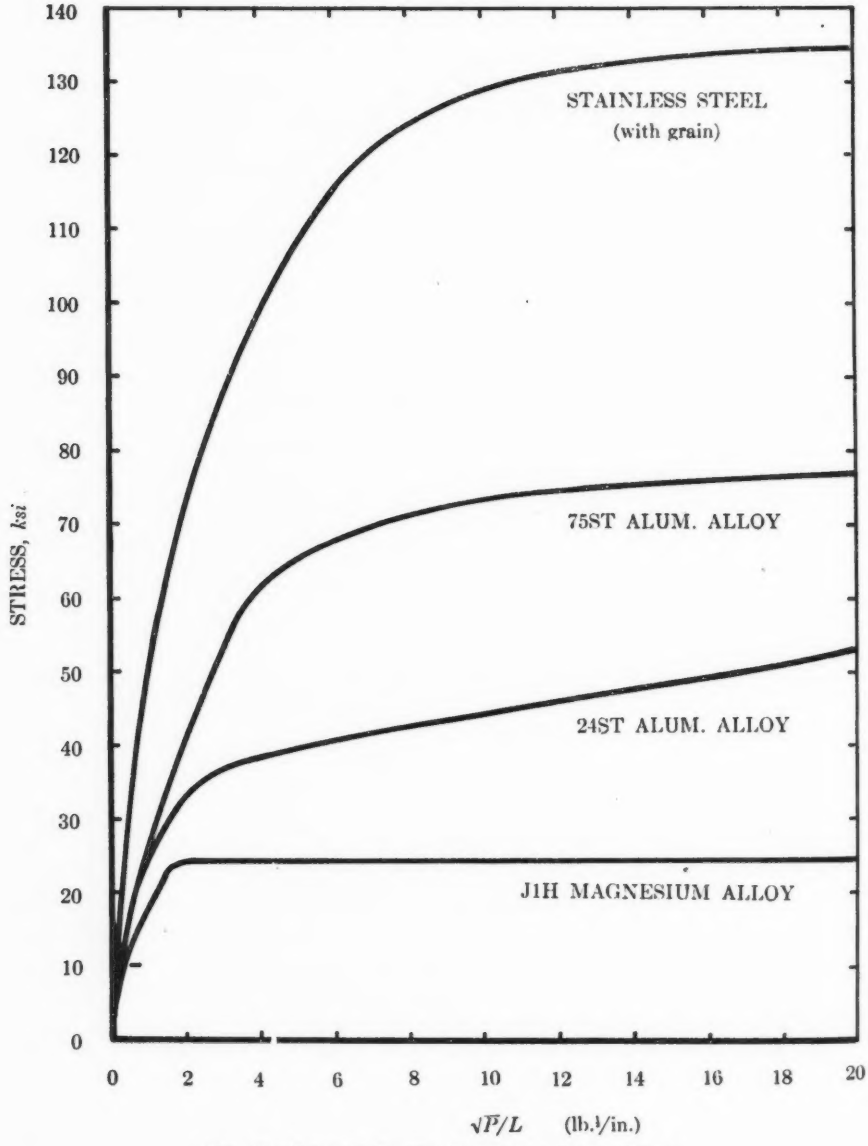

FIG. 17.—OPTIMUM COLUMN STRESS FOR ROUND TUBES

the structural index for the case in question and selecting the optimum stress from such a chart. The load P is divided by the optimum stress to obtain the cross-sectional area. Eq. 43, or curves derived from it, may be used to deter-

mine the optimum ratio D/t, from which the mean diameter may be determined by the following equation for thin-walled tubes:

$$D = \sqrt{\frac{A}{\pi}\frac{D}{t}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

To show how the weight of a column is affected by the material and the loading conditions, the optimum stress may be divided by the relative density. If aluminum alloy is used as a base (density 0.10 lb per cu in.), the curves will

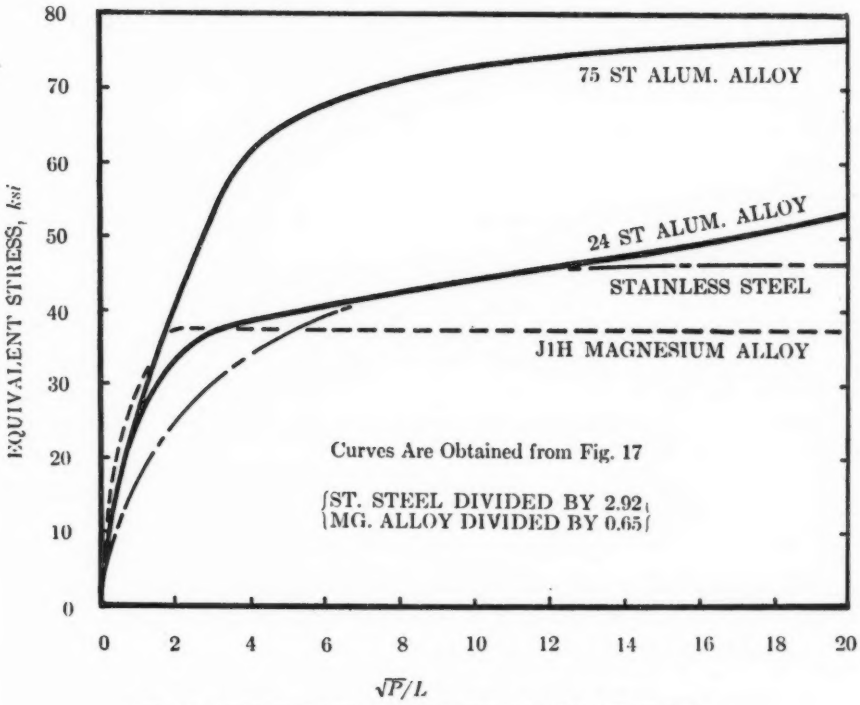

FIG. 18.—OPTIMUM COLUMN STRESS REDUCED TO SAME WEIGHT BASIS (ALUMINUM ALLOY USED AS BASE)

appear as in Fig. 18. (The square root of the structural index has been used in Figs. 17 and 18, to contract the scale.) It is important to note that curves of this type may be plotted directly from test data.[24] If all available data for a given type of loading were plotted on a single chart, the optimum conditions would be indicated by the envelope to the test points.[21] Application of the method, therefore, is not limited by the lack of a theory for optimum design.

To apply the method to members such as those used in steel structures the same shape parameter (Eq. 36) could be used to determine the relationship between the structural index and the column buckling stress, as in Eq. 39. For

[24] "Design Charts for Flat Compression Panels Having Longitudinal Extruded Y-Section Stiffeners and Comparison with Panels Having Formed Z-Section Stiffeners," by N. F. Dow and W. A. Hickman, *Technical Note No. 1389*, National Advisory Committee for Aeronautics, Washington, D. C., August, 1947.

the local buckling stress it would be necessary to use the ratio between one of the major cross-sectional dimensions (such as width) and the thickness of some part of the cross section. Since it is difficult to calculate the local buckling stress with accuracy, for such sections, a series of tests would be advisable in which the width/thickness ratio would be systematically varied. This procedure could be carried out for various basic shapes, such as I-beams, WF columns, and H columns. The results would be used in place of Eq. 42, to obtain curves similar to those in Fig. 17. A systematic program of this type would answer all questions regarding the efficiency of various cross-sectional shapes. It would show, for example, that beyond a certain value of the structural index any attempt to improve efficiency by revising the cross-sectional shape would be futile, at least for structural steel. More research of this type is needed, not only to supply valuable design information, but also to answer questions regarding the relative efficiency of various materials. For any structure that fails through buckling the term "strength/weight ratio" is misleading. The only way in which materials can be properly compared, in such cases, is through the use of optimum design procedures based on the structural index.

The round tube, loaded as a simple column, has been used as an example of the general method of optimum design. H. Cox and H. Smith extended the method to various other types of construction and loading conditions.[22] The National Advisory Committee for Aeronautics has determined the optimum design for flat panels with various types of stiffeners and for several different materials.[24] Some information is also available for shear webs and for shells loaded in bending.[21]

10. Recommendations and Conclusions

Eight general conclusions supported by this paper are:

(*a*) The generalized Euler formula, Eq. 2 (using the tangent modulus), is recommended for the design of columns that fail by primary instability;

(*b*) Specification of a minimum guaranteed compressive yield stress provides for most of the effects of material variations;

(*c*) The effects of small eccentricities may be provided for by the factor of safety;

(*d*) The interaction curves for combined bending and compression may be applied to eccentrically loaded columns;

(*e*) The effects of end restraint should be conservatively estimated in design;

(*f*) The coefficient of fixity should be replaced by an effective length coefficient;

(*g*) Tests are needed to determine interaction curves for various cross sections and materials; and

(*h*) Optimum design methods should be used to evaluate the efficiency of various types of construction and materials.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

AIR-INLET VALVES FOR STEEL PIPE LINES

BY JOHN PARMAKIAN[1]

SYNOPSIS

Analytical basis is presented for determining the size and location of air-inlet valves in steel pipe lines in order to prevent their collapse due to external pressure. The allowable internal pressure for steel pipe shells is first computed from formulas published elsewhere for the collapse of cylindrical shells under external pressure. The conditions under which the hydraulic gradient may fall below atmospheric pressure at some points in the pipe line are then investigated and the size and location of air-inlet valves required to prevent the collapse of the pipe shell are determined on the basis of adiabatic expansion of air. Various charts are presented to facilitate the required computations.

Under certain conditions of operation water may be removed from some sections of a steel pipe line faster than it can be supplied from the reservoir. Such conditions may occur as a result of gate operation, the failure of the pipe line at a low point, or other causes. If the pipe shell is thin, air must be admitted into the pipe to prevent an excessive reduction of the internal pressure and the subsequent collapse of the pipe throughout a considerable length because of the external atmospheric pressure. The strength of the pipe shell under this differential pressure will depend on the length-to-diameter ratio, the thickness-to-diameter ratio, and the modulus of elasticity of the steel. The presence of stiffener rings may exert a considerable influence on the strength of the pipe shell. However, if the distance between stiffener rings is great, they will have no appreciable effect on the section of the pipe shell midway between them and the collapsing pressure will be the same as that of a pipe shell of infinite length. The collapsing pressure of steel pipe shells with and without stiffener rings has been studied and experimentally determined by various investigators and the resulting formulas are given elsewhere.[2]

To obtain the collapsing pressure of steel pipes more conveniently a chart has been prepared (Fig. 1) based on the following instability formulas:[2]

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by November 1, 1949.

[1] Engr., Branch of Design and Constr., Bureau of Reclamation, Denver, Colo.

[2] "Collapse by Instability of Thin Cylindrical Shells Under External Pressure," by Dwight F. Windenburg and Charles Trilling, *Transactions*, A.S.M.E., December, 1934.

For cylindrical shells without stiffener rings,

$$p' = 50.2 \times 10^6 \left(\frac{t}{D}\right)^3 \quad \text{.....(1)}$$

and, for cylindrical shells with stiffener rings,

$$p' = 73.4 \times 10^6 \frac{(t/D)^{5/2}}{L/D} \quad \text{.....(2)}$$

In Eqs. 1 and 2, p' is the collapsing pressure for the pipe shell, in pounds per square inch; t is the thickness of the pipe shell, in inches; D is the inside diameter of the pipe, in inches; and L is the spacing of the stiffener rings, in

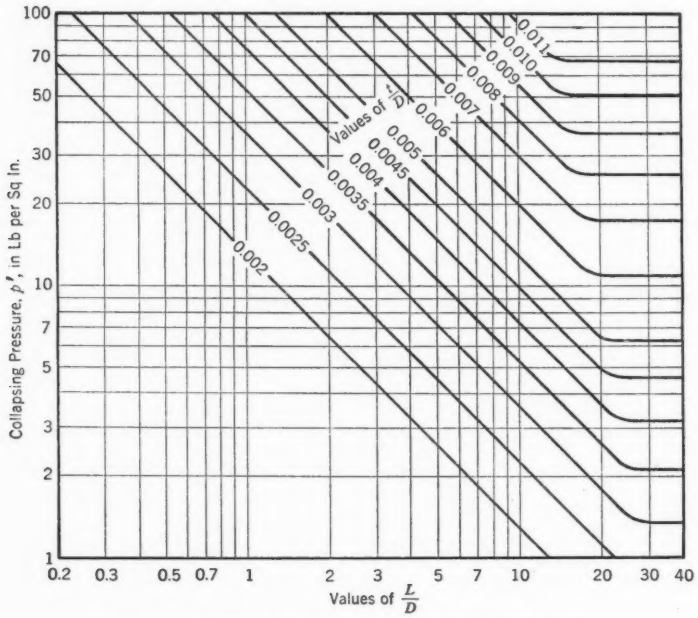

FIG. 1.—COLLAPSING PRESSURES OF STEEL PIPE LINES

inches. To determine the collapsing pressure of a pipe shell with stiffener rings, the $\frac{t}{D}$-ratio and the $\frac{L}{D}$-ratio must first be computed, after which the value of the collapsing pressure may be obtained from Fig. 1. If there are no stiffener rings, or if the $\frac{L}{D}$-ratio is large, the value of the collapsing pressure is given by the horizontal lines on the right-hand side of the chart.

Application of a suitable factor of safety, N, to p' will give p_2, the allowable pressure on the inside of the pipe. Thus,

$$\frac{p'}{N} = p_1 - p_2 \quad \text{.....(3a)}$$

or

$$p_2 = p_1 - \frac{p'}{N} \quad \text{.....(3b)}$$

in which p_1, the atmospheric pressure on the outside of the pipe, may be obtained from Fig. 2. Selection of a value of N depends on the judgment of the designer, who must also consider the probable frequency of the assumed operation which causes a reduction of the internal pressure.

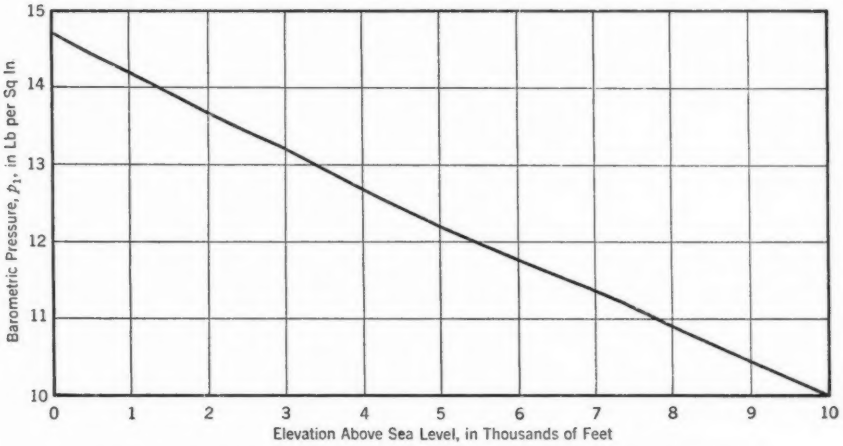

FIG. 2.—BAROMETRIC PRESSURES AT VARIOUS ELEVATIONS ABOVE SEA LEVEL

LIMITING RATIOS FOR WHICH NO VALVE IS REQUIRED

Inasmuch as the largest value of p' that can be exerted on an exposed pipe line is limited to atmospheric pressure, the limiting value of the $\frac{t}{D}$-ratio for a pipe shell without stiffener rings and the $\frac{t}{D}$-ratio and the $\frac{L}{D}$-ratio for a pipe shell with stiffener rings may be readily obtained from Fig. 1. For example, for a pipe shell without stiffener rings, with $N = 1$, at sea level, the value of t/D must not be less than 0.0067. For $N = 2$ at sea level, and $t/D = 0.003$, the required $\frac{L}{D}$-ratio for the stiffener ring spacing must not be greater than 1.3. On some pipe lines it may be found more economical to provide stiffener rings at the thinner sections than to install and maintain air-inlet valves at the various crests in the profile.

HYDRAULIC CONDITIONS AT THE VALVE LOCATION

Under normal conditions of operation the flow in the pipe line is essentially uniform and the hydraulic gradient is above atmospheric pressure throughout the entire length. If the flow is increased abnormally, as a result of a rupture in the line at a low point, or any other cause, the hydraulic gradient may drop considerably below atmospheric pressure at the crests in the pipe line profile.

External atmospheric pressure may then cause collapse of a considerable length of the pipe shell, although this collapse may be prevented by installation of an air-inlet valve of ample size to mantain the pressure inside the pipe at any desired allowable pressure, p_2. The hydraulic flow conditions that will exist inside the pipe at the air-inlet valve location with the valve open may then be determined from the following considerations: The minimum allowable internal pressure at the air-inlet valve is p_2, therefore, the pressure head at the air-inlet valve is $2.31(p_2 - p_1)$. The water velocities in the pipe line leaving and approaching the air-inlet valve location, designated as V_1 and V_2, respectively, may then be computed by means of the Bernoulli energy equation. If $V_1 > V_2$, or V_1 and V_2 are in opposite directions, an air-inlet valve is required to supply enough air to the pipe interior to occupy the space created by the breaking of the water column; that is,

$$Q = W v_2 = \frac{\pi D^2}{4 \times 144} \Delta V \quad \ldots\ldots (4)$$

in which Q is the quantity of air, in cubic feet per second, that must be supplied to the pipe interior at the pressure p_2; W is the weight of air flowing through the air-inlet valve, in pounds per second; v_2 is the specific volume of air, in cubic feet per pound, at pressure p_2; and

$$\Delta V = V_1 - V_2 \quad \ldots\ldots (5a)$$

It should be noted that, if V_2 is opposite in direction from V_1,

$$\Delta V = V_1 - (-V_2) = V_1 + V_2 \quad \ldots\ldots (5b)$$

Flow of Air Through Inlet Valves

The air drawn through the inlet valve will undergo a volumetric change as it passes from atmospheric pressure, p_1, to the lower pressure, p_2, inside the pipe. Ordinarily, this takes place so rapidly that there is little interchange of heat between the water and the air. Therefore, the air flow through the valve may be considered as an adiabatic expansion; that is,

$$144\, p_1 v^{\gamma}_1 = 144\, p_2 v^{\gamma}_2 \quad \ldots\ldots (6)$$

in which v_1 is the specific volume of air, in cubic feet per pound, at pressure p_1; and γ is the ratio, for air, of the specific heat at constant pressure to the specific heat at constant volume.

The weight of air per second flowing through an air-inlet valve is then given by the expressions:[3] For $p_2 > 0.53\, p_1$,

$$W = C \left(\frac{\pi d^2}{4 \times 144} \right) \frac{1}{v_2} \sqrt{2 g (144 p_1) v_1 \left(\frac{\gamma}{\gamma - 1} \right) \left[1 - \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} \right]} \quad \ldots (7a)$$

and, for $p_2 \leqq 0.53\, p_1$,

$$W = C \left(\frac{\pi d^2}{4 \times 144} \right) \left(\frac{2}{\gamma + 1} \right)^{1/(\gamma-1)} \sqrt{2 g \left(\frac{\gamma}{\gamma + 1} \right) \frac{144 p_1}{v_1}} \quad \ldots\ldots (7b)$$

[3] "Mechanical Engineers Handbook," by Lionel S. Marks, McGraw-Hill Book Co., Inc., New York, N. Y., 1941.

in which C is the coefficient of discharge for the air-inlet valve; d is the inside diameter of the valve, in inches; and g is the gravitational acceleration. Substituting Eqs. 4 and 6 in Eqs. 7, the ratio of the air-inlet valve diameter to the pipe diameter is, for $p_2 > 0.53\ p_1$,

$$\frac{d}{D} = \sqrt{\frac{\Delta V}{C}} \left\{ 2\,g\,(144\,p_1)\,v_1 \left(\frac{\gamma}{\gamma - 1}\right)\left[1 - \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} \right] \right\}^{-\frac{1}{4}} \quad \ldots\ldots(8a)$$

and, for $p_2 \leqq 0.53\ p_1$,

$$\frac{d}{D} = \sqrt{\frac{\Delta V}{C}} \left(\frac{p_2}{p_1}\right)^{1/(2\gamma)} \left\{ 2\,g\,(144\,p_1)\,v_1 \left(\frac{\gamma}{\gamma + 1}\right)\left(\frac{2}{\gamma + 1}\right)^{2/(\gamma-1)} \right\}^{-\frac{1}{4}} \quad \ldots(8b)$$

Eqs. 8 may be further simplified by using an average value for the term $[2\,g\,(144\,p_1)\,v_1]^{-\frac{1}{4}}$, as its numerical value is nearly constant regardless of altitude, air temperature, or humidity. The assumed average conditions will be

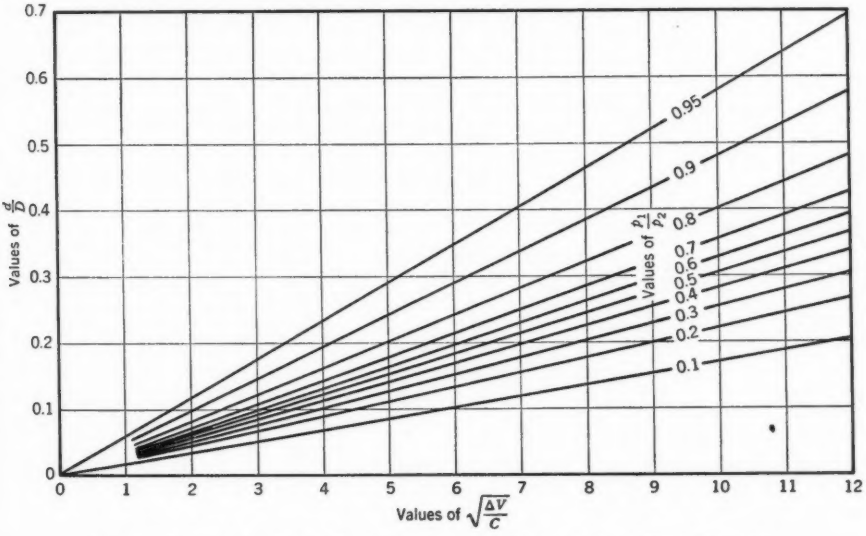

FIG. 3.—SIZE OF AIR-INLET VALVE REQUIRED TO PREVENT PIPE LINE COLLAPSE

taken at 40° latitude, an elevation of 1,000 ft above sea level, an air temperature of 75° F, and 20% humidity, for which $g = 32.155$ ft per sec per sec, $v_1 = 13.969$ cu ft per lb, and $144\,p_1 = 2{,}040$ lb per sq ft. These values for g, p_1, and v_1, together with a value of $\gamma = 1.405$ for air, may be substituted in Eqs. 8 to give, for $p_2 > 0.53\ p_1$,

$$\frac{d}{D} = 0.0199 \sqrt{\frac{\Delta V}{C}} \left[1 - \left(\frac{p_2}{p_1}\right)^{0.288} \right]^{-\frac{1}{4}} \quad \ldots\ldots\ldots\ldots(9a)$$

and, for $p_2 \leqq 0.53\ p_1$,

$$\frac{d}{D} = 0.0391 \sqrt{\frac{\Delta V}{C}} \left(\frac{p_2}{p_1}\right)^{0.356} \quad \ldots\ldots\ldots\ldots\ldots(9b)$$

Determination of the ratio $\frac{d}{D}$ from Eqs. 9*a* and 9*b* is facilitated by a chart (Fig. 3), which is entered with the calculated values of p_2/p_1 and $\sqrt{\Delta V/C}$.

If the value of C, the coefficient of discharge of the air-inlet valve and connecting pipe, is known, its value should be used in Eqs. 9. A value ranging from 0.5 to 0.6 is believed to be reasonable for the ordinary design of air-inlet valves.

Numerical Example

To illustrate the procedure to be followed in determining the size of an air-inlet valve required in an actual pipe line installation, consider the steel siphon whose profile is shown in Fig. 4. In the vicinity of the crest at point B the pipe shell is $\frac{1}{4}$ in. thick and there are stiffener rings spaced 30 ft apart. It is

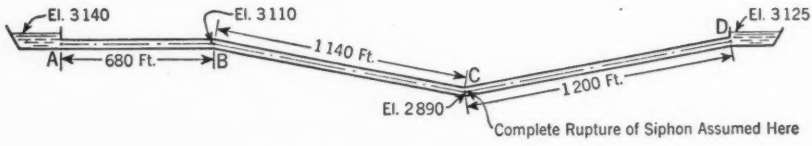

Fig. 4.—Profile of Siphon Installation

desired to calculate the size of the air-inlet valve at point B if a complete rupture of the pipe is assumed to occur at point C. The required factor of safety for this assumed condition of operation is taken as 2, and the coefficient of discharge of the air-inlet valve is taken as 0.5.

Value of p_2/p_1.—Given: $t = 0.25$ in., $D = 78$ in., $t/D = 0.0032$, $L/D = 4.62$, and $N = 2$. From Fig. 1, $p' = 9.2$ lb per sq in.; and, from Fig. 2, $p_1 = 13.1$ lb per sq in. By Eq. 3*b*, $p_2 = 8.5$ lb per sq in., and $p_2/p_1 = 0.65$.

Value of ΔV.—Using the Bernoulli equation for the pipe line from point A to point B, it is found that $V_2 = 27.0$ ft per sec.

Similarly, using the Bernoulli equation for the pipe line from point B to point C it is found that $V_1 = 54.7$ ft per sec. Thus, by Eqs. 5, $\Delta V = V_1 - V_2 = 27.7$ ft per sec.

Size of the Air-Inlet Valve.—Using the assumed value of C and the value of ΔV just found, $\sqrt{\Delta V/C} = 7.44$. From Fig. 3, $d/D = 0.26$. Therefore, the diameter of the air-inlet valve required at point C is $0.26 \times 78 = 20.3$ in.

Acknowledgments

The writer wishes to acknowledge the assistance of W. E. Evans and C. C. Crawford of the Bureau of Reclamation in the preparation of this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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REPORTS

ADVANCEMENT OF SANITARY ENGINEERING

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON ADVANCEMENT OF SANITARY ENGINEERING

This is a progress report. Because of the changing nature of the world and the consequent adaptation of engineering the better to serve its needs, it is unlikely that a final report can ever be written. Some committee on advancement of sanitary engineering should be continued indefinitely, and should itself be changed in membership and function to conform to the conditions of a changing world and an evolving profession. Certain suggestions for improvement in the committee's organization and service are given subsequently in this report. A brief abstract of the report has appeared elsewhere.[1]

This committee was created on January 16, 1934, and for the first year was designated "The Committee on the Promotion of the Status of Sanitary Engineering." At the ASCE Annual Meeting in 1935 the present title of the committee was adopted. Its membership has ranged from five to eight during several changes in the method of constituting the committee. Reports have been submitted to the Sanitary Engineering Division at the ASCE Annual Meetings in 1935, 1937, 1940, 1941, 1942, and 1946. The 1935 report was given verbally by the chairman.[2] The 1937 report was furnished in mimeographed form, but was not published. The report of 1940 was informal and oral, and was for the primary purpose of evoking discussion on several topics under consideration by the committee. A formal report was presented to the ASCE Fall Meeting at Cincinnati, Ohio, later that year, where it was reviewed and discussed. A revision of this report, written with the advice of some leading sanitary engineers from other societies, was presented in 1941, but was not published. However, the 1940 and 1941 reports, after extensive discussions and revisions, resulted in the very comprehensive and lengthy report of 1942,[3] which, because of its formality and wide scope, was designated as a "completion" report. This report set forth the views and recommendations of the committee as then constituted on such a range of subjects as the following: The problems and activities of the sanitary engineer; his education

NOTE.—Please forward all comments on this report directly to Chairman Herman G. Baity, School of Public Health, Univ. of North Carolina, Chapel Hill, N. C.

[1] *Engineering News-Record*, March 31, 1949, p. 49.

[2] *Civil Engineering*, April, 1935, p. 229.

[3] *Proceedings*, ASCE, June, 1942, p. 965–995.

and training; opportunities for his professional activity and public service; appraisal of potential future extensions of his services; and methods of improving his status and opportunity.

As a result of the intervention of World War II the committee was inactive from 1942 to 1945. Then the reconstituted group decided on the principle of confining its future progress reports to the topics and problems which at the time appeared to be of greatest timeliness and import to the profession. Following this idea, the committee's report of 1946 was on the theme: "Opportunities for Sanitary Engineers in the Postwar Period."[4]

During 1948 the committee held three meetings and also has encouraged an exchange of ideas by correspondence. It has decided to continue the principle of discussing in its report a limited number of topics of special importance and pertinence to the quality of sanitary engineering service rather than to attempt to prepare a comprehensive encyclopedic coverage of the field, or to discourse on factors that may be primarily for the personal benefit of sanitary engineers. The subjects that follow are believed to be of more than usual significance at the present time.

Trends in Sanitary Engineering Education

The advent of social security in 1936 and the accompanying provision of funds for the training of needed public health personnel have exerted a profound influence on the education of sanitary engineers. The pattern of education that is developing is only now becoming discernible, because much of the development was masked during the first five years by the temporary and developmental character of the initial objectives of the educational program, and during the next five years by the urgency of superimposed and unrelated military needs. Programs of instruction and research in the following four fields appear to have been in the making:

1. Sanitary engineering;
2. Industrial hygiene;
3. Public health engineering; and
4. Environmental sanitation.

The nature and objectives of these programs are outlined as follows:

1. Sanitary Engineering.—Programs of instruction and research in sanitary engineering are historically the oldest. Originating as undergraduate options in civil engineering, they have successfully broken away from the parent stem only by being placed on a graduate basis. Although this separation first took place many years ago, it did so only in a limited number of institutions and the current structure of graduate instruction did not crystallize until the emergency call for qualified sanitary engineers was issued in 1936. With but one year of training permitted under the Social Security Act, the education of graduate civil engineers in the principles and practice of sanitary engineering provided the only rapid measure for meeting the emergency. Apparently this scheme fits sufficiently well into the pattern of engineering education in the

[4] *Proceedings*, ASCE, May, 1946, p. 647–654.

United States to become more or less fixed and generally accepted—in addition to the continuation of a limited amount of instruction in sanitary engineering in connection with undergraduate programs in civil engineering.

The programs of instruction and research in sanitary engineering that are based on a first degree in civil engineering attempt to offset the primary deficiencies of the civil engineering curriculum by including instruction in bacteriology, parasitology, and chemistry. Some criticism is directed against this practice because of the elementary nature of the subject matter dealt with in comparison with the subject matter based on the engineering sciences that are included in undergraduate civil engineering programs. Two alternatives are being explored to raise all graduate instruction in sanitary engineering subjects to a more obviously advanced level: (*a*) The inclusion of needed preparation in the elementary biological and chemical sciences in the undergraduate program of civil engineering; and (*b*) the requirement that prospective students make up deficiencies before beginning graduate work—for example, by a summer of intensive instruction offered specifically for this purpose.

Although the expansion of instruction in sanitary engineering is traceable to the needs of public health departments, most programs of instruction in sanitary engineering are formulated about engineering subjects rather than about public health subjects for two reasons: (*a*) Adequate knowledge of essential engineering subjects cannot be acquired without effective guidance and laboratory work, whereas the limited number of public health subjects that are of importance to engineers are readily acquired by self-study; and (*b*) programs in sanitary engineering are intended for students who may wish to plan, design, construct, and operate sanitary works as well as for students who are to take part in the administrative control over such works through governmental health agencies.

2. Industrial Hygiene.—Programs of instruction and research in this field are relatively new. Except for industrial safety, these programs have their origin in industrial medicine and continue to be closely associated with this field and that of applied physiology. Need for the prevention of industrial diseases and the creation of satisfactory working conditions eventually brought the engineer into the field. Because much engineering activity is associated with the control of air in relation to its temperature, humidity, motion, pressure, and harmful (largely chemical) impurities, mechanical and chemical engineers were found ordinarily to possess the best preparation for work in industrial hygiene. Like civil engineers in relation to sanitary engineering, however, mechanical and chemical engineers are generally lacking in basic preparation in the biological sciences (bacteriology and physiology). Mechanical engineers, furthermore, seldom possess an adequate knowledge of chemistry. The dilemma of introducing elementary preparatory subjects at the graduate level recurs, therefore. Graduates of programs in industrial hygiene find employment in governmental agencies concerned with industry and public health, in insurance companies, and in industry itself, including mining as well as manufacturing. As yet the field of industrial hygiene has not attracted large numbers of engineers and educational opinion in relation to it is still quite fluid.

3. Public Health Engineering.—Although at least one accredited program in public health engineering was in existence in 1936 in an engineering school (later discontinued) and although some schools of public health had admitted a handful of engineering graduates to study (and even to degrees in) public health, the principal impetus to the development of programs that would presumably prepare engineers specifically for work in public health departments was derived from the Social Security Act, and through public health rather than university authorities. The public health authorities—although conversant with the public health aspects of needed engineering undertakings and aware of the fact that civil engineers who had not taken work in sanitary engineering were lacking in needed knowledge—were not qualified to judge the inadequacy of the sanitary engineering preparation possessed by civil engineers. As a result they believed that programs of instruction in public health closely paralleling those devised for medical, nursing, and laboratory personnel would "fill the bill" for engineers too. At the same time, furthermore (as is always true when practitioners have their "say" in educational matters), the public health authorities stressed the vocational aspects of instruction and included even practical experience in the field as part of the educational system. The resulting programs of instruction contain so little material that is founded on engineering that nonengineering graduates of programs in general science have had no difficulty in mastering the subject matter of instruction and are often admitted to the same graduate degrees as engineers. Such degrees, obviously, have no engineering basis.

Since the instruction offered by schools of public health is not subject to accreditation by the Engineers' Council for Professional Development, there is no check on the engineering content of such programs in public health engineering.

4. Environmental Sanitation.—Programs of instruction in environmental sanitation are newcomers to the educational field. Started as short courses, and still continuing as such here and there, the most ambitious curricula include either four years of undergraduate work in general science and sanitary technology, or a year of graduate work in the latter subject. The best-designed undergraduate programs incorporate sufficient instruction in mathematics and physics to permit an engineering-minded student to make up his deficiencies by about two years of further study, and eventually to become a qualified engineer.

Programs of instruction in environmental sanitation are being developed in liberal arts colleges and even in some medical schools. Graduates are expected to find employment as sanitarians who presumably will work under qualified engineering and medical direction. Since there is a recognized need for personnel of this kind, the educational opportunities that are being set up are justified provided that the programs have proper content and competent direction and instruction. Although there are some dangers that ambitious "sanitarians" will eventually attempt to dilute the profession of sanitary engineering, or that public health agencies will substitute sanitarians where engineers are needed or will employ cheap subprofessional sanitary inspectors in place of competent sanitarians, these are risks that are perhaps best met, not

by opposition to the training of sanitation personnel, but by direction of education and by proper incorporation of such men into the engineering structure as ancillary technical personnel.

From a limited point of view, ASCE should be interested only in the welfare of sanitary engineers. The threat to the unity of sanitary engineering created by current attempts to split off public health engineering from sanitary engineering on the basis of differences in educational needs and offerings, however, is of grave concern to ASCE. The educational and professional progress of the sanitarian is of interest, and should be not only watched but also promoted in such fashion as to insure the advancement of sanitary engineering.

Sanitary Engineering in Governmental Agencies

The complexities of modern society increase the dependence of the average citizen on the protective safeguards afforded by governmental controls. The freedom of isolation that once permitted a maximum of self-determination and decision is disappearing with increased urbanization, and now the safety once afforded by isolation is secured through restrictive controls.

Environment can no longer be chosen freely. It is constantly being changed and modified by others, and individuals can do little or nothing about it, unless they seek protection through governmental group action. It is a well-recognized primary function of government that it protects its citizens from disturbing factors that lie beyond individual control. When the individual can no longer secure the environment needed for health and comfort through his individual action, he finds himself dependent on the protection that can be obtained through group effort expressed as a function of government.

Because sanitary engineering is basically concerned with problems of environmental control that relate directly to the securing of better public health and comfort and because increase in urbanization multiplies these problems, it follows that the future may be expected to bring a corresponding increase in the sanitary engineering problems that will require solution.

The success or failure of the sanitary engineering profession to measure up to this increasing responsibility will depend to a great extent on the ability of those in the profession clearly to envision future problems and to develop sufficient technical competence and capacity for leadership to be effective in the formulation of such governmental policies as may be required to maintain environments that are safe and wholesome from a public health standpoint.

Sanitary engineering problems have developed rapidly during the past several decades. Many of these problems, now unduly complex and complicated, are the result (in part at least) of the lack of sound sanitary engineering advice that should have been given at the policy-making levels of government. Too frequently sanitary engineers have been utilized by governmental agencies to help correct problems and to cure situations that should never have been allowed to develop.

No prediction can be made with regard to how the sanitary engineer of the future will measure up to the responsibilities that will be his, not by any right imposed on him by a fiat act of government but rather by the more

basic right, that of being the best qualified person available to provide a needed service.

The committee is optimistic in its belief that society will afford the qualified sanitary engineer an opportunity to render service at all governmental levels, provided only that he continue to demonstrate the worthwhileness of his services.

Sanitary Engineering in the Military Services

The opportunities for effective sanitary engineering service in the military establishment are not good, at this time—primarily because, in the postwar reorganizations of 1946 and 1947, the needs for sanitary engineers were largely disregarded.

The United States Army.—In the Army there are the following three categories in which, in the past, sanitary engineers have been utilized:

(*a*) The Corps of Engineers.—The Officer Personnel Act of 1947 made vital changes in the Corps of Engineers. There are no education and experience requirements except for employment on the Civil Works Program. Otherwise, the Corps of Engineers is no longer a professional group, and it appears very difficult for a man to remain an engineer and to receive the promotions he must have to retain his commission. The officer on duty with troops is always more advantageously placed. There is no specific place for a sanitary engineer officer.

(*b*) Civilian Personnel.—Although civil service ratings are low, a moderately adequate staff of sanitary engineers is being maintained and the work is apparently well handled. There are some positions overseas that offer experience and from fair to low pay opportunities.

(*c*) Medical Department.—Under the Medical Service Corps Act, passed in 1947, sanitary engineers are grouped with entomologists and other allied medical scientists, as well as with pharmacists, optometrists, and supply and administrative officers, into a Medical Administrative Corps. This corps, unlike the Medical, Dental and Veterinary Corps of the Medical Department, does not rate a general officer for each complement of approximately 200 officers. Currently, furthermore, this corps is permitted only 2% full colonels against 8% for the remaining branches of the Army. The Environmental Sanitation Branch of the Preventive Medicine Division of the Surgeon General's Office is headed by an officer in the Medical Corps who is assisted by an officer detailed to the Surgeon General's Office by the Sanitary Engineering Section of the Medical Service Corps.

The United States Air Force.—The problems of organizing this new component of national defense have not been solved. No one can foresee the developments of the near future. For civilian employment, opportunities are probably about the same as in the Corps of Engineers.

The United States Navy.—The Navy does not utilize sanitary engineers as commissioned officers as far as is known. There are a few openings for civilians as sanitary engineers.

The Public Health Service.—Expanding postwar duties, including the Taft-Barkley Stream Pollution Act, plus a sounder organization of the Sanitary Engineering Division make the Public Health Service especially attractive to sanitary engineers. High standards of education and experience are required for appointment; a big job must be done. For the sanitary engineer who desires to enter one of the services, the Public Health Service offers by far the soundest and most satisfactory opportunity.

The Future.—The expansion authorized in the Army, Navy, and Air Force will certainly result in an increased need for sanitary engineering services. However, there is presently no reason to believe that the conditions of employment and service will have much appeal for qualified engineers. In the circumstances temporary assignments to active duty, too, are not attractive. The only benefit to a reserve officer will be that the service rendered will count toward his eventual retirement under a law passed by the Eightieth Congress.

The postwar reorganization of the national defense establishment appears to represent in large part the resentment of the combat officers against the staff organization that existed during World War II. No doubt the pendulum has swung too far. Until it returns, and the necessity for efficient utilization of technological personnel is again recognized, none of the armed services offers any attraction to the sanitary engineer. It may be years before an adequate balance is again achieved. It is not a question of any vested interest or right that the sanitary engineer may have in the military establishment. He has no such right. It is a question of proper and efficient organization to deal with the manifold problems of environmental health in the Army, the Navy, and the Air Force. Under present conditions there is no incentive to draw qualified personnel into these services or to build a competent sanitary organization in any of them. This situation is serious in peacetime; it might prove calamitous in wartime.

Sanitary Engineers in Consulting and Commercial Organizations

During the depression years of the 1930's, many water works and sewage works projects were held in abeyance because of lack of funds for their construction, and proper maintenance of existing works was deferred for the same reason. During the war years that followed, materials and labor were so scarce that construction of sanitary works was limited almost exclusively to cantonment areas and war industries. A great backlog of normal municipal construction was therefore built up which must be satisfied in this postwar period. In anticipation of possible extensive unemployment at the close of World War II as a result of shutting down war industries, numerous plans were prepared during the closing years of the war in a comprehensive postwar planning program so that contracts for the construction of sanitary works might be let as soon as the war was over. Contrary to expectations, no postwar depression has developed. In fact, business has grown increasingly better since the war. Materials and labor which have been released from war use for the construction of municipal sanitary facilities are inadequate to meet

the demand, with the result that the backlog of needed construction is increasing rather than decreasing.

For a number of years prior to the war, there was a growing public demand for the abatement of pollution in the streams and public waters of the United States. As a result of this demand, legislation has now been enacted in many states, giving the state agencies authority to compel the construction of treatment plants for sewage and industrial wastes. Similar federal legislation which has been under consideration in Congress for many years has also been enacted. This legislative activity has resulted in a great acceleration in pollution abatement programs both by municipalities and by industry.

During the war considerable development work was done in the fields of water treatment and waste treatment largely because of the availability of federal funds to subsidize such development. Most of this work was carried on by commercial and manufacturing interests under the protection of patents. For example, great advances were made in the demineralization of water by synethetic resins. Some other developments were the diatomaceous earth filter for water purification and numerous proprietary processes for the treatment of sewage and industrial wastes. Many of these developments were designed for military use or for use with war industries. During the postwar period there has been a concerted attempt to adapt these new developments to peacetime use in competition with older methods. It has been found, however, that many of these new developments which were eminently successful for wartime use, where cost was no deterrent, are too expensive for normal peacetime use. Nevertheless, the effort to adapt these new developments to normal use has resulted in high-pressure sales methods in commercial organizations. There is evidently a great opportunity for competent sanitary engineers to develop further some of these new methods and processes in an effort to reduce the costs within competitive range of other processes and to find projects for which these new developments are peculiarly suited.

The expanding activity both in the United States and throughout the world in water works, waste disposal, and pollution abatement finds the profession woefully lacking in competent sanitary engineers. For more than two decades, the need for sanitary engineers has greatly exceeded the supply and the situation has grown progressively worse. Because of the current expansion of opportunity in water works and sewage works construction and pollution abatement, many consulting engineers enter this field who formerly did no work in sanitary engineering, and many manufacturing and commercial organizations enter the field who have little or no background for competent service. Some of these consultants and commercial organizations have wisely employed experienced sanitary engineers to assist them, but the supply of competent men is inadequate for the situation.

New Fields of Service for the Sanitary Engineer

Among the most important new tasks to which the sanitary engineer may be expected to address himself are those recently defined in the field of nuclear fission, and the yet undelineated aspects of bacterial warfare. In regard to the latter, one can only speculate on the basis of scanty public information.

However, a perusal of the "Official Report on Biological Warfare" by George W. Merck[5] suggests the probability that sanitary engineers will be called on to devise and operate defensive measures. Among disease agents of possible use, which include those affecting man, animals, and farm crops, those related to food, water, and air-borne transmission would be of special concern to public health workers.

However, there has been no intimation yet—at least publicly—of the kind or scope of probable protective measures. It is not unlikely, however, that conventional tests for the detection of contaminants would have to be re-examined to provide more positive methods for detection of pathegenic organisms during periods of possible attack; and, since the methods adopted for transmission of some diseases could be far different from those used under natural conditions, entirely new techniques for sanitary control might be required.

Nuclear Fission Problems.—In regard to the rôle of the sanitary engineer in problems associated with nuclear fission operations, substantial progress has been made in recent months toward a clearer definition and response. The Atomic Energy Commission (AEC), under whose purview fall all matters relating to such operations, has now found it desirable to acquaint the profession with several aspects of these problems as related to environmental sanitation. In so doing the AEC reversed a policy wherein it appeared that only the most limited participation of the profession in the development of safeguards would be permitted.

It was argued aggressively and persuasively by leaders in the profession that protection of man and his environment against the hazards of radioactivity fell into areas of responsibility that could be serviced by the sanitary engineer. Therefore, it followed that the sanitary engineer should be made acquainted with the problems of nuclear fission operations, especially those relating to the disposal of wastes to the atmosphere, to the soil, and to surface waterways.

Acceptance of this thesis by the AEC has resulted in frank and challenging discussion by engineers associated with the commission. Thus, A. E. Gorman and Abel Wolman, Members, ASCE, have concluded that:

> "* * * (1) the continued expansion of a nuclear fission industry presents many problems and, therefore, an opportunity and a challenge for the trained sanitary engineer; (2) the Atomic Energy Commission is striving actively for a permanent solution of its waste disposal problem; (3) its objective is that this new industry may expand to meet its potentialities without creating objectionable or hazardous conditions at and in the vicinity of its plants as many new industries have done; (4) public health workers should be trained in fundamentals of nuclear physics so that they may have a better understanding of this new industry and its environmental problems; and (5) the need of training sanitary engineers is especially acute considering their responsibilities in a national defense program."[6]

[5] "Official Report on Biological Warfare," by George W. Merck, *Bulletin of the Atomic Scientists*, Vol. 2, Nos. 7 and 8, October 1, 1946.

[6] "Some Public Health Problems in Nuclear Fission Operations," by Arthur E. Gorman and Abel Wolman, *American Journal of Public Health*, April, 1949, pp. 443–453.

Industrial and Commercial Needs.—More orthodox tasks, which nevertheless represent opportunities for the sanitary engineer to extend his usefulness, are those associated with industrial and commercial needs.

High on such a list are the requirements of industry for men who can cope with stream pollution abatement measures. The United States is on the threshold of an era in which top management of industry will be found increasingly sympathetic to the correction of waste disposal abuses. In part, this trend represents the effect of many years of education and agitation directed toward stream cleanup, followed by legislative action in the form of stronger regulatory measures. Equally important has been the growing responsiveness of industry toward taking corrective steps. This is not motivated simply by a higher sense of public duty, but stems also from industry's realistic knowledge that protection of the quality of water supply is essential if industry is to continue operating in its present locations.

Research directed toward development of industrial waste treatment methods coupled with the possible salvage of by-products is already a field of endeavor in which sanitary engineers are finding profitable satisfaction for themselves and in which they are providing important benefits for their employers.

That there are unique opportunities for pioneering in the industrial field is illustrated by work on sewage disposal facilities for rail passenger cars. Many elements of sanitation in airplane transportation likewise command study; and the dimensions of this problem, involving as it does international contacts, are even broader than those relating to the railroads.

The sanitary engineer also finds ready identification with other developments of a world-wide character. Notable examples are provided by the operations of the United States oil companies in foregin lands and by other types of enterprise in which productivity, and hence profit, depend on the development and maintenance of high standards of environmental sanitation. Thus in the program of The Liberia Company, organized as an American private venture to develop that country, a health and sanitation foundation is an essential element and will be financed with 10% of the profits of the company.

The wartime work of the Health and Sanitation Division of the Institute of Inter-American Affairs already has made a profound impact in Latin America; and, in the first meeting of the World Health Organization, sanitary engineering projects were accorded a high place on the agenda of future activities.

National Research Council Activities.—In 1942 a Committee on Sanitary Engineering and Environment of the National Research Council was convened "to render advice and assistance" to agencies concerned with national defense. Operating as it does in a twilight zone where the glare of publicity is shrouded in the fog of secrecy, the committee and much of its work seldom have been topics for discussion because of security reasons. Nevertheless, it has served to coordinate the resources of the entire profession, and its recommendations are exerting a wide influence on technical practice as well.

In this connection it will be recalled that one of the committee projects was a study pointing to the desirability of maintaining a chlorine residual in water distribution systems. Recommendations of the committee in this

regard were adopted at all Army installations and have since influenced practice in many municipal operations. A notable postwar undertaking of the committee was the compilation of all information relating to sewage treatment at military installations.

The Need for Sanitary Engineering Research

The responsibility for the development of the art of water treatment, treatment of wastes, and pollution abatement rests squarely with the sanitary engineering profession. The public looks to the professional sanitary engineer as the legitimate expert in this field. If he is to maintain this position, he must support research and development actively and he must be prepared to examine and test new developments with competence. Sanitary engineering is by tradition a branch of civil engineering but it differs vitally from general civil engineering in that it deals primarily with processes, whereas the latter deals primarily with the building of structures or works. The nature of his work requires that the sanitary engineer be conversant with developments in chemistry, biology, and fluid motion so that he may adapt them to sanitary engineering. The process of adaptation requires fundamental research, applied research, and development.

Prior to World War II most fundamental scientific research was done in Europe. The great industrial development in the United States was due primarily to the adaptation and engineering use of fundamental scientific principles discovered by Europeans. The American contribution to fundamental knowledge was pitifully small. During the war fundamental research in Europe was deferred in favor of active development of newer scientific knowledge for war use. The devastation brought by the war in Europe has practically obliterated scientific centers on the continent and the prospect of basic scientific research in Europe is very poor for the immediate future. If such work is to be conducted anywhere in the world, it must be done in the United States, since this country is the only one which now has adequate facilities and financial resources. During the war the National Defense Research Council was established to aid the armed forces in the development of armament, equipment, and other needs for waging the war. The work of this agency was largely applied research but it was nevertheless responsible for the development of such important items as the atomic bomb and radar. The federal government and American industries are fully alive to the responsibility that must be assumed by the United States in promoting both fundamental and applied research. In line with this obligation there is now under consideration by Congress a bill to establish a National Science Foundation. This organization would be similar to the National Defense Research Council, except that its efforts would be devoted primarily to peacetime research of a fundamental nature. Much of the research work conducted by various governmental agencies during the war primarily for war purposes has been continued for peace purposes and some of it has been expanded. The National Science Foundation, if established, would doubtless coordinate all these activities.

Of particular interest to sanitary engineers is the program of research being promoted by the National Institute of Health through its Division of Research Grants and Fellowships. Grants-in-aid are available for worthy projects relating to public health and comfort. To be eligible for a grant-in-aid, a project must be basic or fundamental research and must be supervised by a competent person. Projects which are of a sanitary engineering nature are reviewed by the Sanitation Study Section of the National Institute of Health. This program has been under way for about two years and during that period 143 projects requesting grants-in-aid in the amount of $1,704,278 have been reviewed by the Sanitation Study Section, out of which 62 in the amount of $553,646 have been approved. The explorations of the Sanitation Study Section during this period have established without question a great need for basic research and development in the field of environmental sanitation. One of the significant findings of the Sanitation Study Section is the dearth of properly qualified engineers and scientists to undertake basic research in this realm. Many of the applicants for grants-in-aid appear unable to distinguish between basic research and ordinary routine surveys and testing. One of the important aims, therefore, of the Research Grants Program is to discover and to develop talent for basic scientific research in the field of environmental sanitation.

The experience of the Sanitation Study Section reveals that most of the talent capable of undertaking basic research in environmental sanitation is connected with the universities. This finding may be considered a reflection on the interest as well as on the competence of practicing sanitary engineers. It is hoped that more consulting engineers will take an active part in research and development, and it is vital that they do so if they are to maintain their leadership in the sanitary engineering profession.

The Sanitary Engineer Must Sell His Services on Merit

For generations most of the design and the supervision of construction of sanitary engineering works has been in the hands of consulting engineering firms which for the most part conduct a general practice in the field of civil engineering. Most of these firms do not have adequately trained sanitary engineers on their staffs. As previously indicated, this situation is growing worse because of the entrance into the field of sanitary engineering of still other firms who formerly did no work in this field. Largely because of the absence of properly trained sanitary engineers on the staff of consulting engineering firms, commercial organizations dealing with processes and equipment for the treatment of water and sewage have increased their practice of selling directly to clients. It has long been the practice of manufacturers and private industries who have process water problems and boiler water problems to buy directly from commercial organizations without the advice of private consulting engineers. This practice has doubtless developed because industrial management, generally, has failed to differentiate between the value of advice offered by the seller of a process or piece of equipment and the advice given by a competent and unbiased consulting engineer. Now that industry is being compelled to take part in the pollution abatement program the manu-

facturers are continuing to look to commercial organizations for the solution of their waste treatment problems rather than to private consulting engineers. This situation is unsatisfactory for the manufacturing industries, because it denies them the opportunity to explore alternate possibilities and frequently leads them to the purchase of an unsatisfactory process or unsatisfactory equipment. It also denies to them the possibility of joint action with other manufacturers or municipalities whereby the cost to them might be reduced or charged off as operating expense such as would be the case with a municipally financed project.

The consulting engineer has a great deal to offer to manufacturing industries in the solution of their water treatment and pollution abatement problems, but he must sell his services to them on merit. It is obvious that he cannot sell his services on merit unless he is a well-qualified sanitary engineer and has an over-all grasp of the essentials. He must be able to view a problem as a whole, to investigate competently all the alternate solutions, to be fully aware of the merits of the newer processes and equipment being offered by commercial interests, and to be able to test and develop these processes and the equipment so as to adapt them to the particular problem at hand. Consulting engineers whose practice has been primarily with municipalities customarily avoid the use of new patented processes or equipment because of the legal compulsion of competitive bidding on construction contracts. These engineers are in general not prepared for pilot plant and laboratory testing of new processes and they are not prepared to pass upon the merits of new processes. Such an attitude on the part of the consulting engineer is not to the best interest of his client, even though that client is a public agency, because it tends to deny to the client the use of a newer and better process or piece of equipment. It is essential that consulting engineers who practice in the field of sanitary engineering be prepared to test and develop new processes. Services of this type are badly needed, and if they are not furnished by the consulting engineers they will be furnished directly to the client by proprietary interests.

The Scattered Allegiance of Sanitary Engineers

Sanitary engineers have no single professional home. Because of their broad interests they are scattered through a dozen or more technical, scientific, and engineering organizations, with the choice of their allegiance depending in large measure on their particular interests and pursuits. Some belong to several of these societies; others to only one or two. Many sanitary engineers probably participate in none. Unfortunately there is not enough coordination or cooperation between these various groups looking toward the improvement of either the technical competence or the professional status of the sanitary engineer—there is no united front or common purpose. This situation may stem from a number of causes, including inertia, diversity of interests, and the absence of some cohesive influence to bring a measure of unity to this branch of the engineering profession. Perhaps the greatest possible service that can be rendered to sanitary engineers and to sanitary engineering would be to bring these diverse professional groups into closer acquaintanceship and understanding. An independent central organization of sanitary engineers would

probably serve little purpose. However, it is believed that great good can be accomplished by joint action of representatives of these various organized groups, with the ASCE acting as the stimulating and motivating agency. Notable examples of accomplishment through joint action of intersociety committees are to be found in the Engineers Council for Professional Development and the Engineers Joint Council.

This committee in a former report[3] called attention to the peculiar dispersion of sanitary engineers throughout many organizations, and the need for bringing unity to these various groups. A recommendation was made that steps be taken to coordinate their interests and activities. Perhaps, in part because of the intervention of war, and in part because of the generality of the recommendation, nothing has been done to carry it into effect. The present committee wants to be more specific in its recommendation and more definite in its suggestion for implementing it, and it urgently requests that these suggestions be brought to the attention of the Executive Committee of the Sanitary Engineering Division and the ASCE Board of Direction for action as soon as possible.

It is recommended that the present divisional Committee on the Advancement of Sanitary Engineering be discharged and that a new joint committee of the same name and purpose be authorized, but with representation to be invited from the following organizations in addition to the Sanitary Engineering Division, ASCE: The Engineering Section of the American Public Health Association, the American Water Works Association, the Federation of Sewage Works Associations, the American Chemical Society, the National Society of Professional Engineers, and the American Society for Engineering Education. There may be others which should be included. It is suggested that the representation from each of these associations be determined by the Executive Committee of the Sanitary Engineering Division, ASCE, and that the joint committee be authorized to organize and conduct its business in the same manner as other similar intersociety committees.

It is believed that over the years the contributions of such a joint committee can be immeasurably greater and more effective in advancing sanitary engineering practice than could the best efforts of any committee representing only a part of this professional group.

Respectfully submitted,

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Committee of the Sanitary Engineering Division on Advancement of Sanitary Engineering

January 20, 1949

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

REFUSE COLLECTION AND DISPOSAL

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON REFUSE COLLECTION AND DISPOSAL

The present committee was appointed in 1947 by the Executive Committee of the Sanitary Engineering Division with the stated purpose:

> "* * * to evaluate present practice in the collection and disposal of refuse and other community wastes, to inquire into the newer processes available for disposal, and to study the use of chemicals and other agents for the control of vermin, odors and other possible nuisances."

A large assignment such as this can best be covered by a series of progress reports. The first of these reports will concern itself with the results of preliminary surveys; outlines of the steps to be taken in completing broader suveys for the collection and analysis of significant data on methods, costs, and trends; and plans adopted by the committee for achieving some of its assigned objectives.

In the period from 1930 to 1949 the art of refuse collection and disposal has witnessed many changes. These may be attributed principally to the renewed application of modern engineering principles and practices to an old problem which was too often neglected by municipal engineers. Mechanical devices for loading and packing refuse into collection trucks, the use of modern earth-moving equipment and other practices to make land fills sanitary, the mechanical stoking of incinerators, and the availability of chemicals for insect and odor control—all of these have contributed markedly toward increasing the efficiency, economy, and safety of this important component of engineering for the public service.

The size of the undertaking that faces sanitary engineers and others engaged in refuse collection and disposal may best be realized by referring to "Nation-Wide Inventory of Sanitation Needs" published by the United States Public Health Service (U.S.P.H.S.) in 1948.[1] This survey shows that, throughout the United States, a total of $166,526,000 should be spent in approximately 8,000 communities for adequate disposal of refuse. These costs are on the basis of an *Engineering News-Record* (ENR) index of 348. Since the ENR index at the end of 1948 is approximately 460, the estimated cost of furnishing adequate refuse collection and disposal facilities would be approximately $220,000,000.

NOTE.—Please forward all comments on this report directly to Chairman Rolf Eliassen, Dept. of Civ. Eng., Massachusetts Inst. of Technology, Cambridge, Mass.

[1] "Nation-Wide Inventory of Sanitation Needs," *Supplement No. 204*, Public Health Repts., U. S. Public Health Service, U. S. Govt. Printing Office, Washington, D. C., 1948.

The U.S.P.H.S. has broken down the items required to include 12,535 refuse collection trucks, 1,090 incinerators, and 5,549 land fill installations. These needs represent those of 8,369 communities having an aggregate population of approximately 70,000,000 persons. It is immediately evident that much work lies ahead in this field for engineers with special training and interest in the application of engineering principles to the collection and disposal of refuse of all types.

Refuse Collection

Refuse collection is fundamentally a problem of engineering management, requiring a high degree of coordination between personnel and equipment for economical operation and the maintenance of high standards of public health. The responsibility for refuse collection belongs to the municipal government, even though many communities depend upon private contractors for these services. The many aspects of refuse collection have been surveyed by a special committee of the American Public Works Association (A.P.W.A.). Its findings were published in 1941[2] in a large volume entitled, "Refuse Collection Practice." This work is recommended to all engineers interested in the subject of refuse collection and is a complete analysis and guide to modern practice. This progress report will not attempt to cover any of the features presented by the A.P.W.A. since the work is so complete.

In 1948 the staff of *Engineering News-Record* made a rather complete survey of some 300 cities in the United States to determine pertinent facts regarding refuse disposal practice and costs.[3] The cities ranged in size from those of 5,000 population to the largest cities in the United States, which fact showed the predominant practice of collecting once or twice a week, with the greater number of collections during the summer months when the volume of ashes is low and when high temperatures are conducive to odor nuisances. Many cities make weekly or biweekly collections in residential areas, with more frequent service in apartment and business districts. Frequency of collection is the controlling factor in determining the number of collection vehicles, personnel, and costs.

In cities larger than 5,000, most of the refuse is collected by municipal employees, using municipally owned equipment. The ENR survey indicated that 54% of the cities surveyed utilized their personnel for this work. Contractors, having firm contracts with municipalities and paid by municipal funds, were utilized for the collection of combined refuse in 13% of the cities, whereas a total of 21% relied on municipal contractors to collect garbage only. Private collectors, paid by individual residents, usually at rates agreed upon by the municipal administration, were responsible for the collection of refuse in 9% of the cities reporting. Even though many cities rely on contractors to haul refuse, the trend appears to be toward more municipalities collecting refuse with their own personnel and equipment.

Developments in mechanical loading devices for collection vehicles have been quite rapid since 1934. The trend in most cities is toward the use of closed

[2] "Refuse Collection Practice," Am. Public Works Assn., 1941.

[3] "Refuse Disposal Practice and Cost," *Engineering News-Record*, Vol. 140, 1948, p. 712.

vehicles with facilities for elevating the refuse from the low point on a truck up into the body of the vehicle. Body capacities up to a maximum of 22 cu yd are utilized, but the average capacity of vehicles is probably between 6 cu yd and 8 cu yd. The selection of suitable vehicles for any municipality involves a study of all types and sizes of equipment available for this work and an analysis of the application of each type of equipment to local conditions which affect the problem of refuse collection. Sanitary conditions, reliability of operation, ease of loading and unloading, and safety for the loading personnel must all be considered in the selection of the most effective and economical equipment for any municipality.

Some equipment manufacturers are now providing mechanisms that will compact the refuse within the truck body at the same time as the refuse is being loaded into the vehicle. In this manner it is possible to decrease the total volume hauled to the disposal site from one and one half to three times, depending on the type of refuse being collected. A number of cities are using these packing devices. This committee could well make a study of the economies and practicability of packing and loading devices on refuse collection trucks, based on actual operating experience in representative cities throughout the United States.

One of the serious situations that has arisen in the selection of mechanical loading equipment on refuse collection trucks is the necessity for the purchase of standard designs, many of which were prepared for large cities such as New York, N. Y., and Chicago, Ill., to satisfy conditions in those cities. One city tried to modify the New York standards by providing for loading the trucks from both sides of the vehicle instead of from the center of the back, and thus facilitate collecting from both sides of the street simultaneously. It was found that the cost of manufacture to accomplish these changes on standard vehicles would be prohibitive.

A definite need has arisen for a series of standard designs of collection vehicles to be promulgated by cooperative effort on the part of organizations such as the ASCE and the A.P.W.A. If these standard designs could be developed in cooperation with equipment manufacturers it might be possible to meet the needs of smaller cities for special-purpose equipment at reasonable cost without having to improvise on equipment designed for other purposes. This cooperative committee could also evaluate the effectiveness of existing types of equipment for various purposes and enable municipal engineers to purchase more judiciously.

Refuse collection presents a special problem in each municipality. The enactment of reasonable ordinances, the establishment of collection frequencies, the planning of collection routes and procedures to utilize personnel and equipment to the highest degree of effectiveness, the persuasion of city officials to appropriate sufficient funds for more personnel and new equipment where needed and for adequate maintenance of existing equipment—all of these factors, and many more, can present a challenge to the ingenuity and resourcefulness of the best municipal engineers. Refuse collection is an engineering problem. As such the salaries allotted to the directors of this work should be sufficient to attract and hold capable engineers.

Incineration

Incineration is one of the most satisfactory methods of destroying combustible refuse when the units are properly designed and operated. Incinerators may be located at points convenient to collection areas to minimize the length of haul and thus reduce collection costs. The bulk of combustible refuse is reduced to from 5% to 20% of its original volume leaving an inorganic residue that can generally be disposed of without difficulty.

The greatest drawback to the installation of incinerators is their cost—both of construction and operation. The U.S.P.H.S. survey showed that 1,090 incinerators are needed for municipalities throughout the nation and that on the basis of 1948 prices the costs would total about $100,000,000. Cost data are not easily evaluated but some contracts for large incinerators, let during 1947 and 1948, showed construction costs ranging from $1,500 to $2,500 per ton of 24-hr capacity. In addition, operating costs may range from 50¢ to $1.50 per ton of refuse destroyed. In view of these costs, many cities have given serious consideration to other methods of disposal.

This committee (with the assistance of Stephen Petro, Jr., and Martin Kornspan, Juniors, ASCE) has undertaken a survey of municipalities that have installed incinerators at one time or another. Letters were written to all the cities listed in the 1931 Report of the United States Chamber of Commerce,[4] the most complete census of incinerators to that time. Incinerator manufacturers cooperated in furnishing information on units installed after 1931. The purposes of the survey were to establish the continuity of incinerator practice in each municipality, determine the extent of abandonment of incinerators for other methods, and obtain a cross section of the life expectancy of incinerators, together with the frequencies of remodeling and enlargement.

TABLE 1.—Abandoned Incinerators and Their Periods of Useful Life

Periods of use (years)	Installations Abandoned	
	Number	Percentage
1 to 10	36	27
10 to 15	27	20
15 to 20	32	24
20 to 30	30	22
30 to 60	9	7
Total reported	134	100

Progress on this survey has been good but data for a final tabulation of results are not complete. Some 400 installations have furnished data from which the following tentative conclusions may be drawn for this progress report:

1. A total of 134 incinerator installations have been abandoned for various reasons, the most prevalent being recourse to another method of disposal or abandonment of older units in favor of newer types of incinerators.
2. Of the total number of installations there has been abandonment of only 12% of these in favor of the sanitary land fill method of refuse disposal.
3. The periods of useful life of the incinerator installations prior to abandonment were as shown in Table 1.

[4] "Refuse Disposal in American Cities," Rept. of the U. S. Chamber of Commerce, Washington, D. C., April, 1931.

5. Of the 400 odd incinerator installations considered in this survey, the percentage of the total constructed in each decade since 1890 was as follows:

Decade	Percentage of total
1890 through 1899	1
1900 through 1909	7
1910 through 1919	12
1920 through 1929	40
1930 through 1939	25
1940 through 1948	15

Refuse Charging.—When completed, the survey of this committee will also include a tabulation showing the methods for charging and for ash removal utilized in each municipality in which incinerators were installed since 1931. Many improvements have been made in methods for handling refuse at the incinerator site. The old method of dumping refuse on the floor of the incinerator and storing it there until burned was unsanitary, required more laborors in some instances, created a fire hazard, and frequently delayed trucks for considerable periods of time.

Larger plants have a large storage pit into which trucks can dump their refuse and from which bridge cranes can obtain the refuse for loading into the feed hoppers of incinerators. Most average-size communities cannot afford an incinerator plant equipped with a bridge crane. However, other designs can readily be made to adapt modern and relatively inexpensive materials handling equipment to smaller plants. Single-line buckets with electric hoists and motorized trolleys have met with considerable success[5] in taking refuse from a dumping pit to a charging hopper. Methods such as these not only should be applied to new incinerators, but also should be given consideration in the remodeling of old plants.

Incinerator Design.—One of the most significant advances in the design of refuse incinerators since 1939 has been the adoption of mechanical stoking equipment by two incinerator manufacturers. This equipment has been installed in circular furnaces from 9 ft to 15 ft in outside diameter. The furnace is stoked by slowly revolving arms that agitate the refuse and gradually move it toward the periphery. The central part of the floor of the furnace consists of a cone approximately 3 ft in diameter by 2 ft high, two or three rotating arms, and a flat circular steel plate which serves as the drying hearth. This hearth serves the same purpose as its counterpart in the conventional hand-stoked furnace and occupies approximately half of the projected area of the furnace. The drying hearth also serves as the bed plate over which the stoking arms rotate. Forming an annular ring around the drying hearth is the burning hearth which consists of a series of grates through which air for combustion is furnished. All the grates are usually of the dumping type to facilitate removal of ashes and cleaning of the furnace with a minimum of manual stoking. This type of furnace does not eliminate hand stoking, but on larger units, particularly those of more than 75-ton capacity, on a 24-hour basis, may serve to lessen manpower requirements.

[5] "Refuse—In and Out of the Incinerator," by H. W. Taylor, *Engineering News-Record*, Vol. 140, 1948, p. 54.

Air is introduced into the mass of refuse over the drying hearth through orifices in the central cone and the rotating arms, as well as through the grates of the burning hearth. It is claimed that this procedure, together with uniformity of stoking, increases the burning capacity of the furnace. This method of introducing air differs from the practice in the conventional hand-stoked incinerators in which air is usually introduced into the mass of refuse over only the grates comprising the burning hearth.

Tests conducted at Detroit, Mich.,[6] Tonawanda, N. Y.,[7] and Winnipeg, Man., Canada,[8] showed that higher burning rates may be utilized than the conventional 50 lb of refuse per hr per sq ft of inside horizontal furnace area. Rates ranging from 78 lb at Winnipeg to 112 lb at Tonawanda and other plants were utilized for design purposes. The plants have been reported to be in successful operation at these rates.

In order that an impartial basis may be estabished for the design of incinerators with mechanical stoking equipment, this committee should undertake a survey and analysis of operating data at the 20 odd incinerator plants using this equipment. The objective of this analysis should be to determine whether circular furnaces with mechanical stokers can destroy average municipal combustible refuse successfully and economically at burning rates greater than those ordinarily used[9] with hand-stoked furnaces of conventional design. In hand-stoked furnaces these rates are usually expressed as 75 lb of refuse per sq ft of open burning grate area per hr and 50 lb per sq ft of total projected area of furnace (projected horizontal area of drying plus burning hearths). If higher burning rates are possible (as preliminary evidence seems to indicate) it is desirable that designing engineers take cognizance of these higher rates in the design of municipal incinerators and thus reduce the size of units required for any given load.

An important problem arises in the evaluation of bids for the construction of mechanical and hand-stoked incinerators. The actual low bid, from the standpoint of total construction cost for furnaces, accessories, and building, may not be the most desirable on the yearly cost basis. Differences may arise in the need for power for blowers and stoking equipment and the number of men required to operate the incinerators. The designing engineer should specify the labor requirements with each type of incinerator, based on the experience of communities utilizing each type, or on guaranteed performance test over extended periods of opeartion. At Winnipeg[10] the specifications stated the basis for bid evaluations—namely, the estimated annual cost, including charges for interest and amortization, labor, power, and fuel costs. Although this is common practice in most fields of engineering it has not been generally followed in incinerator construction.[11]

[6] "Detroit's Mechanically Stoked Incinerators," by R. H. Stellwagen, *American City*, March, 1947, p. 94.

[7] "New 100-Ton Incinerator of Town of Tonawanda, N. Y.," *Water and Sewage Works*, Vol. 92, 1945, p. 281.

[8] "New Winnipeg Incinerator Undergoes Severe Tests," *Sewage Works Engineering*, Vol. 19, 1948, p. 394.

[9] "Standard Specifications—Section 18—Incinerators," by F. S. Friel, Albright and Friel, Philadelphia, Pa., 1947.

[10] "Incinerator Practice—1910 and Today," by Samuel A. Greeley and Samuel M. Clarke, *Engineering News-Record*, October 14, 1948, p. 87.

[11] "What Constitutes a Low Bid on an Incinerator?" by George L. Watson, *American City*, October 1934, p. 66.

Another type of mechanical incinerator utilizes rotary kilns for the burning refuse after shredding to a size less than 1½ in. Atlanta, Ga.,[12] and Los Angeles, Calif.,[13] have installed this type of equipment. Enough information is not available to determine the effectiveness of rotary kilns and auxiliary equipment for the many materials encountered in municipal practice. At Atlanta[12] successful operation and recovery of waste heat for the generation of steam was reported.

Handling of Incinerator Residue.—Proper incinerator design implies an adequate and satisfactory method for the handling of ashes and other residue from the incinerator. Too often fires are cleaned not through dumping grates but by raking them to the stoking room floor or into carts at the front or side of the furnace where they are quenched with a hose. Thus, odors escape with the steam and pass into the atmosphere. Provision should be made for adequate ash hoppers and quenching facilities under the furnace, if at all possible, so that odor nuisances and operating difficulties may be minimized.[14]

Most modern incinerators of any appreciable size have been designed on the three-story layout as the most feasible solution to the ash handling problem. The top floor is for charging facilities; the second floor is used for stoking operations and grate dumping; and the lowest floor provides for ash quenching and storage in hoppers, below which is the passageway for trucks or ash carts to receive quenched residue. For smaller plants, in which the lowest floor would entail too great an expense, ash conveyors submerged in a trough of water have been used.[5] These conveyers may be designed to deliver the quenched residue to an elevated receiving hopper from which trucks may be loaded without manual handling. Other methods can also be devised to utilize the principle of the three-story design without resorting to the construction of a complete basement under the entire stoking floor.

Adequate removal of the main residue from an incinerator or furnace is but a part of the ash problem. Fly ash and heavier ashes cannot be permitted to accumulate in combustion chambers. The basement layout and arrangement of combustion chambers should provide for the delivery of secondary ash to the ash room under the furnaces. Fixed or portable vacuum systems have been used for the removal of secondary residue from preheaters, combustion and and expansion chambers, and flues. The importance of regular cleaning of chambers, and adequate facilities to accomplish this cleaning, warrants careful attention on the part of designing engineers.

Disposal of Ashes.—Although a well-operated incinerator will reduce combustible refuse to a relatively small percentage of its original volume, the problem of ash disposal cannot be neglected. This implies a sanitary method of transporting, dumping, and covering the ashes. In addition, all the noncombustible refuse from a municipality requires intelligent handling and a sanitary method of disposal. The obvious solution of these problems is the sanitary land fill.

Adequate acreage of wasteland within an economical hauling distance from the center of population is frequently not available in larger municipalities.

[12] "Incinerator Revenue," by H. J. Cates, *American City*, February, 1947, p. 78.

[13] "Kiln Furnace for Rubbish Incineration," *Engineering News-Record*, Vol. 137, 1946, p. 793.

[14] "Incinerator Mechanization Wins Increasing Favor," by Rolf Eliassen, *Civil Engineering*, April, 1949, p. 20.

Therefore, incinerators may be located at strategic points to reduce the volume of refuse to relatively small amounts and thus reduce hauling costs and the area of land needed for final disposal of refuse. Only a complete economic study of the entire refuse collection and disposal problem can determine whether incinerators are needed; but the need for a sanitary land fill, either to receive incinerator ash and noncombustible refuse, or to receive all the municipal refuse, is present in most communities.

Sanitary Land Fills

The disposal of refuse on land fills is finding increasing favor among municipal engineers, many of whom are being pressed by the growing consciousness of taxpayers in the public health significance of refuse disposal. Open dumps, burning dumps, and other assorted methods which give rise to nuisances from odors and smoke, provide harborage and food for rats and insects, and are the source of a number of public health hazards—the cry of the citizenry, newspapers, engineers, and public health officials is for their removal. This trend is evident from the results of the U.S.P.H.S. survey which shows that 5,549 communities will be utilizing sanitary land fills within the near future.

The principles of operation of sanitary land fills are well known—disposal of refuse on marginal or wasteland (the depth of refuse varying from 3 ft to 15 ft) the compaction of this refuse by trucks and bulldozers, and the daily covering of the compacted refuse with earth (the final depth of cover to be a minimum of 2 ft). The foremost thoughts in the minds of municipal engineers must be on the sanitary aspects of the fill to distinguish it from a common dump. Public health officials, the Supreme Court of the State of New York,[15] and responsible engineers have recognized that a properly operated sanitary land fill is a satisfactory method of refuse disposal. Recently the Health Advisory Council of the United States Chamber of Commerce issued a brochure to its members urging the replacement of open dumps by modern sanitary land fill methods in the interests of public health.

In order to secure proper operation, the application of well-established practices of the civil engineering profession is necessary. Modern earth-moving equipment such as bulldozers, "bull-clam" shovels, and draglines is called for, in addition to the intelligent use of this equipment. If a civil engineer will go to extremes in using the most modern earth-moving equipment for the construction of a huge earth fill dam, with a volume of about 5,000,000 cu yd, how much more necessary it is to utilize the same engineering principles and equipment for the disposal of 23,000,000 cu yd of refuse each year from a city the size of New York! The same need, even though on a smaller scale, applies to thousands of other municipalities in the United States.

Some of the greatest problems arise in the securing of satisfactory cover material, particularly in northern states, with their long and severe winters. The most generally adopted method is to excavate a trench and utilize the excavated material to cover the refuse placed in the adjacent trench.[16] Suc-

[15] "Health Aspects of Land Fills," by J. L. Rice and S. Pincus, *American Journal of Public Health*, Vol. 30, 1940, p. 1391.

[16] "Summarizing the Sanitary Fill," by William S. Foster, *American City*, April, 1947, p. 92.

cessive trenches from 8 ft to 100 ft in width have been used, the width depending on the daily volume of refuse deposited in the fill and the type of excavating equipment available. Where suitable cover material is not available at the land fill site it is necessary to haul from another source. This may increase the expense of operation to the point where this method of disposal no longer becomes economical.

Since the subject is such a broad one, this progress report can mention only the significance of sanitary land fills and cannot elaborate as to details of planning and operation. Members of this committee have been approached by numerous municipal engineers who have asked for detailed instructions on the conduct of the sanitary land fill method, the equipment necessary, the costs of disposal, the land requirements, and other pertinent data to enable them to plan their operations and secure the approval of municipal authorities for the use of this method. Such information is not available in any one publication. Although the task is a long and arduous one, this committee is considering the preparation of a manual on sanitary land fills for the use of ASCE members and municipal engineers in general.

Use of Completed Fills.—Sanitary land fills have proved to be an excellent method for the reclamation of swampland and other low-lying areas of little or no value. However, the use to which this reclaimed land can be placed is limited by the characteristics of the refuse comprising the fill.

Foremost among these characteristics is the heterogeneous nature of the organic matter, such as paper and garbage, mixed with ashes, boxes, street sweepings, and sundry materials. Decomposition of this mixture has been studied at New York University, in New York,[17] in connection with some of the fills placed by the New York Department of Sanitation. Samples taken of fill materials with ages from three months to twenty-five years all showed active decomposition. The rate of digestion is very slow and will continue for many decades. Gases of decomposition, notably carbon dioxide and methane will continue to be given off during this long period of time.

Rates of subsidence are dependent upon the initial degree of compaction during the placing of the refuse. Studies[18] show that, although the initial rate is fairly high (perhaps 30% in three to six months), after two years very little general settling will occur, except in isolated spots. Loading tests indicate fairly high bearing values for established fills—2,000 lb per sq ft giving a nominal settling. Only buildings with broad mat foundations should be considered to minimize differential settling.

Best use of the reclaimed land is for parks and other recreational areas. Only buildings needed as functional units of the park should be constructed over the fill. The areas have also been used as parts of airfields, preferably not for runways. Storage yards for equipment and materials that can be left in the open have also been developed over completed fills.

Once a fill is sealed with 2 ft of cover no attempt should be made to open it. Odors from the decomposing refuse do not escape to the atmosphere under

[17] "Decomposition of Land Fills," by Rolf Eliassen, *American Journal of Public Health*, Vol. 32, 1942, p. 1029.

[18] "Load-Bearing Characteristics of Landfills," by Rolf Eliassen, *Engineering News-Record*, Vol. 129, 1942, p. 369.

ordinary conditions because the gases of decomposition are mostly odorless, being principally carbon dioxide and methane, both of which have no odor. However, when refuse is exposed in a fill, the inherent odor of the partly decomposed refuse is so strong that conditions become unbearable for workmen and the surrounding community.

Experiences in the construction of a veterans' housing project over a five-year-old sanitary land fill have been reported in the literature.[19] Excavation for utilities trenches over an area of 100 acres of fill led to mass protests from workers and residents half a mile away, none of whom had been bothered to any extent by odors during the original placing of the fill. Odors were controlled by spraying the sides and bottoms of trenches with a powerful oxidizing solution.[20] It was found that if the exposed surfaces of garbage were oxidized rapidly the odors subsided.

Other difficulties in constructing buildings over land fills are experienced as a result of the escaping gases. Since the rate of decomposition is very low in a normal fill, the diffusion of gas through the earth cover into the atmosphere is so slow that no effect is noted. However, if buildings have entrapped areas under them, the gases may concentrate and lead to an explosive mixture. Unless the venting of the undersides of the buildings is sufficient and the floors are tightly sealed, particularly around the gas, water, and sewage pipes, dangers may arise within the buildings themselves. Some unfortunate incidents of this nature have occurred.

Therefore, the use of completed sanitary land fills should be restricted to open areas such as parks, airfields, and storage yards. Buildings should not be constructed unless special precautions are taken and structural loadings are held to a minimum. This matter of ultimate use should not detract from the consideration of sanitary land fills for the reclamation of otherwise useless land. Parks and recreational areas are needed in virtually every community—far more than mosquito-breeding swamps.

Other Methods of Refuse Disposal

Although incinerators and sanitary land fills are accepted as methods of refuse disposal which, when properly operated, can meet exacting public health requirements, other less satisfactory methods are still practiced. The ENR survey[3] showed that, of 300 cities with populations greater than 5,000 and geographical representation from all over the nation, 28% of the cities utilized incinerators and 25% maintained sanitary land fills. The survey also showed that 41% disposed of all or part of their garbage by hog feeding, 50% had open dumps, and only a few used miscellaneous methods such as garbage reduction, dumping in near-by waters, and central grinding stations.

Hog Feeding.—Hog feeding with raw garbage has been condemned by public health authorities as a distinct health menace in encouraging the transmission of trichinosis.[21] It is estimated that approximately 5% of garbage-fed hogs

[19] "Housing Construction on Refuse Landfills," by Rolf Eliassen, *Engineering News-Record*, Vol. 138, 1947, p. 756.

[20] "It Doesn't Have to Smell," by Rolf Eliassen, *American City*, December, 1947, p. 92.

[21] "Control of Trichinosis," Public Health Repts., U. S. Public Health Service, U. S. Govt. Printing Office, Washington, D. C., April 9, 1948, p. 478.

are infected with the organisms of this disease. Reports by the U.S.P.H.S.[21] have indicated that garbage should either be boiled or frozen before being fed to hogs to kill the organisms. Some cities are utilizing these methods, with attending increases in the costs of garbage handling.

No solution to the problem is close at hand as long as municipal officials can see a reduction in their refuse collection and disposal budgets by permitting garbage feeders to collect garbage from householders or by collecting garbage from municipal transfer stations. Until a dollar value can be placed on the menace of trichinosis it will be difficult for many municipal authorities to evaluate the significance of the disinfection of garbage or the abandonment of hog feeding in favor of more sanitary measures for garbage disposal.

Dual Disposal.—Much has been written concerning the feasibility and economics of disposing of garbage with sewage.[22] Household garbage grinders have come into wide acceptance, with the installation of approximately 250,000 of these units since 1939. The effect of these may be an eventual increase in the strength of municipal sewage. By grinding garbage with shredding knives, or hammer mill grinders, to a fine size, it is possible to mix it with sewage at central grinding stations or with raw sludge at the sewage treatment plant. On the average, the size of sludge digesters will have to be doubled if all the garbage in a municipality is disposed of in this manner.

Experiments have disclosed[23] a great increase in gas production. If this gas is utilized for the generation of power or for the operation of pumps and blowers in the sewage treatment plant, some return might be realized on the investment for larger digesters, garbage receiving stations, and grinding equipment. This may be a logical step forward in offering municipalities an alternative to hog feeding with the garbage serving some useful purpose such as power generation.

TABLE 2.—UNIT COST FOR REFUSE COLLECTION BASED ON DATA FROM 75 CITIES

Size of city (population)	Number represented	DOLLARS PER YEAR PER CAPITA		
		Maximum	Median	Minimum
More than 1,000,000	1	$0.633	$0.633	$0.633
250,000 to 500,000	7	2.059	1.344	1.064
100,000 to 150,000	11	3.080	1.236	0.436
50,000 to 100,000	13	5.027	1.801	0.285
25,000 to 50,000	21	3.472	1.230	0.600
10,000 to 25,000	22	2.945	0.961	0.171

Financing Refuse Collection and Disposal.—With the ever-changing cost picture it will not be possible to make any survey of costs of refuse collection and disposal until some semblance of stabilization is reached. Since municipal salaries are prone to lag far behind those in construction and other industries well represented by labor organizations, municipal costs can be expected to continue on the upswing for a number of years.

[22] "Ground Garbage—Its Effect Upon the Sewer System and Sewage Treatment Plant," *Sewage Works Journal*, Vol. 19, 1947, p. 441.

[23] "Operating and Experimental Observations on the Digestion of Garbage with Sewage Solids," by David P. Backmeyer, *ibid.*, p. 48.

The latest data on refuse collection costs were presented in the ENR survey for 1947.[3] Not only rising prices, but also frequency of collection, length of haul dictated by the methods utilized for refuse disposal, amount of segregation necessitating multiple collection, and other factors, will determine ultimate costs. These factors accouut for the wide spread in the basic cost-per-year-per capita data noted in Table 2.

Similar cost data are not readily available on the operation of the various methods of refuse disposal. These can only be obtained by an extensive cost survey of municipalities known to have accurate cost accounting methods for each item of refuse collection and disposal. Cost accounting procedures are far from standardized and far from being accurate in many cities—even the larger ones. This is one of the great needs of municipal engineers to enable them to control their own costs and make comparisons with other municipalities.

Funds for the operation of refuse collection and disposal organizations are raised mainly by general taxation. The ENR survey showed that 72% of the cities relied on this method whereas 15% used special service charges and 11% permitted private contractors to collect from property owners for services rendered.

Summary

The Committee on Refuse Collection and Disposal has prepared this Progress Report to include the highlights of recent developments, the need for special studies and reports by the committee, and the need for the application of modern principles and practices of civil engineering to achieve efficiency and economy and to maintain the highest standards of public health engineering.

Respectfully submitted,

Francis S. Friel
Norman W. Nester
Newell L. Nussbaumer
Warren A. Schneider
Henry W. Taylor
Rolf Eliassen, *Chairman*

Committee of the Sanitary Engineering Division
on Refuse Collection and Disposal

January 20, 1949

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

A CENTROIDAL METHOD OF RIGID-FRAME ANALYSIS

Discussion

BY S. J. BELL

S. J. BELL,[14] ASSOC. M. ASCE.—The discussions of the paper are appreciated by the writer. No claim was made that the method would replace all other methods of ring analysis; however, the hope that it would prove to be an interesting addition to them seems to be justified.

Professor Grinter points out that the centroidal method "* * * involves the use of common principles" and that the "* * * principles of ring analysis are few." These statements are true because all the methods of ring analysis are attempts to simplify the application of the same fundamental principles. Professor Grinter's comparisons of the centroidal method with the neutral-point method and with the column analogy are interesting. The writer agrees that the methods should be judged by

> "* * * clarity or ease of understanding, convenience in making routine calculations, and visual characteristics that aid one both to understand and to remember the procedures."

Mr. Gluckmann notes that results obtained by the centroidal method are similar to those obtained by the ellipse of elasticity which arrives at the amount and direction of the thrust by using the properties of the ellipse. The centroidal method obtains the same result by using Mohr's circle of inertia which is a graphical method of solving elliptical equations.

Mr. Yao shows that the components F_x and F_y of the thrust F_o may be found by using displacements parallel to the principal axes. There are many variations in the application of part or all of the method.

Professor Ferguson believes that the method will be found more useful for situations where "* * * movements of supports are the basic data and lack of symmetry is common." The writer has found this statement to be true. Some of the applications shown in the paper (for instance, applications to structures with both ends hinged) were included to give a more complete picture of how the procedure would apply to all types of end conditions.

NOTE.—This paper by S. J. Bell was published in November, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: June, 1948, by L. E. Grinter, Alexis L. Gluckmann, Yi-Mai Yao, and Phil M. Ferguson; and September, 1948, by Leroy A. Beaufoy, and William A. Conwell.

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Mr. Beaufoy finds it difficult to follow the paper because of the variability of dimensions in the quantities employed. The method was developed for a structure with constant values of E and I, the latter without subscript being the sectional moment of inertia. An explanation of how to apply the equations to structures with variable values of E and I is given in the paragraphs following Eqs. 20. If desired, Eqs. 21 may all be written for variable values of both E and I in which case the structural moment of inertia would be (compare with Eq. 21*b*)

$$I_s = Z^2 \frac{ds}{E\,I} \qquad (44a)$$

and the structural length would be (compare with Eq. 21*a*)

$$L_o = \frac{ds}{E\,I} \qquad (44b)$$

The writer has found it convenient to factor out the product $E\,I$ when it is constant throughout the structure.

The writer believes that there is nothing fundamentally wrong with Section 6, "Application to Structures with Both Ends Hinged." Moving the hinges A and B 2 in. closer together is a relative movement and any or all of the movement may be assumed to take place at either end. In any case the total force F_o through the ends will be the same, but the actual movement of the ends will be different. The movements in Fig. 14 are shown correctly for the assumption that the relative movement is equally divided between the two ends. The actual rotations of ends A and B in moving to positions A′ and B′ is equal to 0.0088 radian each. The rotations of the ends with reference to a line through the final positions A′ and B′ will be those shown in the text following Eqs. 42 of Mr. Beaufoy's discussion.

In Section 7, point D was chosen for the determination of both deflection and rotation because it was easy to illustrate the determinate moments, as may be noted in Fig. 17. The final displacements of the point are very small and the values found in Example 8 are not exact. The examples shown in the paper are for the purpose of illustrating the application of the method and no attempt has been made to check exact solutions. All the original illustrations were drawn on 8½-in. by 11-in. paper. If greater accuracy is desired it may be obtained by using larger scales.

Mr. Conwell suggests that it might have been advantageous to use the geometry of flexure in developing the method. The writer found that the development in the paper was shorter and took less explanation than any other he could devise.

The centroidal method was developed soon after the column analogy was published and before it had come to the attention of the writer. Had he been familiar with the column analogy at that time, in all probability, the centroidal method would not have been developed. The writer believes that both methods have points in their favor, and that individual experience and preference in each case will determine which will be used.

The writer wishes to thank all those who discussed the paper and is pleased that most of them considered it of interest.

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DISCUSSIONS

UNUSUAL DESIGN PROBLEMS—SECOND TACOMA NARROWS BRIDGE

Discussion

BY CHARLES E. ANDREW

CHARLES E. ANDREW,[5] M. ASCE.—As written, the paper had as a primary object the enumeration of problems encountered at Tacoma which are not ordinarily present in bridge design and construction, such as fitting a new, heavier, and wider bridge to existing piers and abutments and, of course, attempting to improve aerodynamic stability.

The failure of the former bridge attracted world-wide attention and, to say the least, created grave doubt in the public mind, if not in all engineers' minds, as to the safety of certain types of suspension bridges. In consequence, engineers responsible for the new design felt that all possible safeguards must be incorporated to be doubly sure that the new bridge would be a safe structure. These safeguards were the result of much study and extensive and costly experiments, and represent the best judgment of those responsible. Some of these safeguards may be unnecessary but, it is believed, they must be considered further insurance at a relatively low cost. Their significance was manifest in the importance which bond investors attached to them and in the writer's opinion the extensive studies and the safeguards that resulted were largely responsible for the successful consummaiton of an otherwise difficult financing problem.

In any original research, such as the aerodynamic problem in suspension bridges, the investigators encounter difficulty and practical failures in many of their first guesses, abandon them, and try some other course of action. The problem involving the second Tacoma Bridge was no different. The very first preliminary design based on available knowledge of aerodynamics, coupled with judgment when tested by model, proved to be a great improvement over the design for the original bridge. This design included the same greatly increased stiffening truss and practically the same weight as does the final design; yet it showed decided instability in certain velocities and angles of wind. As a next step

NOTE.—This paper by Charles E. Andrew was published in December, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1948, by O. H. Ammann, Charles A. Ellis, and F. H. Frankland.

[5] Cons. Engr., Washington Toll Bridge Authority, Tacoma Narrows Bridge, Tacoma, Wash.

in seeking added stability, sundry modifications of the rail and leading edges were tried. Some of these developed a relatively small amount of added stability in certain winds. The streamlined rail was incorporated in the final design but none of the leading edge modifications were. Finally, the original rail was changed to one less streamlined but retaining the same percentage of openings and practically the same aerodynamic properties. Not until the roadway slots were incorporated did the tests indicate a very decided improvement in stability. In fact, almost complete stability resulted and the very minor movement that occurred became noncatastrophic whereas all former motions were catastrophic. Surely such an improvement is worthy of consideration and incorporation in the structure.

As to placing too much reliance on past experience in suspension bridges and setting up design criteria from former performance, it should be called to the designer's attention that the first Tacoma Narrows Bridge was designed by the best talent of the day, guided by accepted past practice; yet the design, unfortunately, was inadequate. Thus the writer contends that each suspension bridge is an individual case that must be designed to suit its site and the traffic that it may be called on to carry. Many may need no consideration of aerodynamic problems, others need it badly. Borderline cases will be more numerous in the future—hence, the necessity for caution.

The writer wishes to correct Mr. Frankland's impression that a hold-down cable parted on November 7, 1940, the day of failure. One of these ropes did part at an included weak link about one week prior to the failure under pure vertical oscillation, but on the day of failure, which was the first time that torsion occurred, all hold-down cables were intact. The cable sag of the new design is 1/10.

The writer did not go into the many and various ramifications of why all features of the design were adopted, simply because the limit of such a paper would not allow the vast amount of data required. Final reports (five volumes) are now (1949) being written on the subject.

The writer wishes to thank Messrs. Ammann, Ellis, and Frankland for their interest and discussion.

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DISCUSSIONS

HIGHWAY BRIDGE FLOORS
A SYMPOSIUM

Discussion

BY N. M. NEWMARK, F. E. RICHART, AND C. P. SIESS

N. M. NEWMARK[40] AND F. E. RICHART,[41] MEMBERS, ASCE, AND C. P. SIESS,[42] ASSOC. M. ASCE.—The entire discussion appears to have centered on the problems of design of I-beam bridges and of the behavior of composite beams. Several discussers have made important contributions to the knowledge of these problems. Their comments and suggestions are appreciated.

An interesting summary of his early work is given by Mr. Slack. He calls attention to the partial interaction shown by field measurements, as indicated by the fact that measured deflections were only from 50% to 75% of the "computed" deflections. Part of the discrepancy, however, can be ascribed to incorrect assumptions in the computation of deflection. The design specifications for highway bridges, even to the present time, do not take full account of the lateral distribution of the loading produced by the roadway slab. This deficiency is shown by the values of s in Table 2, compared with the corresponding value of 5 ft in the "Standard Specifications for Highway Bridges"[18] of the American Association of State Highway Officials. These factors are for truck loading or lane loading. The lateral distribution of a single concentration, sometimes used in tests, is even greater.

Nevertheless, good evidence supports the belief that in actual structures there is some partial interaction between the steel beams and the concrete slab, even if it is due to friction alone. Unfortunately, other evidence also indicates that such partial interaction can cease to exist, under unfavorable conditions, unless positive methods are provided to hold the two elements of the structure in contact. The data contributed by Mr. Slack agree with this view. For example, from Table 4, one can compute the increments in measured strains due

NOTE.—This Symposium was published in March, 1948, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1948, by Searcy B. Slack; October, 1948, by E. W. Wendell, and H. Tachau; November, 1948, by Reto Furrer; and April, 1949, by Louis Balog.

[40] Research Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

[41] Research Prof., Eng. Materials, Univ. of Illinois, Urbana, Ill.

[42] Special Research Asst. Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

[18] "Standard Specifications for Highway Bridges," AASHO, Washington, D. C., 4th Ed., 1944.

to the application of the live load. The results are given in Table 7. The strains are determined by dividing the increments in stresses by the values of the modulus of elasticity given by Mr. Slack.

It is apparent from the data in Table 7 that in the composite beam the neutral axis is above the center at roughly the place where one would expect it, and that the slab acts with the beam as an additional top flange with a greater strain than the top flange of the beam. However, in the other beam the neutral axis is about in the middle of the depth, and the strains show evidence of considerable slip between the concrete and the steel.

TABLE 7.—MEASURED STRAINS FOR LIVE LOAD ONLY
COMPUTED FROM TABLE 4

Strains, in microinches per inch	Composite beam; beam 2	Noncomposite beam; beam 4
Top of slab	−25	−16
Top flange of beam	−20	−51
Bottom flange of beam	+54	+49

Because of such experiences, the writers do not feel that bond alone can be relied on to insure composite action unless the entire upper flange and possibly part of the web of the beam are encased in the concrete of the slab.

It appears to be common sense and good engineering to accept composite action only as a bonus or as an added factor of safety unless some positive mechanical method of transferring shear is provided. The procedure would depend on circumstances and judgment. Many expedients are available and some have been described in the papers and in the discussions.

The writers concur with Mr. Slack that simple design methods are preferable, and that unusual refinements may be unwarranted in view of the uncertainties and the large number of variables that must be considered.

Mr. Wendell raises a number of questions of great practical importance. His comments on the economic factors involved in the design of composite bridges are particularly valuable. He points out that a considerable additional saving in weight is possible by using a thin web with stiffeners in place of the thick web, one eighty-seventh the depth, used in the studies reported in the third Symposium paper. However, such additional savings are possible only for beams deeper than those considered in that paper. The studies reported were limited in scope to a maximum simple-span length of 90 ft, a maximum beam spacing of 7 ft, and a maximum live loading of H-20. These limitations were necessary to obtain bridges in which rolled wide-flange beams could be used. Otherwise, the comparisons in Table 3 for the several types of beams could not have been made. The maximum over-all depth of welded beam considered was 42 in.; and, for depths not greater than this, the savings in weight from the use of a thin web with stiffeners will probably not be enough to offset the cost of additional welding. Actually, the built-up welded beam is not limited to this depth; and, for longer spans and consequently greater depths, the thin webs with stiffeners may be more economical, as has been indicated by Mr. Wendell.

Mr. Wendell expresses some skepticism regarding the possibilities of the steel companies rolling unsymmetrical beams, and he may well be right. It is of interest to note, however, that the studies made to determine the optimum proportions for such beams were undertaken only after a tentative and unofficial request for such information had been received from a steel company.

The effect of plastic flow on the concrete slab was mentioned by Messrs. Slack and Wendell. Mr. Slack raised the question of plastic flow of composite bridges built with temporary supports in which the dead-load moments are resisted by the composite section. The action of such a structure was discussed in the third Symposium paper, by simply assuming that the effects of plastic flow could be estimated by using a value of $n = 30$ rather than one of $n = 10$. This approach is purely analytical, of course; but it does not seem likely that much evidence concerning plastic flow can be obtained from laboratory tests on small-scale models. Observations on actual structures would be preferable in this case, but even these may not be too fruitful because of the complications introduced by deformations caused by shrinkage of the concrete, changes in ambient temperature, and radiant heating.

Mr. Wendell is not concerned about plastic flow in the slab. His opinion is no doubt associated with the fact that he does not approve of the use of temporary supports. Where only live load is carried by the composite section, plastic flow occurs only as the result of any dead load added after the structure is built; and Mr. Wendell does not think that this condition requires serious consideration. In this opinion he is in agreement with the opinions expressed by the writers.

The lack of real economy in the use of temporary supports or shores for composite bridges is cited by Mr. Wendell. He calls attention to the fact that the cost of providing temporary supports will offset, in general, any saving in weight in the structure. This reason is not the only one why the use of shores may be inadvisable. It was stated in the second Symposium paper (Section 4*c*) that a structure designed to be constructed with temporary supports will have a reduced ultimate strength and a lower usable capacity, and therefore the saving in weight is gained at the expense of a loss in the real factor of safety.

The contributions to the subject made by Mr. Wendell emphasize the fact that experience as well as analysis and tests must be used in approaching the solution of an engineering problem. One of the major reasons for the success of the program of study of highway bridge floors at the University of Illinois in Urbana has been the cooperation of highway bridge engineers and others who have contributed freely from their background of practical knowledge.

Two interesting studies have been presented by Mr. Tachau. It might be helpful to note that the relative beam stiffness H is dependent on the span length as well as on the type of construction. In general, the longer the span the greater is the value of H. If reasonable limits are set on H in Fig. 37, a roughly horizontal band, covering the range from about 85% to 95%, is found to represent the proposed design moments. This result is essentially the basis for the values reported in Table 2.

The selection of the proper beam spacing is an important point, which has been treated by Mr. Tachau. It should not be concluded from Fig. 38 that the

most economical beam spacing is between 4 ft and 5 ft, however. It might have been more useful to compare the weights of the steel beams and of the concrete slab separately, as a function of beam spacing. Part of the trend obtained by Mr. Tachau is due to the fact that the slab, which is the heaviest part of the structure, increases in weight with the spacing of the beams, according to Mr. Tachau's assumptions. The weight of steel would probably be a minimum for somewhat wider spacings, and the minimum cost would occur at intermediate spacings. It is only fair to assume that the minimum cost lies somewhere in the range in spacing from 5 ft to 8 ft because most I-beam bridges are constructed in this range of dimensions.

An interesting attempt to account for the measured strain distribution in composite beams having small amounts of slip is presented by Mr. Furrer. He uses an analytical procedure developed by F. Stüssi, and applies it to the beam shown in Fig. 23 to explain the results indicated in Fig. 25. It is unfortunate, however, that the greatly condensed data given in the paper were not sufficient for his purpose. For example, the beam shown in Fig. 23 is typical only of beam T3; the shear connectors used in beams T1 and T2 were spaced at varying distances, ranging from $3\frac{3}{4}$ in. at midspan to 2 in. at the ends. Furthermore, the correct values of the modular ratio, n, were 7.3, 10.5, and 8.7 for beams T1, T2, and T3, respectively, as compared to the values of 9.4, 14.0, and 12.4 assumed by Mr. Furrer. His values of the shear connector stiffness, C, are likewise in error. As a result, the comparisons presented in Table 6 are not too reliable. In addition, the values given in Table 6 for the strains or stresses observed in the test and computed according to the classical theory for full composite action do not agree with those given in Fig. 25.

It was mentioned in Part B, Section 4, of the third Symposium paper that an analysis had been made of the behavior of a composite beam in which slip can take place. This analysis was developed by Professor Newmark and was described in an unpublished progress report in 1943. It is somewhat more approximate in its assumptions than is the method of Professor Stüssi. In it, the shear connection is assumed to be continuous along the length of the beam, and the resulting behavior of the beam is characterized by a differential equation for which a general solution was obtained in terms of an infinite trigonometric series. For certain specific cases the solution may be expressed in finite form. Comparative analyses have been made with both Professor Newmark's and Professor Stüssi's methods, and in both cases using the correct values of n and C for the three T-beams tested. The differences between the stresses computed by either method and those observed in the tests were usually less than the experimental errors of observation. In general, though, Professor Newmark's procedure seemed to give results in somewhat better agreement with the trends of the tests.

The foregoing discussion of methods of analysis should not obscure a much more important principle:

> If the shear connectors in an I-beam highway bridge are properly designed, the difference between the actual behavior of the structure and that computed for the assumption of full composite action will be so small that it need not be considered in design.

Consequently, little of a practical nature is to be learned from analyses of the type just discussed; and, for this reason, the analysis developed by Professor Newmark was never prepared for publication. The foregoing statements do not diminish in any way the importance of these analytical tools as applied to other types of composite structures.

The major part of the discussion by Mr. Balog does not appear to be pertinent to the papers. The second Symposium paper was specifically limited to recommendations for the design of I-beam highway bridges, consisting of a concrete roadway slab continuous over steel stringers in the direction of traffic. In the second paragraph of the paper, it was stated that:

> "The bridges considered, in general, are supported by four or more parallel beams of equal stiffness at a constant spacing of the beams between the limits of 5 ft and 8 ft."

In setting these limitations, Professor Newmark had in mind the necessity for fixing on a specific type of bridge if a design specification were to be possible. It was not intended that all highway bridges should be constructed in this fashion or that they should be designed by the recommendations if they fell outside these specific limitations. Other types of framing may certainly be used, and possibly under certain conditions may be more economical. The I-beam bridge has certain advantages, however, that make it now one of the most widely used types of highway bridge in the United States. It seemed, therefore, worthwhile to study this type of bridge in great detail.

It is impossible to consider the Stillhorn-Neuland (Germany) bridge with 20.34-ft girder spacing, or the "Arpád" bridge (over the Danube River in Budapest, Hungary) with comparable spacing, as an I-beam bridge. These bridges are so far from the proportions considered that there is no point in applying to them the results of the Illinois tests and analyses, even if specific limitations excluding such bridges had not been set in the paper. The writers are entirely at a loss to understand Mr. Balog's purpose in discussing such additional extraneous systems as

> "* * * widely spaced girders and widely spaced, load-distributing, rigid floor beams (diaphragms), between which grids consisting of shallow floor beams and stringers are provided."

Of course the recommendations stated in the Symposium papers do not apply; they were never intended to apply to anything but the simple ordinary I-beam bridge which Mr. Balog apparently considers only as a trivial structure.

Since Mr. Balog agrees that Professor Newmark's statement (which he quotes in his first paragraph), regarding diaphragms in I-beam bridges, "* * * applies only to small bridges of the type investigated by the author," there seems to be no point in discussing the matter of transverse framing in general, since such a discussion is outside the stated purpose of the Symposium, and would warrant a separate paper or a series of papers.

However, in his presentation of the data for the Xenia (Ohio) bridge, Fig. 43, Mr. Balog makes an unfortunate and serious error to which attention must be called. The error results from confusion regarding the so-called "distribu-

tion" of a concentrated load on a bridge. This matter is discussed in some detail in Sections 2*b*, 2*d*, and 2*e* of the second Symposium paper. In Fig. 18, there are shown the different types of "distributions" that one obtains from deflections and from moments in the beams of an I-beam bridge. It is entirely unsafe to use the "distribution" computed from deflections in determining stresses or moments.

In the Xenia bridge tests to which Mr. Balog refers, only deflections of the beams were measured. These measured deflections are actually in good agreement with computed deflections for this bridge. However, the moments in the beams are not proportional to the deflections of the beams. The difference is virtually that shown in Fig. 43 between the measured (deflection) values, and the curve marked "Newmark" which was obtained from Fig. 15, and which applies to the moments.

TABLE 8.—RELATIVE PROPORTIONS OF LOAD ON BEAM B OF XENIA (OHIO) BRIDGE TO ACCOUNT FOR DEFLECTIONS AND MOMENTS PRODUCED BY A CONCENTRATED LOAD

Quantity	Proportion of Load Determined from	
	Computations	Measurements
Moment.....	0.428	.....
Deflection.....	0.339	0.309

To illustrate the difference more clearly, in Table 8 a summary is given of the relative proportions of the load applied to beam B, in Fig. 43, required to account for: (1) The computed moments; (2) the computed deflections; and (3) the measured deflections. The last value is taken from Mr. Balog's discussion. The first may be obtained from Fig. 15, and the second value, for $b/a = 0.1$, $H = 5$, may be determined from a report published in 1942.[43]

It can be noted from Table 8 that the agreement between the proportions of load carried by the loaded beam, on the basis of computed and measured deflections, is quite good. The small discrepancy can be attributed to the fact that the exact proportions of the structure were not used in the calculations. However, for the same structure, the computations indicate a 26% greater load to account for the moment in the beam than is required to account for the deflection.

As a matter of fact, if one makes proper allowance for the difference in results obtained from deflection measurement rather than from strain measurement, the Xenia bridge tests verify rather than contradict the studies and recommendations given in the second Symposium paper.

In his discussion of the Xenia tests, Mr. Balog is quite correct in stating that the measurements indicate that the slab acts as a diaphragm. He is not correct in stating that one properly connected diaphragm can achieve the same effect. However, several properly connected diaphragms can achieve the same effect—at least three, and possibly more, are required to replace a slab so as to keep the maximum moment, for any position of the loads (on or off the diaphragm), at the same value. However, since the slab has to be there, why put the other

[43] "Moments in I-Beam Bridges," by N. M. Newmark and C. P. Siess, *Bulletin No. 336*, Univ. of Illinois Eng. Experiment station, Urbana, Ill., 1942, p. 121, Table 100.

diaphragms in? There is no need to do so except (see Section 2*b*) "Where it is expedient or desirable to make the slab thin, and therefore flexible * * *."

However, it should not be inferred that the use of diaphragms is to be discouraged. They can help somewhat in any case, even when they are not necessary; and, certainly if pre-cast floor systems or steel grid floors are to be used, transverse framing is necessary to make the structure act as an integral unit. The writers are glad that Mr. Balog called attention to this point.

Since the paper was written, some additional data have become available regarding the action of channel shear connectors. A full-size composite I-beam has been tested, which was designed as a composite beam of 37.5-ft span, with one H-20 wheel load at midspan, including 30% impact. It was designed for 6/5.3 wheel loads. The steel beam was a 24-in. WF 76-lb-per-ft beam and the slab was 6 in. thick and 6 ft wide.

Shear connectors were 6-in. lengths of 4-in., 5.4-lb channels, uniformly spaced throughout the length of the span at 18 in. The top flange of the beam was covered with a heavy coat of white lead paint before pouring the slab to insure that there would be no bond between the slab and beam. This coating seemed to be very effective, because slip between the slab and beam was measured from the start of tests. The sharp break, that is usually found when adhesive bond is broken, was not observed.

Strains were measured at five points on several shear connectors with type A-7, SR-4 electrical strain gages. Four gages were placed on the back of the connector and one in the fillet. The gages were waterproofed with an asphaltic compound ("Petrolastic").

In the test to failure, the highest load reached was 111,000 lb or 4.7 live loads. No indication of shear connector failure was observed other than a slight yielding in the fillet of the connector. The shear connectors were continuing to take load, and there was no break in the total load-slip curves at this load. At the maximum load there was absolutely no indication of inadequate shear connection except a slight amount of slip.

TABLE 9.—MAXIMUM STRESS IN SHEAR CONNECTOR COMPUTED BY VARIOUS METHODS

Load on connector (lb)	Computed stress (lb per sq in.)	Method of computation
31,300	32,400	Based on measured strains.
	61,000	Computed by Eq. 15*b*.
	144,000	Computed on the basis of linear pressure distribution on the back of the channel, from zero at the top to a maximum at the bottom.

A comparison of the measured strains with various methods of computing stress in the connectors is given in Table 9. The comparison is made at a load of 80,000 lb, or 3.4 live loads, at midspan. This point of comparison was chosen because it represents a load at which the beam was just beginning to yield in the lower flange at midspan. The connector load X was computed by the usual

formula, $X = \frac{V\,Q}{I}$ times connector spacing. The connector selected was one near a support and the one in which the measured strain was largest.

Obviously, the last method of computing stress in the connector is far too wasteful and conservative, when used as a basis for design. Even the formula in the third paper, Eq. 15*b*, is extremely conservative. Full composite action can be attained with even less adequate shear devices when the natural bond of the concrete with the steel is not destroyed. In any case the design procedure given in the third Symposium paper is amply conservative for channel shear connectors.

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DISCUSSIONS

A SIMPLIFIED METHOD OF ANALYZING SUSPENSION BRIDGES

Discussion

BY LING-HI TSIEN

LING-HI TSIEN.[18]—There being little disagreement in the discussions with the method proposed in the paper, few closing comments appear to be needed. The writer wishes to express his indebtedness for the clarifying effect of the discussion.

Professor Gravina, Mr. Ramaswamy, and Professor Liu each offer an approach to the derivation of formulas for the horizontal components of cable tension. They derive the same results as those presented in Table 1. Professors Gravina and Liu both use the trigonometrical series and are rigorous in their procedures. Mr. Ramaswamy suggests that the case of a moving concentrated load on the span should be treated first, and that the other cases of loading could easily be deduced by integration. The idea of influence lines could be used only after the applicability of the principle of superposition is demonstrated. Although superposition is proved to be applicable, approximately, for distributed loadings by Theorem 3 (a conclusion drawn from Theorems 1 and 2), Eqs. 17, thus deduced for the case of concentrated load, should be used with care. As pointed out by Professor Gravina:

> "* * * in the extreme case of a concentrated load it is no longer possible to consider only the first term of the series, as was done in the case of the distributed load; if this were done, the resulting error would be considerable."

Eqs. 17 could be considered only as corollaries to the cases of distributed loading, rather than as fundamental formulas as Mr. Ramaswamy proposes.

In fact, there are other approaches to the derivation of simplified formulas for H. For instance, at first the writer was preoccupied by the realization that the classical but implicit and lengthy formulas from the deflection theory[19] may be greatly simplified and reduced to the same form as those given in the paper.

NOTE.—This paper by Ling-hi Tsien was published in September, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1949, by Pedro B. J. Gravina, and G. S. Ramaswamy; and April, 1949, by Hui-Sien Liu.

[18] Prof., Civ. Eng., National Univ. of Chekiang, Hangchow, China.

[19] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1929, pp. 264–267.

This result can be achieved by expanding terms in series, and by introducing some minor approximations. However, the writer prefers the demonstration in the paper for the following reasons: By comparing the two extreme types of suspension bridges (namely, the unstiffened and the very rigidly stiffened types), it reveals immediately the fact that, although the horizontal component of cable tension may be appreciably different in magnitude, its law of variation remains practically the same for all types of suspension bridges when the live load moves

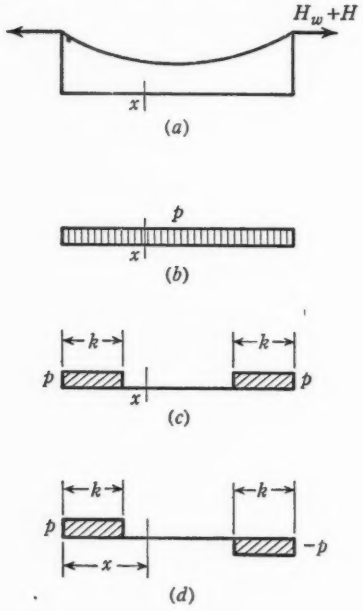

FIG. 11

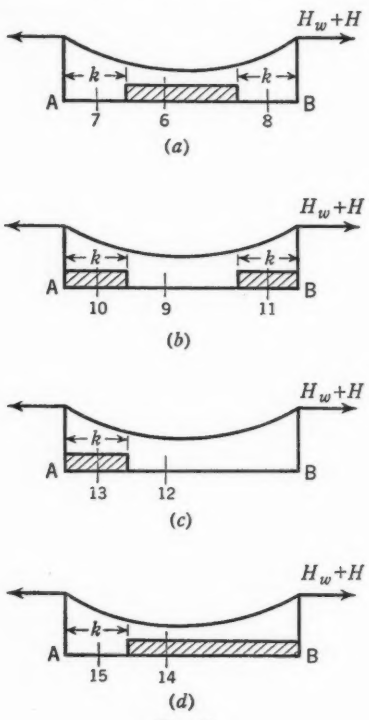

FIG. 12

arbitrarily on the span. The writer emphasizes the fundamental fact that this structural behavior of cable tension should be the key point of the analysis, and demonstrates his point by a simple comparison. The remaining task is to determine the correction factor D for the effects of side span interaction, cable stretch, and temperature changes. Each step furnishes its own physical meaning and reveals its own relative weight, or importance.

Professor Liu's discussion is especially valuable for its precise point of view on the problem of suspension bridges. His general equation (Eq. 138) for the horizontal component of cable tension reveals the applicability of the principle of superposition, although this is partly demonstrated in the paper by Theorem 3.

Incidentally, a possible mode of superposition can be also found even for the bending moment in a stiffening truss. By inspecting the formulas in Table 2,

it will be shown that the M-equation for different loadings can be obtained by summation of five moment components. Let

$$m = M \frac{c^2 l^2}{p l^2} \quad \text{(141a)}$$

$$Q = \frac{H}{H_w} \quad \text{(141b)}$$

and

$$A_x = \frac{\cosh c \left(\frac{l}{2} - x \right)}{\cosh \frac{c l}{2}} \quad \text{(141c)}$$

In Eq. 141c, A_x is a symmetrical function of x which varies from 0 to l; and it remains the same for section x whether x is measured from one end or the other of the span. Five moment components are as follows:

1. m_1 due to the direct effect of cable tension (Fig. 11(a))—

$$m_1 = -Q(1 - A_x) \quad \text{(142a)}$$

2. m_2 due to full uniform live load without considering the direct effect of cable tension (Fig. 11(b))—

$$m_2 = 1 - A_x \quad \text{(142b)}$$

3. m_3 due to symmetrical partial loading from both ends without considering the direct effect of cable tension (Fig. 11(c))—

$$m_3 = A_x (\cosh c\,k - 1) \quad \text{(142c)}$$

(Section x is in the unloaded length measured from either end of the span.)

4. m_4 due to pure couple loading (Fig. 11(d))—

$$m_4 = \frac{\sinh c\,(\tfrac{1}{2} l - x)}{\sinh \tfrac{1}{2} c\,l} (\cosh c\,k - 1) \quad \text{(142d)}$$

(Section x is in the unloaded length measured from the left end of the span.)

5. Whenever section x is shifted from an unloaded part (m) to a loaded part (m'), the value of m is reduced by a component m_5, and vice versa—

$$m_5 = m - m' = \cosh c\,(k - x) - 1 \quad \text{(142e)}$$

(To shift section x from a central region to a side (Figs. 12(a) and 12(b)) the distance x must be measured from the nearer ends of the span to this side.)

The m-value at sections noted in Fig. 12 can be obtained by superposition as follows:

$$m_6 = m_1 + m_2 - m_3 \ (x \text{ measured from either end A or end B}) \quad \text{(143a)}$$

$$m_7 = m_6 + m_5 \ (x \text{ measured from end A}) \quad \text{(143b)}$$

$$m_8 = m_6 + m_5 \ (x \text{ measured from end B}) \quad \text{(143c)}$$

$$m_9 = m_1 + m_3 \text{ (x measured from either end A or end B)} \quad (143d)$$

$$m_{10} = m_9 - m_5 \text{ (x measured from end A)} \quad (143e)$$

$$m_{11} = m_9 - m_5 \text{ (x measured from end B)} \quad (143f)$$

$$m_{12} = m_1 + \tfrac{1}{2}(m_3 + m_4) \text{ (x measured from end A)} \quad (143g)$$

$$m_{13} = m_{12} - m_5 \text{ (x measured from end A)} \quad (143h)$$

$$m_{14} = m_1 + m_2 - \tfrac{1}{2}(m_3 + m_4) \text{ (x measured from end A)} \quad (143i)$$

$$m_{15} = m_{14} + m_5 \text{ (x measured from end A)} \quad (143j)$$

The principle of superposition can be applied only to the derivation of expressions of m, not to the numerical values of m.

Professor Liu declares that the determination of the section of maximum moment is not justifiable because Eqs. 72 and 75 are not rigorously exact. Eq. 72 is exact, but Eq. 75 is obtained with the assumption of a constant c, and the method of solving these two simultaneous equations is also approximate. By checking several existing bridges, it has proved to be quite accurate for practical purposes. Since the variation of moments near the maximum point is small, the procedure given in the paper for determining the maximum moment can be judged justifiable for its simplicity and its directness.

It is hoped that the theory and the design procedure presented in the paper will be tested in practical designs.

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DISCUSSIONS

EXPANSION JOINT PRACTICE IN HIGHWAY CONSTRUCTION

Discussion

BY A. A. ANDERSON

A. A. ANDERSON,[23] ASSOC. M. ASCE.—The lively interest in the subject of joints in concrete pavement continues after 60 years of experimentation, invention, and use under service conditions. The comments made by those who discussed this paper have added to the store of useful knowledge.

Mr. Gardner suggests an extension of the discussion into phases of the subject that were beyond the original intent of the writer. It is true that all the factors mentioned by Mr. Gardner do have a bearing on the subject, but a more comprehensive coverage would not have changed the conclusions.

Regarding Mr. Gardner's six predictions, the writer is in only partial agreement. Experience to date indicates that grooves cut to form contraction joints after the concrete has set do not perform their intended function. The tensile stresses that result from shrinkage will have come into existence before the concrete is sufficiently hard to permit the operation of the cutting machines. At this early age the concrete has little strength to resist tensile stresses, and cracks are likely to have formed before the grooves are cut. Unless the grooves can be cut much sooner than now appears possible, this method of forming contraction joints is not practical.

The installation of tie bars across contraction joints makes it impossible for the joints to relieve tensile stresses in the concrete. Tie bars thus placed will still relieve warping stresses; but, unless distributed steel is used in the pavement, open cracks may be expected to form in the slabs adjacent to the tied joints. If distributed steel is used, contraction joints are tied, and expansion joints are omitted, the result will be a continuously reinforced slab. The percentage of distributed steel that is required will be increased materially. Continuously reinforced pavements are currently under investigation, but tests have not progressed far enough to permit definite conclusions to be drawn.

It is agreed that, if tie bars are used across transverse joints (which cannot then be called contraction joints), load transfer devices may be omitted if the

NOTE.—This paper by A. A. Anderson was published in September, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1948, by Frank H. Gardner; March, 1949, by K. B. Woods, and J. K. Knoerle; and April, 1949, by William A. Conwell.

[23] Mgr., Highways & Municipal Bureau, Portland Cement Assn., Chicago, Ill.

joint is the dummy groove type so that aggregate interlock is present. This procedure could not be followed if the joint were formed by a full-depth, non-deformed metal plate.

Certainly there is opportunity for development of joint sealing materials which will be better than most bituminous materials commonly used. Asphalts that are not too soft at summer temperatures are too hard and brittle at winter temperatures when watertight joints are of greatest importance. The thermoplastic rubber compounds recently developed give promise of much superior performance in this respect.

The need or effectiveness of subsealing in areas adjacent to joints on new construction is questioned. Presumably, in predicting this development, Mr. Gardner has in mind the problem of slab end pumping. Although subsealing has proved effective in correcting pumping on old pavements where steps were not taken at the time of construction to prevent it, other inexpensive and effective methods are available which will insure freedom from this difficulty if applied at the time of construction.

Use of distributed reinforcement will permit slab lengths to be increased as predicted by Mr. Gardner. The longer the slabs are made, the greater the weight of steel that will be required, and the greater the costs will become. Furthermore, the longer slabs will result in more movement at slab ends, increasing difficulties of joint construction and maintenance. For greatest economy and best performance, the joint spacings suggested in the paper are believed to be correct.

No conflict is found between the statements in the paper and conditions reported by Professor Woods or those in the paper[15] to which he refers. The Indiana roads covered in the latter paper[15] included 2,623 miles built without expansion joints. All were 10 years old or more at the time the report was made; thus, they were of ample age to have developed blowups if the blowups were going to appear in more than normal volume. Actually there were 851 miles with no blowups and another 864 miles with only 203 blowups. In contrast, on 284 miles of pavement with five coarse aggregates, there were 1,188 blowups. Nearly 30% of all blowups in the state were on 97 miles of pavement in which coarse aggregate from one source had been used.

It seems fairly evident that the Indiana pavements having excessive blowups were not built of materials with normal expansion characteristics. Thus, the facts agree with the writer's conclusion that little or no expansion space is needed if, among other qualifications, the pavement is constructed of materials having normal expansion characteristics. The abnormal behavior of the Indiana pavements may have been accentuated by capillary moisture supplied by some subgrades, but the difficulty was definitely connected with certain aggregates. It is understood that these unsatisfactory aggregates are no longer used in Indiana concrete pavements.

On other pavements covered in the Indiana survey—pavements not equipped with contraction joints designed for effective sealing—the number of blowups was small. With effective sealing of transverse contraction joints, the

[15] "Pavement Blowups Correlated with Source of Coarse Aggregate," by K. B. Woods, H. S. Sweet, and T. E. Shelburne, *Proceedings*, Highway Research Board, National Research Council, January, 1946, pp. 147–168.

infiltration of inert, noncompressible material into open transverse cracks would be so limited that there would have been even fewer blowups than the small number reported.

At the same time, experience indicates that the use of expansion joints in the pavement having abnormal expansion properties would not have prevented the "mapcracking" or the blowups. The expansion likely to occur with abnormal aggregates cannot be predicted accurately but it may be too great to handle in this manner. Furthermore, internal stresses resulting from the abnormal expansion would have caused deterioration of the concrete even if it had been possible to place joints that produced complete freedom from restraint. Thus, even in this exceptional concrete, expansion joints probably would not have given permanent relief.

Mr. Knoerle suggests the use of distributed steel in proportions that will permit the spacing of contraction joints at 46.5 ft to 61.5 ft, stating that this method results in the lowest over-all cost of joint material and distributed steel. The writer's studies have led to the conclusion that the lowest over-all cost of joints and distributed steel results from a spacing of 28 ft to 35 ft, depending on the cost of joints. Moreover, observations in the field (and particularly on one of the test roads referred to in the paper) lead to the conclusion that movement at slab ends makes it difficult to prevent infiltration into joints where slab lengths are much in excess of 30 ft, so that it may be unsafe to omit expansion joints with the longer contraction joint spacings.

The difficulties of constructing and maintaining contraction joints do not seem as great as implied by Mr. Knoerle; and, as far as maintenance is concerned, the difficulties should be increased by the longer spacings. It is agreed that skilled workmen, good inspection, and supervision are necessary to insure proper construction, but these requirements would hold even if the number of joints were reduced as Mr. Knoerle suggests.

The writer recognizes the advantages accruing from the complete elimination of all transverse joints, requiring the use of fairly heavy continuous reinforcement if tests now (1949) under way demonstrate that design to be practical. Although these tests have not progressed far enough to permit definite conclusions, there are preliminary indications that continuously reinforced pavement is not a panacea for all joint problems.

Mr. Conwell is correct, of course, in pointing out that no material is "incompressible." The term is used to describe material that is compressible to approximately the same degree as concrete; therefore it offers no relief to the concrete. The same objection is applicable to the words "insoluble," "nonconductor," and numerous other terms widely used in technical literature. As long as there is no doubt as to meaning, it is questioned whether the advantages of greater accuracy are sufficient to justify the awkward wordiness required for technical exactitude.

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DISCUSSIONS

ANALYSIS OF PILE FOUNDATIONS WITH BATTER PILES

Discussion

BY JACOB KAROL

JACOB KAROL,[9] M. ASCE.—In his desire to make the method of analysis simpler to use, Professor Hrennikoff has introduced reduced foundation constants into the basic equations of equilibrium from which the pile loads are more easily determined, and has developed a criterion, $\Delta'_x \leq P_a$ (Eq. 25), which presumably indicates the adequacy of the design. Considering the cases in Table 3 for which r_2 and r_3 are zero (pin-ended piles), the criterion indicates that the even-numbered cases, 2*a* to 12*a*, are weaker than their respective odd-numbered counterparts, 3*a* to 13*a*. It will subsequently be shown that, considering displacements only, the opposite is true. Cases 2*a* and 8*a* must be deemed inadequate because the allowable axial pile load of 40 kips has been exceeded.

The actual displacements of the foundation are an inverse function of n. Thus, if the piles were infinitely rigid axially (a case approximated by very short piles driven to rock), the lateral displacement of the foundation even in case 1 would be zero, since, by Eq. 11*a*, $\Delta_x = \Delta'_x/n$. Furthermore, considering the example in Fig. 1, if it is assumed that the axial displacement of the pile is $-3/80$ in., or one tenth of the original value, it follows that the lateral displacement of the foundation will be reduced to one tenth of 0.928 in., or 0.093 in. Most designers would probably decide that the foundation was adequate for both load and lateral displacement, even though the latter was 2.6 times as great as the vertical displacement. The examples illustrate the important fact mentioned by the author, but insufficiently emphasized, that the pile loads are functions of the "relative" values of the pile constants (pile ratios), whereas the foundation displacements are functions of the "absolute" values of these constants. Since the adequacy of a foundation is determined by both the pile loads and the foundation displacements, the designer must establish the absolute values as accurately as possible for any particular design.

NOTE.—This paper by A. Hrennikoff was published in February, 1949, *Proceedings*.

[9] Design Engr., Howard-Needles-Tammen and Bergendoff, Kansas City, Mo.

Dividing through the reduced foundation movements in Table 3 by the values of n from Table 2, the actual foundation movements are obtained and are shown in Table 4. As a matter of interest, the movement of the top of the retaining wall at the inner face is also given, assuming that the wall rotates as a rigid body. The elastic deformation of the stem would have to be added to the lateral movement shown to obtain the final displacement of the top of the wall.

TABLE 4.—MOVEMENTS OF WALL IN FIG. 13 UNDER VARIOUS COMBINATIONS OF PILE RATIOS

Case No.	n	Foundation Movements			Movements at Top of Wall	
		Δ_x (in.)	Δ_y (in.)	α (radians)	Δ_x (in.)	Δ_y (in.)
2*a*	265	−0.870	−0.090	+0.00156	−0.308	−0.076
3*a*	60	−2.164	−0.027	+0.00196	−1.458	−0.009
4*a*	530	−0.307	−0.016	+0.00040	−0.164	−0.012
5*a*	60	−0.650	+0.301	+0.00238	+0.207	+0.322
6*a*	530	−0.104	+0.028	−0.00018	−0.170	+0.026
7*a*	60	−0.211	+0.393	−0.00360	−1.507	+0.361
8*a*	1,000	−0.173	−0.011	+0.00024	−0.086	−0.009
9*a*	280	−0.298	+0.030	+0.00006	−0.277	+0.030
10*a*	2,000	−0.049	+0.003	+0.00001	−0.044	+0.003
11*a*	280	−0.086	+0.076	−0.00066	−0.324	+0.070
12*a*	2,000	−0.014	+0.010	−0.00009	−0.045	+0.009
13*a*	280	−0.034	+0.085	−0.00078	−0.317	+0.078

If a solution of the pile loads is obtained by the conventional method for vertical pile groups using Eq. 16, and a part of the transverse shear is assumed to be carried by the horizontal components of the axial loads in the batter piles, the resulting axial loads are practically the same as those shown for cases 7*a* and 13*a*. The unbalanced transverse shear, as indicated by these cases, could be assumed to be carried by the vertical piles only. Therefore, the conventional method of analysis, with this slight modification, may be considered satisfactory for "medium" soils. In addition, the lateral movement of the foundation is given by the expression:

$$\Delta_x = \frac{Q}{t_\delta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

in which Q is the unbalanced transverse shear carried by one vertical pile.

The error in the interpretation of the results of Table 3 is due to the faulty definition of the value of r_1 as the ratio of the allowable lateral load to the allowable axial load on the pile, and to the use of this value in developing the criterion. This definition arbitrarily assumes that the allowable transverse and vertical displacements of the pile head are the same. For example, for timber piles in medium soil, this results in a value of Q_a for case 6*a* of 1.18

kips and for case 7*a* of 10.4 kips. There is no reason why the value of Q_a should not be the same in the two cases, since the pile and the soil are the same. The discrepancy indicates that in general the allowable lateral displacement of the pile will not be equal to the allowable vertical displacement. Hence the value of r_1 as originally defined in Eq. 8*a*, should be used in the analysis, and the criteria in Eqs. 23 should be used to determine the adequacy of the foundation. Despite these criticisms, Professor Hrennikoff has made a notable contribution to the analysis of pile foundations.

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DISCUSSIONS

CONTINUOUS ARCHES AND BENTS ANALYZED BY COLUMN ANALOGY

Discussion

BY JACK R. BENJAMIN

JACK R. BENJAMIN,[9] ASSOC. M. ASCE.—The simplicity of the examples somewhat handicaps the paper. The two examples are easier to solve by other techniques; and, in order to show the method, obvious short cuts are not used. As an illustration, note by inspection in Example 1 that:

$$\delta_{yy} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

$$\delta_{xy} = \delta_{yx} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13b)$$

and

$$\delta_{yz} = \delta_{zy} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13c)$$

However, to explain the method, simple problems are normally desirable, but not problems that can be solved more simply by other techniques.

Normally the column analogy considers only moment distortion. This is not a requirement. The δ-terms can be made to include direct stress and shear deformations. If the elastic properties are considered sufficiently defined by moment distortions, the load distortions can still be made to include direct stress or shear terms. The modification to consider direct stress can be applied very easily. In Example 1, the direct stress deformation in column CD could be significant. It can be considered as a shortening of CD, and the moments resulting from it can be assumed to be the same as if the support at C settled an equal amount.

The imposition of deformations rather than of forces to determine the rigidity of the substitute member will shorten the work in some cases and give additional information. In the application of relaxation procedures, forces resulting from unit displacements are desired. Structures such as those in Fig. 1 can be solved by a combination of the column analogy with relaxation, or by algebraic procedures in which not only forces but deflections will be simultaneously obtained.

The author is to be complimented in bringing the column analogy again to the attention of the profession. The author's procedure seems to offer the most competition to current techniques and to be at its best when applied to continuous arches on tall flexible piers and similar complicated problems.

NOTE.—This paper by T. M. Yu was published in February, 1949, *Proceedings*.

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DISCUSSIONS

THE LOGICAL SYSTEM OF FREIGHT RATES

Discussion

BY HAYWOOD R. FAISON, AND MALCOLM ELLIOTT

HAYWOOD R. FAISON,[5] M. ASCE.—At first glance the design of a logical scale of transportation charges might appear to fall outside the province of the engineer, but a close look at the existing rate structure reveals inequalities that add complications to practically every problem that confronts him. Transportation seems to be the only important element in the entire process of production, assembly, manufacture, and distribution that cannot be measured in terms of a standard yardstick of uniform value. In 1947 public carriers transported 995,000,000,000 ton-miles of freight at a cost to the American people of $13,000,000,000,[6] but under those impressive totals lies an almost hopeless tangle of conflicting rate bases. Under a logical scale of charges the concentration of fuels and raw materials and the distribution of the end products could be expected to occur at rates established to minimize the need for transportation, in recognition of the natural accessibility of sources and markets to the milling plants. Under the prevailing system, however, these processes are controlled by rates adjusted by the carriers, in so far as regulation permits, to encourage long hauls, to discourage competitive movements, and to exact as much revenue from the remainder of the freight as the traffic will bear. There is nothing astonishing in this policy of the carriers thus to promote the commodity which they have to sell, as long as legislative and regulatory bodies seem to encourage a view of transportation as an end in itself, rather than as a means to an end. There is room for legitimate question, however, whether this has been the policy best adapted to the development of the United States along sound lines.

Before discussing any shortcomings in the author's formula it may be appropriate to examine the flaws which he seeks to cure. The original system, under which transport agencies were supposed to "take care" of the producers and shippers on their respective lines by independent rate adjustments, was inherited by the Interstate Commerce Commission before 1890 and many of its

NOTE.—This paper by Charles L. Hall was published in November, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1949, by Raphael H. Courland; February, 1949, by Otto Meyer; and April, 1949, by Alexander Markowitz.

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[6] Annual Report of the Interstate Commerce Commission, Washington, D. C., 1948, p. 14.

defects still remain to plague the shipping public. Apparently the majority regard this benevolent dominion over its welfare as just another unnecessary and unpredictable business risk. The commission's efforts to eliminate such discrimination have been hampered not only by the fact that its function has not been to establish rates but rather to pass judgment upon them, but also by the fact that the original, basic rates available for measuring comparative reasonableness may, in themselves, have been unfair and discriminatory.

According to prevailing practice, the selling price of transportation represents a three-way compromise among conflicting requirements—the demand for the service, the revenue needs of the carrier, and the ability of the traffic to pay. Only in the aggregate must earnings cover the full cost of operation, fixed charges, and return on investment. Demand for the service is measured by the difference in price of the goods at origin and destination, and this spread, in turn, is determined by the carrier's test of what the traffic will bear. Within regulatory restrictions, rates are continuously adjusted and readjusted in response to what these agencies believe to be to their best interests. The doubtful economic soundness of this policy is defended on the so-called "out-of-pocket-cost" or "added traffic" principle. This theory recognizes two distinct kinds of expense—(1) those constant, general costs that can be assigned only to the conduct of the enterprise as a whole, and (2) those variable expenses that are directly allocable to specific commodity movements. Costs in the first group do not vary with temporary fluctuations in tonnage; those in the second group are added as the traffic is added and are saved when the freight is not hauled. Advocates of this basis of rate making, chiefly among the railroads, claim that with the transportation plant functioning and the supervisory staffs employed, a few more tons can always be added to a car, or a few more cars added to a train, without incurring a commensurate increase in total carrier expense. In accordance with this philosophy they can successfully justify the establishment of competitive rates covering only a fraction of the fully distributed costs, and yet term them compensatory.

The theory clearly requires the assumption of ample unused freight carrying capacity and its validity obviously depends upon whether the long-term result is sought. There would be no profit in merely meeting competition on a permanent scale of charges fixed below fully compensatory levels because, in the long run, the maintenance of such a scale would constitute a drain on invested capital. The only logical objective would be the effective discouragement of competition followed by the restoration of monopolistic rates. The flaw in the logic of this policy is readily seen.

Railroad ton-mileage has increased more than fivefold since 1900. In the normal development of each major line there has come a point of growth when one more ton required an additional car; when one more car required an additional train; and when one more train called for double-tracking the main line—each additional step being accompanied by a commensurate expansion of terminal and yard facilities and full overhead. Hence, over the long-term period, out-of-pocket costs have practically coincided with full costs, and economic justification for noncompensatory rates on the "added traffic" basis disappears. Despite its obvious unsoundness, however, the principle still is invoked in

support of most rate cases fought before regulatory agencies. Since the burden of meeting constant overhead expenses is merely shifted by such distortion of rates from competitive tonnage to noncompetitive tonnage, the public pays the bill in any case. Notwithstanding the skepticism with which the rate experts view suggestions from the outside, it can be safely stated that a thesis of such spurious validity would not long survive ordinary engineering analysis.

Added to the illogical character of the existing rate system is the companion defect of instability. No sooner have general adjustments been decreed than the jockeying for special advantage begins all over again. In 1947 no less than 109,965 tariffs containing changes or exceptions to freight, express, pipe line, and forwarder rates were published, and this number does not include the thousands of intrastate tariffs.[7] In order for the validity of one of these rates to be challenged successfully, the ponderous machinery of the Interstate Commerce Commission must be set in motion by the complainants. Some of the more comprehensive rate proceedings may grind on for years, until the situations have changed so radically as to render the dockets obsolete. To claim that such cumbersome procedure is in any appreciable degree responsive to current and changing needs of business would be unrealistic. The author proposes to cure these defects with a highly simplified, rigid, mechanical system of fixing rates on the basis of the full cost of providing the transportation service.

The science of cost finding in transportation has developed to a point where it may be called well nigh perfect. Objection to the use of costs comes, not from the shipping public, but from the firmly established carriers and rate making specialists, learned in the rituals of the existing labyrinthian system. Nevertheless there is a clearly discernible trend not only in rate regulation but in the conduct of modern transportation to rely more and more on cost as a guide to reasonable and profitable rates. Such authorities as J. B. Eastman (Petroleum Products Case (194-I.C.C.-31)), former chairman of the Interstate Commerce Commission and wartime Director of Defense Transportation; G. E. Talmage, Jr.,[8] Director of the Water Carrier Division of the Interstate Commerce Commisssion; *Traffic World*;[9] F. E. Asher,[10] representative of an influential group of midwestern motor carriers; and the Senate Subcommittee on Small Business,[11] have expressed themselves as believing that a system of rates for all carriers based on cost plus a reasonable profit would be better for all concerned, including the nation itself and its industries, than any system of competitive rates. As a method of offsetting government subsidies, E. A. Jack,[12] General Traffic Manager of Aluminum Company of America suggests the enforcement of rate differentials among the respective forms of transportation, arrived at by using the bare cost of operating each facility, exclusive of roadway expense. This proposal drew a sharp rejoinder from A. Markowitz,[13] Jun. ASCE, spokes-

[7] Annual Report of the Interstate Commerce Commission, Washington, D. C., 1948.
[8] *Marine News.*, October, 1946.
[9] *Traffic World*, June 15, 1946.
[10] *Ibid.*, September 14, 1946.
[11] *Print No. 14*, 79th Cong., 2d Session, 1946.
[12] *Traffic World*, November 6, 1948.
[13] *Ibid.*, December 11, 1948.

man for a New Jersey motor express company, who offered as a counter-suggestion the dropping of the controversial subject of subsidies in favor of a general improvement in service under a rate policy, making rates upon the fully allocated costs of each form of transportation.

Despite this evidence of fairly general concurrence on full carrier costs as the appropriate basis for establishing rates, this writer does not agree that it would be practicable or expedient to impose a rigid service cost basis on the transportation system of the United States all at once. He believes that realization of its manifest advantages should be set up as a long-range objective of rate regulation. It would be attained by gradual and cautious advances, avoiding, on the one hand, abrupt dislocations of the industrial pattern that would entail extensive sacrifices of invested capital, and, on the other hand, the letting down of the bars to the usual reactionary forces of disintegration that have always heretofore frustrated efforts toward uniform standardization of the rate structure.

To this critic, the ingenious mechanism designed by Colonel Hall to implement his proposal seems to meet many of the practical requirements for a workable rate formula. Its manifold advantages should not be discounted in emphasizing its defects. The writer must confess that he cannot see all the way through to a happy solution of all the complexities that might arise in the application of the proposed formula. He is not convinced that the element of demand for the service can be safely ignored altogether. He is not certain that the distance scales should not be modified by factors that recognize regional differences in density of population and functional differences in through and local hauls. He believes that certain commodities need milling-in-transit privileges and the the liberalizing of transit arrangements to eliminate preference for particular types of carrier will prove difficult. However, many refinements to take care of special situations undoubtedly could, and would, be made in the gradual adaptation of a uniform policy. The writer is inclined to leave the mechanical adjustment of the system to the traffic experts and to concentrate on what he believes to be the most significant passage in the paper under discussion.

The author puts his finger on the most serious deterrent to the adoption of his proposed system when he recognizes the inconsistency in the existing competitive situation in which some modes of transportation must pay for their roadbeds, whereas the routes of others are partly or wholly subsidized by federal or state governments. He contends that a subsidy to any carrier, or its operation with public assistance at an admitted loss, unbalances the entire setup. Only if carriers meet all their expenses out of their own receipts will the logical system he proposes work smoothly. Since it appears highly improbable that public aids to highway, airway, and waterway development will ever be entirely abandoned as a national policy, the only way that carriers operating over these types of improvements could be brought into line with the author's requirements would be by the assessment of tonnage taxes on cargo or cargo capacity sufficient to pay the carrying charges on the facilities they use. The author infers that until this is done, there can be no just and reasonable application of a uniform service cost basis of rates.

The writer holds that the converse is of equal significance, in that there can be no just and effective application of the tolls principle to motor or water transportation unless, and until, a rigid cost-of-service basis of rates is uniformly imposed on all types of carrier. Because of their ubiquity, railroads, for instance, can handle any commodity movement open to any other mode of transportation in intercity carriage. Under the present system they can and do depress their rates on any block of competitive traffic they choose to select; and they raise rates proportionately on other traffic out of reach of the restricted types of carrier. Barge lines, in particular, have no such blocks of noncompetitive traffic within the limited scope of their transportation activities upon which they could shift the burden by levying higher rates. They have no noncompetitive freight. All their tonnage movements must carry their full share of the total expense if these enterprises are to prosper. If tonnage taxes were imposed on their operations, such taxes would have to be passed along to the shippers in proportional additions to each rate. It thus becomes clearly apparent that these water lines would be at the mercy of carriers with diversified traffic and could be eliminated from the competitive field, one by one, by selective rate cutting. Under a tolls system, their only defense would be a rigidly enforced adherence on the part of all modes of transportation to some such system as that proposed by the author, in which each commodity movement would be charged its commensurate share of full carrier expense.

On certain vital aspects of transportation there is practically unanimous concurrence. It is universally conceded, for instance, that adequate carrier potential must be maintained on a sound and stable footing to enable it to handle the national freight burden in time of war. The transportation agencies not only must be prepared to move the ordinary goods of commerce along established routes in sharply increased volume, but must be ready for prompt expansion of facilities to handle uncommon types of matériel and munitions between unaccustomed terminals on short notice and with minimum delay and congestion. As indicated by the author there is opportunity here for sane compromise between the needs of peace and of war. There is no justification for setting the controlling scale of rates so high in peacetime as to perpetuate the penalties of unsound conception, faulty design, and extravagant financing that characterize many marginal transportation units. Most of these high cost properties furnish an unnecessarily expensive peacetime surplus of freight carrying capacity, basically incapable of profitable operation on any rational basis and representing their maximum recoverable value as salvage. Under the rate structure proposed by Colonel Hall, these wasteful enterprises would be eliminated from the competitive field and the national transportation system made healthier by such economic surgery.

Any reliable industrial map of the United States shows that the principal manufacturing centers (which, according to Colonel Hall, should be located with a view to their accessibility to raw materials and markets) have long since become almost irrevocably concentrated by other considerations into a relatively few limited areas. Under the existing rate structure, with its low charges on long hauls favoring these concentrations, there is but little economic incentive toward changes in location. Doubtless, had the author's rate making

formula been adopted a century ago, development of national resources would have followed a more rational design and the problem of sunk capital would not now count so heavily against dispersal. The principal grounds for hope for a more favorable distribution of processing sites in accord with a balanced transportation pattern seem to lie in the manifest trend of national activity since 1945 toward general decentralization. The writer does not see this trend as wholly accounted for by considerations of security, but deems it reasonable to assume that the ever-increasing proportion of total production costs represented in freight charges is also becoming a compelling force toward dispersal of plant sites. With the gradual establishment of a logical system of freight rates, the writer believes that a progressive migration not only of maunfacturers but of consumers to the areas nearer the sources of supply might well be set in motion to the obvious net benefit of the nation at large. Assuredly, any workable procedure that gives reasonable promise of bringing about simplification of the rate system together with reduction in the net transportation bill of the nation to the practical minimum deserves serious consideration.

MALCOLM ELLIOTT,[14] M. ASCE.—Colonel Hall defines the illogical features of the present system of freight rates and makes a convincing plea for a new system. If one were to accept what appears to be the author's fundamental assumption—that freight rates shall be based wholly on cost to carriers of rendering the service—his formula might well be regarded as a logical approach to determination of rates on that basis.

In the writer's opinion, however, the problem is not that simple. Actually the rates for transportation of persons and commodities are based in part on factors other than the cost of rendering service. There are to be considered such factors as competition with other forms of transportation, the necessity (in the interest of national welfare or defense) of subsidies for some or all means of transportation, the effects of changing freight rates on established industries—especially in cases where such changes might drive a heavily capitalized business out of existence—and possibly other complicating factors.

One of the most important factors in determining the price for rendering any kind of service is "what the traffic will bear." Practically all competitive business in the United States—as well as in any other country in which "free enterprise" is the guiding principle—bases prices to some extent on what the public is willing to pay. Otherwise the law of supply and demand would have no place in the economy. In the transportation business, however, the determination of rates on a wholly competitive basis is thought to be impracticable. Freight rate stabilization is required by law even though uniformity of prices is a crime in other industries when accomplished by agreement between competitors. Rate stabilization by government regulation, however, should take some account of competitive situations. It might even go so far as to allow a transportation company to carry some of its cargo at a loss when to do otherwise would force the company to abandon service that might be vitally important to established communities or to public welfare.

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The author's desire to establish a factual basis for determining transportation costs is commendable. To a large extent these costs should have an influence on the establishment of rates. Nevertheless, it would appear that these costs—by whatever formulas they are developed into rates for various classification of commodities—cannot be adopted as the sole factor in determining freight rates.

Granting that a more logical system of utilizing costs in determining rates is desirable, one may question whether the preparation of the corresponding rate book would not be attended by many more complications than are foreseen in the paper. What would become of the "processing-in-transit" rates whereby wheat, for example, is shipped to a milling city, processed there into flour, and then shipped out to a final destination under one bill-of-lading whereby the proportionate rate to the milling city is lower than the rate for the same haul for local consumption? Under the author's system of basing rates wholly on costs, the rate from point of origin to the milling city would be the same whether the wheat were intended for local consumption or for processing in transit. It is not the writer's intention to discuss here whether such a change would be desirable, but it would certainly cause some concern to the milling industry.

With rates based wholly on cost is it intended to require that all rates should be remunerative? Enforcement of such a criterion might wipe some communities "off the map."

These difficulties and others of like character would have to be given very careful consideration in revamping the rate structure along the lines suggested by the author. Of course, there should be no objection to a realistic approach to the subject, as is suggested in this paper; and, even if it is found that costs cannot be the sole basis for fixing rates, such an approach should show what adjustments and subsidies are involved in financing and operating a transportation system.

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DISCUSSIONS

INTEGRATING THE EQUATION OF NONUNIFORM FLOW

Discussion

BY A. H. DAVISON, AND STEPONAS KOLUPAILA

A. H. DAVISON,[14] M. ASCE.—Agreement with the author's general conclusions presents no difficulty. Practical problems almost never offer a case of strictly uniform flow. The nearest case to a uniform flow problem in the writer's experience was the determination of flow conditions in a power canal that was designed to conform to the capacity output of the hydroelectric plant at the lower end of the canal. Considering peak periods, capacity plant operation occurs for a total of about 2,000 hours per annum which is less than 25% of the total time; and even during the periods of capacity plant operation the stage in the canal does not often remain constant.

A proposed method of working with nonuniform flow should be both convenient and accurate. A method that may be convenient to the specialist who encounters many engineering problems of the same general type may be quite inconvenient and formidable to the engineer in general practice who must achieve a sound engineering solution without spending too much time on a preliminary refresher course in involved technique.

To achieve accuracy, much depends on a correct evaluation of the factor n, the well known coefficient of roughness which appears in the Manning and Kutter formulas.

With respect to artificial channels, the initial evaluation of n can doubtless be a quite accurate assumption based on laboratory experiments; but these experiments were conducted under uniform flow conditions. If actual operating conditions vary both ways from the uniform flow condition as an average, probably the assumed value of n is realistic enough except for effects of frost and weather exposure with the passage of time. If actual operating conditions are otherwise, perhaps some consideration should be given to the probability that the value of n will vary with wetted perimeter.

In cases of natural streams, an assumed value of n is never reliable particularly for high water or flood conditions in pond areas above a dam. In such

NOTE.—This paper by M. E. Von Seggern was published in January, 1949, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1949, by Philip Z. Kirpich and Paul V. Hodges.

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areas the backwater curve is convex upward, similar to the author's M_2-curve. For this type of curve, a realistic value of n which may be 15% higher than the assumed value means that the length of the backwater curve will decrease from 40% to 60%, depending on circumstances, for the same value of Q. This is an indication that the actual backwater curve will shoot up much more abruptly than expected, facing upstream. An assumed value of n is often too low by 50%. This can have serious results as many power companies have discovered to their embarrassment, when faced by unexpected flood-damage claims involving property that was considered safely located.

The first step in analyzing nonuniform flow problems is to compute the value or values of n corresponding to the range in discharge, from consideration of high water marks and channel conditions along the reach being studied. Such computed values of n will be much more realistic than any assumptions. The technique and computations applied in analyzing practical nonuniform flow problems should be in such form as to emphasize, rather than obscure, this important factor, which is admittedly difficult to evaluate correctly.

Consider a reach bounded by two channel cross sections (section 1 and section 2, reading downstream) which are separated by the distance L. The equation of nonuniform flow is

$$S_e = S_f - \frac{V^2_1 - V^2_2}{2\,g\,L} \quad \ldots\ldots (34a)$$

$$\frac{V^2_1 - V^2_2}{2\,g\,L} = \frac{1}{2\,g\,L}\left(\frac{Q^2}{A^2_1} - \frac{Q^2}{A^2_2}\right) \quad \ldots\ldots (34b)$$

and

$$Q = \frac{1.486}{n} A\, R^{2/3}\, S^{1/2}_f \quad \ldots\ldots (35)$$

Eq. 35 is the Manning formula, with the factors A and $R^{2/3}$ representing the mean of the corresponding values applicable to the two cross sections of the reach, and the factor n signifying the coefficient of roughness. The expression for the slope of the water surface is

$$S_e = Q^2\left[\left(\frac{n}{1.486\,A\,R^{2/3}}\right)^2 - \frac{1}{2\,g\,L}\left(\frac{1}{A^2_1} - \frac{1}{A^2_2}\right)\right] \quad \ldots\ldots (36)$$

For uniform flow, the second term in the brackets in Eq. 36 becomes zero, and

$$S_e = \left(\frac{Q\,n}{1.486\,A\,R^{2/3}}\right)^2 \quad \ldots\ldots (37)$$

For critical discharge,

$$Q = A\sqrt{\frac{A}{b_w}}\sqrt{g} \quad \ldots\ldots (38)$$

For the backwater curve, Eq. 36 may be written

$$\frac{1}{2\,g}\left(\frac{Q^2}{A^2_1} - \frac{Q^2}{A^2_2}\right) = L\left[\left(\frac{Q\,n}{1.486\,A\,R^{2/3}}\right)^2 - S_e\right] \quad \ldots\ldots (39)$$

substituting $S_s = \frac{d_1 - d_2}{L} + S_0$,

$$(d_1 - d_2) + \frac{1}{2g}\left(\frac{Q}{A_1} - \frac{Q}{A_2}\right)\left(\frac{Q}{A_1} + \frac{Q}{A_2}\right) = L\left[\left(\frac{Q\,n}{1.486\,A\,R^{2/3}}\right)^2 - S_0\right] \quad (40)$$

In Eq. 40, the factors d_1 and d_2 are the depths corresponding to A_1 and A_2. Except as otherwise stated, other factors in Eqs. 34 to 40 are the same as defined in the paper. Application of these formulas as required, with assistance from tables giving the physical and hydraulic properties of rectangular and trapezoidal sections, produces the solution of a practical problem with slide-rule computations. A certain amount of tabulation is required but the procedure is less tedious than appears at first glance.

No difficulty was experienced in checking closely the coordinates computed by the author for backwater or drop-down curves illustrated by Examples 1, 2, and 7 of the paper. There seems little point in computing the total over-all length of the surface curve in Examples 1 and 2. The author states, "Theoretically the surface curve would be infinitely long * * *." Usually all that is required is to develop the profile of the surface curve within practical limits.

With respect to Example 7, the writer's computation procedure, using Eqs. 34 to 40, established the factors without resort to "trial and error" included in the author's procedure. The slight discrepancy between the resulting rating curves as computed by the two procedures (see Fig. 11) would disappear if

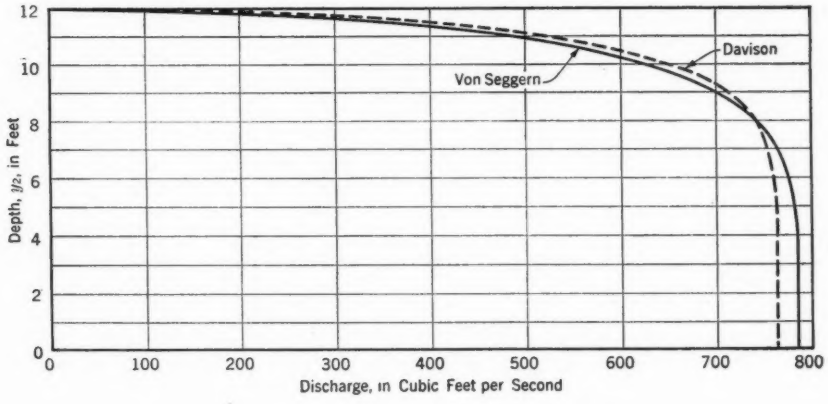

FIG. 11.—COMPARISON OF DISCHARGE CURVES.

the writer had used values of n varying from 0.027 for 200 cu ft per sec, to 0.0256 for 700 cu ft per sec, instead of the constant value of $n = 0.025$ throughout. Since the difference in these n-values is well within the range of allowable difference of opinion with respect to assumptions, the discrepancy in results is unimportant.

STEPONAS KOLUPAILA.[15]—The method concerning the integration of the equation of nonuniform flow in prismatic channels, proposed in the paper, is an interesting extension of the well known method advanced by Boris A. Bakhmeteff, Hon. M. ASCE, in 1912 and 1932. The application of that method, even when facilitated by use of the tables of the varied flow function, is still very complicated, because the expression $(1 - \beta)$ depends on the critical depth. Mr. Bakhmeteff suggests that the variation of $(1 - \beta)$ versus the water depth may be represented graphically and then approximated by successive mean values for the various increments.[16]

The method suggested by Mr. Von Seggern is more universal and convenient, as he provides the necessary tables, as well as the diagrams, for determining values of the hydraulic exponents for trapezoidal cross sections of channel. The exponents can be found in these diagrams in terms of the side slope of channel and the ratio of water depth to the bottom width.

The writer would like to suggest that the author search for a possible simple relation between two hydraulic exponents—one for the normal depth and a second for the critical depth. Such a relation would facilitate the procedure of computation.

Mr. Von Seggern's method still requires the determination of the critical depth for each particular discharge. For trapezoidal cross sections, this is rather difficult. The determination of critical depth may be simplified appreciably by the use of diagrams, such as those used by Russian engineers.[17,18]

In eastern Europe the practice is to attempt to simplify the otherwise difficult backwater computation by use of tables, which cover cases of positive slope and negative slope and for the horizontal bottom. In other methods sometimes used, as that of N. N. Pavlovskij,[19] no tables are employed, but each case is solved by integration.

Corrections for *Transactions:* In January, 1949, *Proceedings*, pages 111 and 112, the symbol b_w in the ordinate captions of Figs. 1 and 2 should be b.

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[16] "Hydraulics of Open Channels," by Boris A. Bakhmeteff, McGraw-Hill, Book Co., Inc., New York, N. Y., 1932, p. 53.

[17] "Brief Consulting Book of Hydraulics," by N. N. Pavlovskij, Kratkij Gidravličeskij Spravočnik, Techgosizdat, Leningrad, 1940 (in Russian).

[18] "Hydraulics," by Steponas Kolupaila, Kempten, 1947 (in Lithuanian).

[19] "Upon the Nonuniform Flow at Horizontal Bottom of the Channel," by N. N. Pavlovskij, Izvestija Naučno-Melioracionnogo Instituta, 1930, No. 20, Leningrad (in Russian).

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DISCUSSIONS

DYNAMIC INSTABILITY OF TRUSS-STIFFENED SUSPENSION BRIDGES UNDER WIND ACTION

Discussion

BY F. B. FARQUHARSON, GEORGE S. VINCENT, AND ROBERT E. MCHUGH, JR., AND FRIEDRICH BLEICH

F. B. FARQUHARSON[21] AND GEORGE S. VINCENT,[22] MEMBERS, ASCE, AND ROBERT E. MCHUGH, JR.,[23] JUN. ASCE.—At the request of the Advisory Board on the Investigation of Suspension Bridges, tests have been made at the University of Washington in Seattle for the purpose of verifying, experimentally, the flutter analysis presented by Mr. Bleich. The purpose of the tests was:

(1) To verify the pure flutter analysis as applied to a thin flat plate in a horizontal wind;
(2) To verify the combined flutter and vortex force analysis as applied to a flat plate with blunt edges or rudimentary girders; and
(3) To determine the degree to which the analysis, developed for a horizontal wind, will indicate the behavior of the section in an inclined wind.

The basic model was a built-up section 12 in. wide, 60.75 in. long, and $\frac{1}{4}$ in. thick. For models B and C (Table 3), the edges were semicircular and sharp, respectively, to minimize vortex discharge in a horizontal wind and thus approach the theoretical conditions of an infinitely thin flat plate. For model A, the edges were square, giving a depth-to-width ratio of 0.0208 and bringing into play small vortex forces. For models G-1 and G-2, model A was used with rudimentary girder webs of fiberboard 0.012 in. thick, attached to the edges to give depth-to-width ratios of 0.05 and 0.067, respectively.

The plates were built up of magnesium with copper edges to give values of the ratio of mass radius of gyration to width within the range generally found for suspension bridges. A wood model H, 13.21 in. by 1.58 in., with semi-

NOTE.—This paper by Friedrich Bleich was published in October, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1949, by A. G. Pugsley, and S. K. Ghaswala.

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[23] Engr. Assistant, Univ. of Washington, Suspension Bridge Investigation, Seattle, Wash.

circular edges, was available and was tested for the purpose of comparison, although its depth-to-width ratio was 0.12.

The mass properties of the various models (including attachments and one third of the spring) are shown in Table 3. They differed little and all models were tested on the same set of coil springs, with freedom to move vertically as well as to rotate about a longitudinal axis. However, models G-1 and G-2 were also tested with added mass to alter the vertical and torsional frequencies, and again with torsional spring action of three different strengths acting in parallel with the vertical springs, in order to modify further the relative vertical and torsional frequencies.

TABLE 3.—COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

Model[a]	Weight (lb)	r (in.)	$\left(\frac{\omega_2}{\omega_1}\right)^2$	Flutter Frequencies ω (Radians per Sec)		Critical Velocity v_c (Ft per Sec)		k_c (Critical Value of k)		Theoretical[b] Air Force Function, f_4	
				Observed	Theoretical	Observed	Theoretical	Observed	Theoretical	A	B
A	15.68	4.784	1.744	8.168	7.900[c]	11.33	13.58[c]	0.360	0.291[c]	0.0271	0.0360
B	15.37	4.770	1.786	8.273	8.032[c]	12.15	13.94[c]	0.340	0.288[c]	0.0164	0.0314
C	14.32	4.639	1.848	8.650	8.357[c]	12.32	14.26[c]	0.351	0.293[c]	0.0112	0.0354
H	14.54	4.314	1.971	8.671	....	11.67	....	0.409	....	0.0588	0.0251
G-1	15.68	4.784	1.744	8.273	....	8.83	....	0.468	....	0.1680	0.0297
			2.849	10.263	....	12.29	....	0.418	....	0.1707	0.0552
			3.129	10.891	....	13.16	....	0.414	....	0.1221	0.0675
			3.327	11.331	....	14.32	....	0.396	....	0.0667	0.0751
G-1	17.31	4.569	1.877	8.168	....	9.15	....	0.446	....	0.1552	0.0445
G-2	15.68	4.784	1.744	8.482	8.584[d]	6.84	6.66[d]	0.620	0.644[d]	0.1900	0.0672
			2.856	10.660	10.650[d]	8.66	8.63[d]	0.616	0.617[d]	0.2037	0.0896
			3.149	11.205	11.236[d]	9.32	9.15[d]	0.601	0.614[d]	0.2472	0.0838
			3.357	11.624	11.624[d]	9.42	9.49[d]	0.617	0.612[d]	0.2557	0.0867
G-2	17.11	5.228	1.477	7.540	7.605[d]	6.00	5.78[d]	0.628	0.658[d]	0.1712	0.0594

[a] Descriptions: Model A, square edges; model B, rounded edges; model C, sharp edges; model H, thick plate; model G-1, 0.6-in. girder; and model G-2, 0.8-in. girder. [b] Theoretical value of f_4 based on the corresponding observed value of k_c. [c] Theoretical values, assuming pure flutter—that is, $f_4 = 0$. [d] Theoretical values based on values of f_4 taken from Fig. 18.

The motion of the model was recorded on a Brush oscillograph, receiving the amplified output of a Statham gage incorporated in the support of one of the springs.

The values of ω_1 and ω_2, the circular frequencies (radians per second) of the vertical and torsional oscillations of the models, were determined by manual excitation of these modes in still air. When the model was manually excited in torsion in a wind stream, the frequency was less than in still air and the axis of rotation was no longer on the longitudinal axis of the plate but was shifted upwind several inches. The frequency, ω, at the critical velocity, v_c (feet per second), is recorded in Table 3 for each model, together with the observed value of $k_c = \frac{\omega_b}{v_c}$.

The logarithmic decrements, δ_o, were determined in still air. These averaged about 0.015 for the vertical mode and 0.005 for the torsional mode. These values of δ_o result mostly from air resistance. Previous tests of the same springs with streamlined weights of about the same mass as these models

showed δ_s, the logarithmic decrement created by structural damping, to be about 0.002. The tests demonstrated that the aerodynamic damping as measured by the logarithmic decrement increases with amplitude—that is, with the velocity of vibration. This increase is an indication that the air forces opposing the oscillation vary as the square of the velocity in accordance with Froude's law. Tests of bridge models have indicated that only a small part of the air damping has a constant logarithmic decrement which is characteristic of viscous damping, on which Reynolds' law is based.

For the flat plate models A, B, and C, the theoretical flutter velocity, v_c, and flutter frequency, ω, were computed using Eq. 59 with $D = 1$. (Since the section model is not distorted but moves as a unit, the vertical and torsional motions have the same displacement form.) The logarithmic decrement of structural damping, $\delta_s = 0.002$, was only one twenty fifth of that damping, which accounted for 6% increase in critical velocity in the author's illustrative example (see text discussion of Figs. 10 and 11). It was therefore neglected in these calculations.

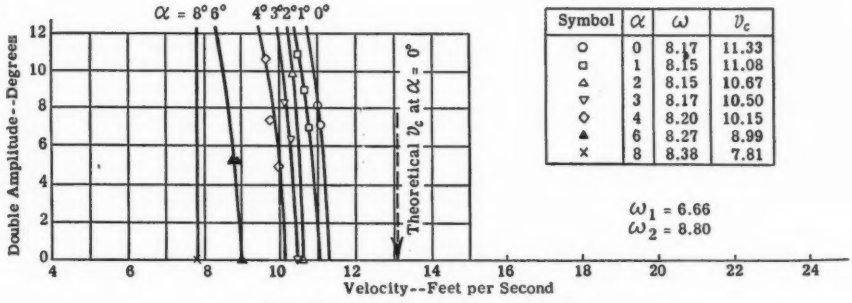

α = 8° 6° 4° 3° 2° 1° 0°
Double Amplitude--Degrees
Velocity--Feet per Second
Theoretical v_c at α = 0°
Symbol | α | ω | v_c
○ 0 8.17 11.33
□ 1 8.15 11.08
△ 2 8.15 10.67
▽ 3 8.17 10.50
◇ 4 8.20 10.15
▲ 6 8.27 8.99
× 8 8.38 7.81
ω_1 = 6.66
ω_2 = 8.80

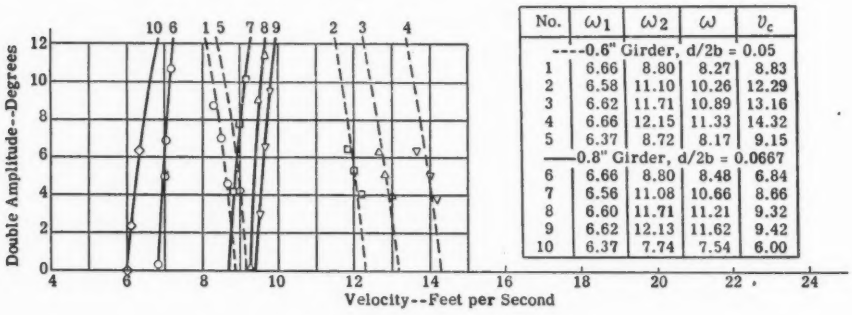
FIG. 15.—FLAT PLATE, SQUARE EDGES

The curves in Figs. 15 and 16 show the amplitude plotted against the wind velocity for typical tests. The curves that are vertical or slope backward indicate that as the velocity is slowly increased no oscillation occurs until the critical velocity is reached, whereupon motion begins spontaneously and the amplitude increases indefinitely. The condition is one of instability—that is, there is no steady-state amplitude. This characterizes the flutter phenomenon. This is the sense in which the author uses the term "instability" as, for example, in the paragraphs following Eq. 86. He does not mean that a "stable" section remains stationary but, rather, that it maintains a stable condition of either rest or steady-state vibration.

A backward sloping curve also indicates that if, at velocities slightly below the critical value, the model is disturbed in any way so that its displacement reaches the amplitude indicated by the curve for that velocity, then the motion will increase indefinitely. In other words, an increase in either velocity or displacement which plots to the right or above the curve precipitates instability.

A response curve that slopes forward has an entirely different meaning. At any velocity above the critical, it shows the steady-state amplitude that

will be built up and maintained until the velocity is increased or decreased. Although this type of response often becomes catastrophic when the velocity increases, its stability or steadiness of amplitude at a given velocity is in contrast to the instability of pure flutter. It is attributed to the combined action of the flutter forces and other forces induced by vortex action discharged from the leading edge. (For different shapes, the relative influence of flutter and vortex forces can vary over a wide range, including many shapes for which flutter is essentially nonexistent.) These vortices, readily visualized by smoke stream tests on stationary and oscillating models, may at first be discharged at some multiple of the natural frequency of the model, but when vibration begins they quickly take on the frequency of the vibration, giving rise to zones of modified pressure moving periodically across the section.



No.	ω_1	ω_2	ω	v_c
----0.6" Girder, d/2b = 0.05				
1	6.66	8.80	8.27	8.83
2	6.58	11.10	10.26	12.29
3	6.62	11.71	10.89	13.16
4	6.66	12.15	11.33	14.32
5	6.37	8.72	8.17	9.15
——0.8" Girder, d/2b = 0.0667				
6	6.66	8.80	8.48	6.84
7	6.56	11.08	10.66	8.66
8	6.60	11.71	11.21	9.32
9	6.62	12.13	11.62	9.42
10	6.37	7.74	7.54	6.00

FIG. 16.—GIRDER SECTIONS ($\alpha = 0°$)

This second type of response curve (with a positive slope) was found in a horizontal wind on only model G-2 (0.8-in. girder, Fig. 16), having a $\frac{d}{2b}$-ratio of 0.0667. When the angle of attack was increased to 8°, it appeared also on model B, the round-edged plate. The vortex forces influence all the other response curves also, even though they do not reverse the slopes. A measure of their influence is indicated by the difference between the observed critical velocity and the theoretical value that was computed by solving Eq. 59 (as in the author's example under the heading, "Illustrative Problem," in Section 5) and is shown in Fig. 15.

In Fig. 17 the amplitude response curves for the various sections in a horizontal wind are plotted against k, which places them on a common basis, except for the small effect of minor differences in mass distribution. The presence of vortex forces tends to reduce the critical velocity and increase k_c, this effect naturally being greatest for the blunt sections and least for the round-edged and sharp-edged ones. The marked influence of streamlining is shown by the relative position of the curve for model H, which has a $\frac{d}{2b}$-ratio nearly twice that of the deeper girder section, yet shows a vortex effect only slightly greater than does the thin, square-edged model A. It should not be concluded, however, that streamlining affords the solution to objectionable oscillation. Al-

though it does reduce the vortex forces and therefore increases the critical velocity, it does not insure a safe section if a violent flutter, which it cannot eliminate, falls within the range of possible wind velocity.

The magnitude and phase of the vortex force as represented by the coefficients, A and B, in Eqs. 71 and 72 were computed using the experimental values of ω, ω_1, ω_2, and k_c and solving Eq. 82, in which Eq. 59 was substituted for

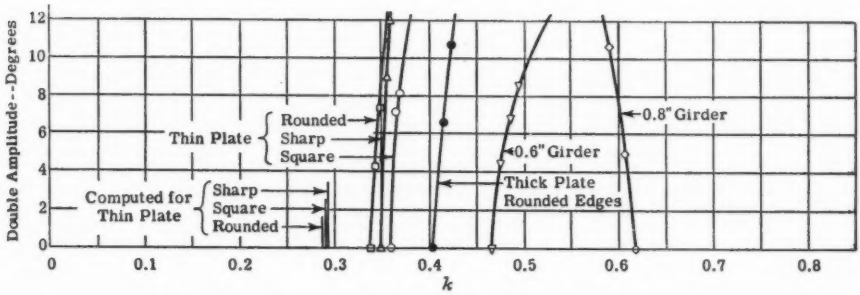


FIG. 17.—FLAT PLATE AND SHALLOW GIRDER SECTIONS ($\alpha = 0°$)

Δ_c. These computed values for model G-2 are plotted in Fig. 18. They show a somewhat random variation with k_c over the limited range of k_c-values covered by the different spring conditions. It was determined that the probable variations in test values (of the order of 0.2 ft per sec for v_c and 0.2 cycles per min for frequency) could account for only a small part of the variations of A and B. Both the test values and the calculations are sensitive to changes in mass distribution. Perhaps the simplifications of Eq. 81 account for some

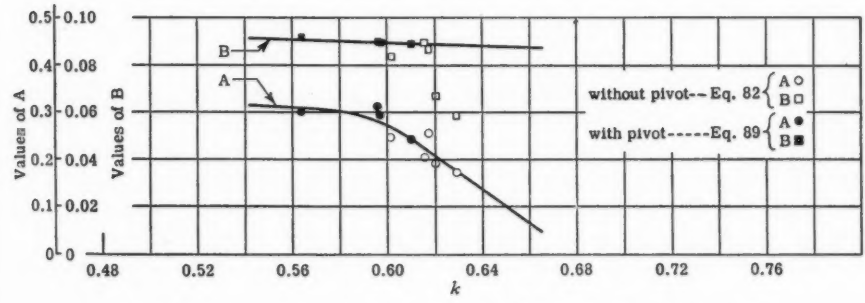


FIG. 18.—COMPUTED VORTEX FORCE $f_v = A + Bi$ (0.8-IN. GIRDER; $\alpha = 0°$)

discrepancy. In an attempt at closer refinement, model G-2 was tested with pivots placed on the longitudinal axis of symmetry to prevent vertical motion so that the values of A and B could be computed by Eq. 89 (earlier tests had shown that the pivots increase the damping by only a small amount). Fig. 18 shows these computed values also plotted against k_c. For coefficient A the values determined for the free model by Eq. 82 show considerable consistency and were taken into account in drawing the curve. For B the values from the

free model calculations were more erratic and the curve was drawn through the points obtained for the pivoted model. The theoretical values of k_c, ω, and v_c for the unpivoted tests on model G-2 were computed using values of A and B taken from the curves of Fig. 18. These theoretical values, shown in Table 3, agree very well with the observed values.

For angles of attack up to 2° the response differs little from that in a horizontal wind but for greater angles the difference becomes considerable. From the few observations available, it is believed the wind at some bridge sites may have an upward angle up to 6° or 8°. (Values of this order were measured on the Golden Gate Bridge, San Francisco Bay, California, during August, 1944.)

In all the tests reported in this discussion, "end plates" were used to prevent air circulation around the ends of the test specimens. Parallel tests were also made without the end plates, and it was found that the differences were small and principally accounted for by the change in mass which they caused. This agrees with section model tests made at the laboratory, indicating that with an aspect ratio (length to width) of as much as 4.5 to 5.0 the end effect is negligible.

It was expected that the models with sharp, thin edges or with rounded edges would show results much nearer the theoretical values and they did, and considerable effort was made to perfect the technique to reduce any chance vortex excitation. Knowing that angularity of wind causes vortex discharge, small, unmeasurable angle changes in the vicinity of 0° were tried with the thought that a perfectly horizontal wind stream would show the highest critical velocity. This was unfruitful, as might be anticipated, because the angle of attack can be measured to an accuracy of about $\frac{1}{4}$°.

The conclusions seem inescapable that any suspension bridge will be subject to vortex forces in addition to the flutter forces and that it will be necessary to run wind tunnel tests on an oscillating model to determine the values of A and B in the expression, $f_4 = A + B\,i$.

The author's mathematical treatment of the problem is especially valuable because it takes into account both the flutter and vortex forces in the varying proportions in which they may act on a given structure, particularly when trusses or shallow girders are used. It is a distinct advantage that his method can make use of data from aerodynamic section model tests; it does not rely on purely theoretical considerations for determining the important and sensitive parameters representing the vortex influence. The analysis should be extended to cover vertical angularity of the wind, which will be needed to assure the stability of bridges at some sites.

One limitation of the analysis is that, although it determines the wind velocity at which motion will occur, it gives no indication of the seriousness of the motion. In many cases, particularly where flutter is dominant, this information is sufficient since the motion is so violent that the possibility of its occurence must be avoided. However, there are many cases in which a vortex-excited motion, revealed at a moderate velocity, may be difficult to avoid but may be noncastastrophic, causing limited amplitude and being subject to effective control, if desirable, through damping or other methods. A major

objective of the current program of the Cooperative Research Project at the University of Washington is the development of a combined mathematical and experimental method in which the results of aerodynamic tests on section models are used to predict the steady-state amplitude that will develop on the bridge in the corresponding wind stream against a given damping. However, little is known concerning the structural damping of suspension bridges since the mass of data collected on other types of bridges is inapplicable. The development of technique for determining the damping of different types of suspension bridges is another major objective of the project.

[The corroborative tests reported in this discussion were made as a cooperative research project of the University of Washington and the U. S. Public Roads Administration, under the direction of Professor Farquharson and with the supervision and active participation of Mr. Vincent. Mr. McHugh made the tests and plotted and tabulated the data. Graduate students assisted in the computations involved.—Ed.]

FRIEDRICH BLEICH,[24] M. ASCE.—Professor Pugsley's criticism is concerned with some general aspects of the solution of the problem of self-excitation in suspension bridges presented in the paper. The writer does not agree with Professor Pugsley's statement that most of the critical speeds observed may be attributed primarily to eddy resonance. In so far as truss-stiffened bridges or bridges with very shallow stiffening girders are concerned, all wind tunnel experiments strongly indicate that flutter always has an important bearing on the phenomenon of catastrophic instability but that, to a greater or less extent, the critical velocity also is influenced by a vortex shedding effect at the leading edge. This general concept cannot be simplified further, as Professor Pugsley suggests, without impairing any theory of self-excitation in suspension bridges. Any simplified concept would deviate from the real facts, as observed in wind tunnel tests, and would lead to an unrealistic theory incapable of explaining the actual behavior of a suspension bridge under wind action.

The discussion by Messrs. Farquharson, Vincent, and McHugh is especially welcome since the experimental verifications which they report are as satisfactory as can be expected with a subject of such complexity as the problem under consideration. The five tests on model G-2 with substantial variations of the spring constant prove, without any doubt, that dependable results can be obtained by the combined theoretical and experimental method as suggested in the paper. It is regrettable that, within the limits of wind velocity available in the wind tunnel, no excitations could be observed on paprallel tests with a pivoted model G-1 in order to determine, with sufficient accuracy, the values of the parameters A and B which define the aerodynamic force F_v at the leading edge.

The attempt made by Messrs. Farquharson, Vincent, and McHugh to derive the parameters A and B, from observations of flutter frequency and critical wind speed on the unpivoted models, led to more or less random values of the characteristics A and B. This can be seen from an inspection of the last column under the heading f_4 in which the A-values and B-values for the models G-1 and

[24] With Frankland and Lienhard, Cons. Engrs., New York, N. Y.

G-2 in each group are widely scattered. The failure to arrive at a series of consistent values A and B in each of these two series of tests is explained as follows: The stability equation (Eq. 82) from which A and B have been computed is completely inappropriate for the determination of these two characteristics. The two equations that follow from Eq. 82 are so sensitive, when solved for A and B (for given values of the flutter frequency ω and critical wind velocity v_c), that a slight variation of less than 1% in ω or v_c suffices to produce very substantial changes of the values of the computed A and B. The investigators admit that the probable error in the observed values of v_c is about of the order of 0.2 ft per sec—that is, about 1.5% to 2% of the test values. This fact, and only this fact, explains the considerable scatter of the values in the last two columns.

Therefore, no conclusion as to the actual values of the air force characteristics A and B can be drawn from these data. There is no method of obtaining reliable values of these parameters other than computing them from observations on a model with a fixed center of rotation (pivoted model) by means of Eq. 89. This equation was especially devised to provide a dependable derivation of the characteristics A and B and its application in the case of model G-2 led to satisfactory results which showed fairly good agreement between the observed and computed values of critical velocity and flutter frequency.

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DISCUSSIONS

THE MUNICIPAL ASPHALT PLANT, BOROUGH OF MANHATTAN

Discussion

BY CLARKSON H. OGLESBY, CLARK D. HENDERSON, AND OSCAR C. HOLMES, JR., AND J. H. CREED

CLARKSON H. OGLESBY,[8] ASSOC. M. ASCE, CLARK D. HENDERSON,[9] AND OSCAR C. HOLMES, JR.,[10] JUN. ASCE.—The description of the Municipal Asphalt Plant is interesting in that it records the developments and advances that have been made. The author is to be commended for the careful description of the problems encountered and the measures taken toward their solutions.

As presented, however, the suitability of the cost data for comparison with the prices of competitive producers is open to question. It seems to the writers that the data cannot be used to measure production economy.

In arriving at total annual overhead costs, it is essential that a proper allowance be made for capital recovery with a return commensurate with the risk. The appropriate method of determining annual capital recovery cost is by the exact method recommended by E. L. Grant,[11] M. ASCE:

$$R = (P - L)\frac{i(1+i)^n}{(1+i)^n - 1} + L\,i \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which R is the uniform annual capital recovery amount; P is the total investment including depreciable assets and land; L represents the salvage value and the land value estimated to be realizable at the end of the useful economic life; n is the expected useful economic life, in years; and i is the interest rate appropriate for the risk involved.

Straight-line depreciation plus average interest may give a satisfactory approximation of R when i and n are small. The author has used straight-line depreciation plus interest on the initial cost of depreciable assets. This method overstates the interest cost by neglecting the declining balance.

NOTE.—This paper by L. Csanyi was published in November, 1948, *Proceedings*.

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[9] Instr. in Industrial Eng., Stanford Univ., Stanford University, Calif.

[10] Acting Instr. in Civ. Eng., Stanford Univ., Stanford University, Calif.

[11] "Principles of Engineering Economy," by E. L. Grant, The Ronald Press Co., New York, N. Y., 1938, p. 90.

Interest on land value is omitted. Although the land was not purchased, it could have been sold or leased and the proceeds could have been used by the local government. Clearly, therefore, the use of this land involves a sacrifice to the city which should be added to the cost of plant operation. In the absence of a basis for decision to the contrary, it is common practice to assume that the present value will remain unchanged and recoverable.

Prospective lives and applicable interest rates can rarely be fixed precisely on the basis of facts. In the typical case, judgment plays an important part. Where genuine reason for doubt exists, conservative practice dictates that the higher interest rate and the shorter life be chosen. The author's estimates seem far too optimistic.

Considering the vast changes in paving practices since the beginning of the twentieth century, recent developments in petroleum refining, and the prospective shortages of petroleum oils, a capital recovery period of 40 years for an asphalt plant cannot be considered as safe. The complete elimination of depreciation on the structures comes from the erroneous assumption that, because a structure is as useful as ever, it is as valuable as ever. Considering the location and the highly specialized nature of this plant, the assumptions of perpetual useful life and constant realizable value are not warranted.

With regard to the choice of length of life for equipment, it is inaccurate to assume a length of life (20 years), compute annual depreciation charges on that basis, and then use all or part of that depreciation for preventive maintenance and replacements, thereby doubling the life of the asset and reducing by half the annual depreciation charge. By traveling around this circle again and again, the depreciation charge soon approaches zero and the life approaches infinity.

Depreciation charges and preventive maintenance expense are separate items of cost. Although a good maintenance policy will tend to prolong the life of the equipment, the original investment must still be recovered through adequate depreciation charges. The inadequacy of the author's combined provision can be demonstrated in this way. If the amount of the depreciation charge ($10,176 per yr) were to be deposited in a sinking fund and compounded annually at 2%, the sum available at the end of 20 years would be slightly less than $250,000, but machinery costing initially $400,000 would be worn out.

The cost of borrowed money (which apparently the author has used) is rarely, if ever, applicable in the determination of cost. The 2% interest rate does not reflect the risk involved in a public asphalt plant, but reflects the investing public's opinion of the general credit of the local government and the worth of the federal income tax exemption on local government bond interest. A suitable interest rate should be determined for this particular venture and it undoubtedly will be considerably greater than 2%.

Certain essential costs are not included in the paper, such as administrative costs, insurance, taxes, maintenance on structures, etc. Administrative services, such as purchasing, cost and pay roll accounting, personnel administration, and a great variety of miscellaneous items, do not appear in the cost estimate presented.

A property risk exists in this plant and, if it is insured, the rate is probably rather high. If it is not insured, the risk is borne by the local government in

its self-insurance plan. In either case, the cost should be allocated to the product.

The omission of property taxes is an error of considerable importance. The effect on the taxpayer must be kept in mind for he is the ultimate purchaser of the paving material. If taxes are omitted in computing the cost of the paving material, the equivalent sum must be paid in other ways. Since private manufacturers are among the taxpayers, it is obviously unfair to omit taxes for the public manufacturer but to include them, as is necessary, in competitive prices. Maintenance expense on the structures is also omitted; and this expense is unlikely to be negligible over a long period of time.

Although most of these comments are directed toward understatements of costs, they do not necessarily imply that accurately determined cost would compare unfavorably with the prices of private manufacturers. Furthermore, an unfavorable direct cost comparison must be tempered by consideration of the value of extra services not reducible to money terms.

Summary.—In the opinion of the writers, the cost data developed by the author are unsuited for comparative purposes because of his treatment of capital recovery (depreciation); the omission of the present realizable value of the land and of depreciation on structures; the use of unwarranted long lives; the confusion of depreciation charges and maintenance expense; the adoption of too low an interest rate; and the failure to include administrative costs, insurance, taxes, and maintenance on structures.

J. H. CREED,[12] Assoc. M. ASCE.—The frank duscussion of the troubles encountered in a Municipal Asphalt Plant in New York, N. Y., is refreshing. Too often such things are somewhat glossed over or avoided entirely.

The selection of a site for an asphalt plant in any settled area always presents many difficulties, which have been met excellently in this case. One of the major costs of an asphalt pavement is frequently that of hauling the hot mix from the plant to the street. The central location minimizes this cost, and can well offset the extra cost of a location on expensive property. Even the prevailing wind seems to have favored the site, blowing the dust across the river.

The materials seem to be well handled, although the capacity of 100 tons per hr might seem a little small for the rock and sand conveyer. The bulk handling of powdered limestone or other finely ground mineral dust always presents many problems, the principal of which are to keep it aerated and thus to prevent packing. A lower silo would help somewhat, as it would reduce the pressure and minimize avalanching.

The 700-ton storage would give a 20-day run at 5% batching of 700 tons per day—indicating that shallower and smaller bins might suffice. The two mixers are arranged quite conveniently, assuring a sure supply of mix in case of the breakdown of one of the units. Of course, this arrangement increases the cost of the installation, as well as the space occupied—quite an item in a central location, on expensive property.

The setup for aggregate handling is good. The system of feeding the aggregate from several bins onto a belt makes for good control of the grading of the

[12] Senior Highway Engr., California Div. of Highways, Bishop, Calif.

cold feed. The capacity of 80 tons per hr would seem a little small, but with large plant bins and an irregular demand, it may suffice. A pan feeder under the sand bins is very satisfactory, feeding even rather wet, fine sand quite well, and obviating the necessity for vibrators. Ordinarily a direct feed from the belt to the cold elevator pit will give a very uniform feed.

The double-fired type of drier is unfamiliar to the writer. Ordinarily a jet type of burner with a pressure of about 150 lb per sq in. on the fuel oil, and 100 lb of steam or compressed air, is installed. From two to three jets are generally set up, which are used to develop the proper heat with the most efficient operation of the burners. The suction of a fan, with a capacity from 25,000 cu ft to 40,000 cu ft, on the dust collector will draw the fire well into the front end of the drum, with very little, if any, tendency for back draft.

Since coating in the combustion chamber of a drier is almost unavoidable, its removal is considered a routine maintenance operation on most plants. There seems to be no real substitute for a good fireman when it comes to controlling drier temperatures. A system whereby the cold feed can be kept uniform, both as to quantity and as to moisture content, and close correlation between the operation of the cold feed and the fire are essential. A correctly set pyrometer in the drier discharge is helpful, but this instrument has a tendency to lag. The vertical hot elevator is always troublesome for the reasons given by the author, and should be avoided, even at the expense of the extra space required by the inclined type. The choice of vibrating screens is a wise one. Much better screening can be obtained than with revolving screens.

Insulated hot bins are a great help, particularly in a plant where there is much stand-by time. Big bins, such as those in the Municipal Asphalt Plant, are also invaluable, giving as they do so much surge space. Continuous running of the drier is important in maintaining even mix temperatures, and big bin storage mininizes drier shutdowns. In the writer's experience, it has been found necessary to have at least three bins; and, on coarser mixes, four bins are preferable, to obtain a properly graded mix.

The asphaltic cement (AC) seems to be excellently handled in the plant. In the writer's experience, asphalt has generally been heated with steam coils in the tanks; but, the "calrod" system may well have many advantages.

The size of the mixers is rather startling to one who is accustomed to plants using mixers of from 3,000-lb to 4,000-lb batch capacities. In the writer's experience mixing times of 45 sec for asphaltic concrete and 30 sec for plant mix have always proved adequate to coat the aggregate thoroughly. In a properly set-up plant, 45-sec and 60-sec cycles, respectively—or a capacity of 120 tons per hr for the 4,000 lb plant running asphaltic concrete—are necessary.

The writer has usually found it advisable to weigh the batch of asphalt, using a hanging bucket and a dial scale. The bucket is allowed to fill to the desired weight while the aggregate is being batched into the weigh box. When the mixer is dumped and the discharge gate closed, the aggregate for the new batch is dumped into the mixer and then the asphalt is dumped in, using either a gravity dump or a pump. Generally the gravity dump produces satisfactory results if the distributor pan is properly adjusted, and it has the advantage of not breaking down, which is not always the case with a pump. Asphalt is fed

to the weigh bucket with a two-way valve, which by-passes the asphalt back to the storage tank while the feed to the bucket is shut off. Metering has the inherent weakness that the volume of asphalt will vary with temperature changes. Also, meters have a tendency to break down at times.

With only a "multiclone" precipitator, it is remarkable that there has not been more trouble with dust. Numerous operations have been shut down by injunction, and damage suits sustained on the strength of dust from plants in much less populous areas than New York. Electrical precipitation will undoubtedly clean up the situation. It would have been interesting to have included the considerations which led to the adoption of electrical precipitation rather than the more familiar steam or water spray methods.

The costs of producing material seem to be quite reasonable, considering the necessary operating conditions. The crew of twenty one seems rather large for a highly mechanized operation such as this. It would be interesting to see a breakdown of duties. One fallacy which enters the cost picture is the failure to charge an item for taxes. If the city sold the property to a private individual, he would pay taxes to the city. Therefore it is costing the city that much money to own the property and the taxes should be included to obtain a true cost picture.

All in all, Mr. Csanyi has written a very interesting and instructive paper, and merits the thanks of those in the profession whose work involves the use of asphaltic paving.

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DISCUSSIONS

MINERAL LAND SURVEYS

Discussion

BY JOHN V. MELDRUM, AND ARTHUR D. KIDDER

JOHN V. MELDRUM.[24]—This excellent paper on mineral surveys is, as the author states, the first technical discussion presented in many years that goes beyond the bounds of the "Manual of Instructions for the Survey of the Public Lands of the United States."[2] Although in 1948 there were 118 commissioned mineral surveyors, it must be realized that many of these men are in the higher age brackets. There is room in the field for younger men; hence, the paper is timely. Furthermore, since changes in the mining laws are being considered, the paper offers an excellent basis for appraising the present status of patented mineral lands.

The original paper was being prepared during the reorganization of the old General Land Office, which was in effect a merger of that office with the Grazing Service under the new Bureau of Land Management. Therefore, the following may serve to clarify several conflicting statements.

Mineral surveys in the western United States, together with all public land surveys, were made originally under the direction of a surveyor general's office for each state or territory, this office being a division of the General Land Office. In 1925 the several offices of surveyors general were abolished and in their stead public survey offices were established under the supervision of an office cadastral engineer. At that time the supervisor of surveys (a position established in 1910 for coordinating cadastral engineering work) assumed direct responsibility for the functioning of the public survey offices and reported directly to the commissioner of the General Land Office. As a result of President Harry S. Truman's Reorganization Plan No. 3, effected July 16, 1946, the position of supervisor of surveys was abolished along with the General Land Office. The United States including Alaska was divided into regions, each headed by a regional administrator to whom, in accordance with the

NOTE.—This paper by Arthur J. McNair was published in November, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1949, by Charles P. Eager, and Robert Lenon.

[24] Mineral Engr., Public Survey Office, Bureau of Land Management, Denver, Colo.

[2] "Manual of Instructions for the Survey of the Public Lands of the United States," Bureau of Land Management, U. S. Dept. of the Interior, U. S. Govt. Printing Office, Washington, D. C., 1947.

general theme of decentralization, has been delegated much of the authority and many of the duties formerly centered at Washington, D. C. The Bureau of Land Management, which, in addition to surveying, has assumed all the former functions of the General Land Office and the Grazing Service, is headed by a director at Washington. The public survey offices have remained essentially the same, and the office cadastral engineer accepts applications for, and approves, all mineral surveys. All records of mineral surveys are on file in these offices. Although appointments of mineral surveyors are made by the regional administrator, applications are directed to the proper public survey office.

The technical information contained in the paper is exceptionally accurate, and any criticism offered could only be in a picayune manner. However, the author states (under the heading, "Introduction") that "* * * since 1935 patents for homesteads and for stock raising homesteads have been essentially stopped * * *" and again he writes in the "Conclusion:" "* * * this land [mineral land] is the only type of public domain patentable in the United States * * *." It is true that mining claims may be located anywhere on the public domain except in obvious reservations such as national parks and monuments and areas withdrawn from entry for a specific purpose, such as power site withdrawals.

It seems advisable to point out that the homestead laws are still in effect and that the decline in homesteading activity may be attributed mainly to the lack of desirable land In accordance with the strong policy of conservation being so vigorously prosecuted at this time (1949) in order to assure a continued high standard of living for the United States and because of the many demands for other uses of the land, patents are not granted promiscuously to any applicant without close scrutiny of his qualifications under the public land laws. This fact, coupled with the character of available land, explains the lessening number of patents issued. However, applications under the various land laws are welcomed and isolated tracts of public domain that cannot be administered effectively by the Bureau of Land Management are offered at public auction.

ARTHUR D. KIDDER,[25] M. ASCE.—A substantial contribution to the subject of the mineral land surveys within the western states of the former public domain has been made by Professor McNair. The paper supplies basic information that is needed by both the claimant and his surveyor where new discoveries of mineral are made on the vacant public land and, also, for the identification of the boundaries of the older mineral claims where that may be required. There has been much less activity in this field since the issue of the "Manual of Instructions for the Survey of the Mineral Lands of the United States," in 1909.[26] That manual contained the rules of survey and entry then in effect, together with citations of the United States statutes.

[25] Kidder and Thoma, Terre Haute, Ind., Commr. of the Supreme Court of the United States, designated for the marking of the boundary between the states of Colorado and New Mexico; formerly Cadastral Engr., Gen. Land Office and Bureau of Land Management, U. S. Dept. of the Interior.

[26] "Manual of Instructions for the Survey of the Mineral Lands of the United States," Gen. Land Office, U. S. Dept of the Interior, U. S. Govt. Printing Office, Washington, D. C., 1909.

Concurrently with the period from 1895 to 1909, a most valuable article on the subject of the problems and field procedure incident to the making of mineral surveys was prepared[27] by Robert S. Stockton and Edward P. Arthur, Jr. Appendix B[28] of the textbook in which this chapter appeared was a reprint, in full, of the official "Mineral Manual"[8] of 1895. Chapter XI[27] and Appendix B[28] of the textbook (which was prepared at a time of great activity in mining), together with the elucidation of the subject by Professor McNair nearly 50 years later, are veritable "gold mines" for tracing the history and practice in the execution of mineral surveys. Literature in this category is rare, and difficult to find at hand just when needed.

The administrative work incident to the mineral surveys in the offices of the surveyors general, and after 1925 in the public survey offices, General Land Office, United States Department of the Interior, decreased very materially between the years 1909 and 1930. In 1930 the technical rules, regulations, and practice then in effect for the making of the mineral patent surveys were published as Chapter X in the general "Manual of Instructions for the Survey of the Public Lands of the United States," in 1930.[29] No historical data regarding mineral surveys were supplied in the 1930 manual. At that time the greater part of the regulations concerning the making of a mineral location on the vacant public lands was issued in the form of departmental circulars, each limited to a particular scope of the subject. The circulars were brought up to date from time to time keeping pace with new legislation or departmental or exceutive orders, and the decisions in mineral contest cases. The general manual of 1947[2] brought the technical practice up to date. At this time the deletions from the earlier manuals were those parts on nontechnical subjects, which are better treated in circular form as required to conform with current legislation.

Basically, the technical procedure for the making of the mineral patent survey as expressed in the general manual of 1947 is little different from the practice of 1895 to 1909. However, the paper is indeed a most important treatment for those who now for the first time are brought face to face with these questions.

It is well to bear in mind that contest cases between mineral claimants concerning the right of entry are subject to adjudication by the director, Bureau of Land Management, under the supervision of, and the right of appeal to, the Secretary of the Interior. After a patent has been issued, any possible controversy concerning the position of the boundaries, and other questions relating to the title to the property, come under the state law and the jurisdiction of the courts of that particular mining district or county, which possess the sole authority to adjudicate such cases. In this respect mineral suits are similar to other boundary and title litigation, except for the rules that give the right to

[27] "The Theory and Practice of Surveying," by J. B. Johnson, John Wiley & Sons, Inc., New York, N. Y., 16th ed., 1902, Chapter XI.

[28] *Ibid.*, Appendix B.

[8] "Manual of Instructions for the Survey of the Mineral Lands of the United States," Gen. Land Office, U. S. Dept. of the Interior, U. S. Govt. Printing Office, Washington, D. C., 1895.

[29] "Manual of Instructions for the Survey of the Public Lands of the United States," Gen. Land Office, U. S. Dept. of the Interior, U. S. Govt. Printing Office, Washington, D. C., 1930, Chapter X.

follow the lateral inclination or dip of the lode underground beyond the vertical projection of the side lines.

A point of much importance in understanding the practice on mineral surveys is that the early discoveries of free gold in California, Colorado, South Dakota, and other western states were made during the frontier days, in some cases even preceding territorial status for those districts. Each new discovery brought a rush of prospectors. Primitive rules of law and order were set up by the miners themselves, including their own plan for staking out a mineral claim, and posting a public notice. Later, to perfect the title, legislation was enacted by Congress to set up a uniform legal right of entry under which patent by the United States might issue to the rightful claimant. Thus, in effect, the miners themselves initiated the first rules. These were then clarified and codified into the general mining laws of 1872, as stated.

The present general laws relating to the public lands, and the regulations thereunder, are available in the "Code of Federal Regulations, Title 43—Public Lands, Interior," the most recent of which are printed in the "Cumulative Supplement."[30]

The federal code includes many subjects such as the legal right of mining or development of the natural resources of oil and gas, potash, phosphate, coal, and other minerals to be found on the vacant public lands, and also the procedure, if authorized by law, for the exploitation of deposits that may be discovered within the various reserved public lands. Another important consideration is also suggested: A large number of patents that have issued in recent years for entries under the general land laws contain a clause that reserves the mineral right in the United States, the entryman thus gaining title to the surface right only. The prospector, geologist, mining engineer, mineral surveyor, or others concerned, when confronted with these questions, should consult the nearest public survey or regional office of the Bureau of Land Management or request the information from the Washington (D. C.) office.

The research of record data should include an examination of the field notes and a plat of the township survey, if the area has been subdivided, as these will show the location of the mineral surveys that were in existence at the time of the subdivisional survey. Also, the meaning and importance of the General Land Office "supplemental plats" within these townships should be noted. The supplemental plats were prepared from time to time as needed to accommodate one or more nonmineral entries on the public land that adjoined the patented mineral lands. In these cases new lottings were designated in order to give a suitable description for the nonmineral fractional lots. No new field work was done for the preparation of the supplemental plat, the latter being based on the field note record of the mineral patent survey when platted through the direction and the length of the lines connecting to the subdivisional survey.

In an active field, new supplemental plats were frequently required as new mineral discoveries were made, but only as some nonmineral applicant initiated an entry on the adjoining land. To a lesser extent, and only in recent years,

[30] "Code of Federal Regulations, Title 43—Public Lands, Interior," Cumulative Supplement: Chapter I, Subchapter L, and Pt. 185, Supt. of Documents, U. S. Govt. Printing Office, Washington, D. C.

"mineral segregation" surveys have been made under the special administrative instructions of the Department of the Interior where needed in order to accommodate a nonmineral entryman on lands adjacent to a known valid mineral claim where the latter had not proceeded to mineral patent.

Looking ahead, it is almost certain that new discoveries will be made, some of them on the surface, and that others will be found much deeper in the ground within the boundaries of the older mineral patents, which are possibly the parent source of the early surface discoveries.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

NOMOGRAPHIC ANALYSIS OF RECTANGULAR SECTIONS OF REINFORCED CONCRETE

Discussion

BY WINFIELD A. MCCRACKEN, KING BURGHARDT, AND ROBERT V. HAUER

WINFIELD A. MCCRACKEN,[11] ASSOC. M. ASCE.—Simplification of Eqs. 1*b* and 1*c* and the addition of one nomogram for determining **B** and **C** make the set of nomograms presented in this paper an excellent design tool.

The moments used in Eqs. 1*b* and 1*c* are taken about the positive and negative reinforcement, respectively. The equations can be simplified by restatement:

$$\mathbf{B} = \frac{N}{f_c\, b\, d^2}\left(e + \frac{d - d'}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (25a)$$

and

$$\mathbf{C} = \frac{N}{f_c\, b\, d^2}\left(e - \frac{d - d'}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (25b)$$

KING BURGHARDT,[12] M. ASCE.—The nomograms presented in Figs. 1, 2, and 3 are considered a decided improvement over the numerous diagrams usually included in treatments of this topic. If conservation of space is important, only Figs. 2 and 3 are necessary, as the economical balance of tension and compression steel may be determined by a "cut and try" method similar to that proposed for checking stresses. However, the inclusion of two additional nomograms would furnish direct solutions for checking stresses or for designing with equal steel in the tension and compression face.

The utility of Fig. 1 is somewhat diminished by the recent tendency to consider that initial compression in reinforcing steel is introduced by shrinkage of the concrete. The 1949 edition of "Standard Specifications for Highway Bridges"[13] of the American Association of State Highway Officials provides for a doubling of the stress and the resisting moment of the compression steel over

NOTE.—This paper by W. P. Linton was published in January, 1949, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1949, by Maurice Barron, and M. Hirschthal.

[11] Cons. Engr., Houston, Tex.

[12] Senior Highway Bridge Engr., U. S. Public Roads Administration, Denver, Colo.

[13] "Standard Specifications for Highway Bridges," A.A.S.H.O., Washington, D. C., 1949.

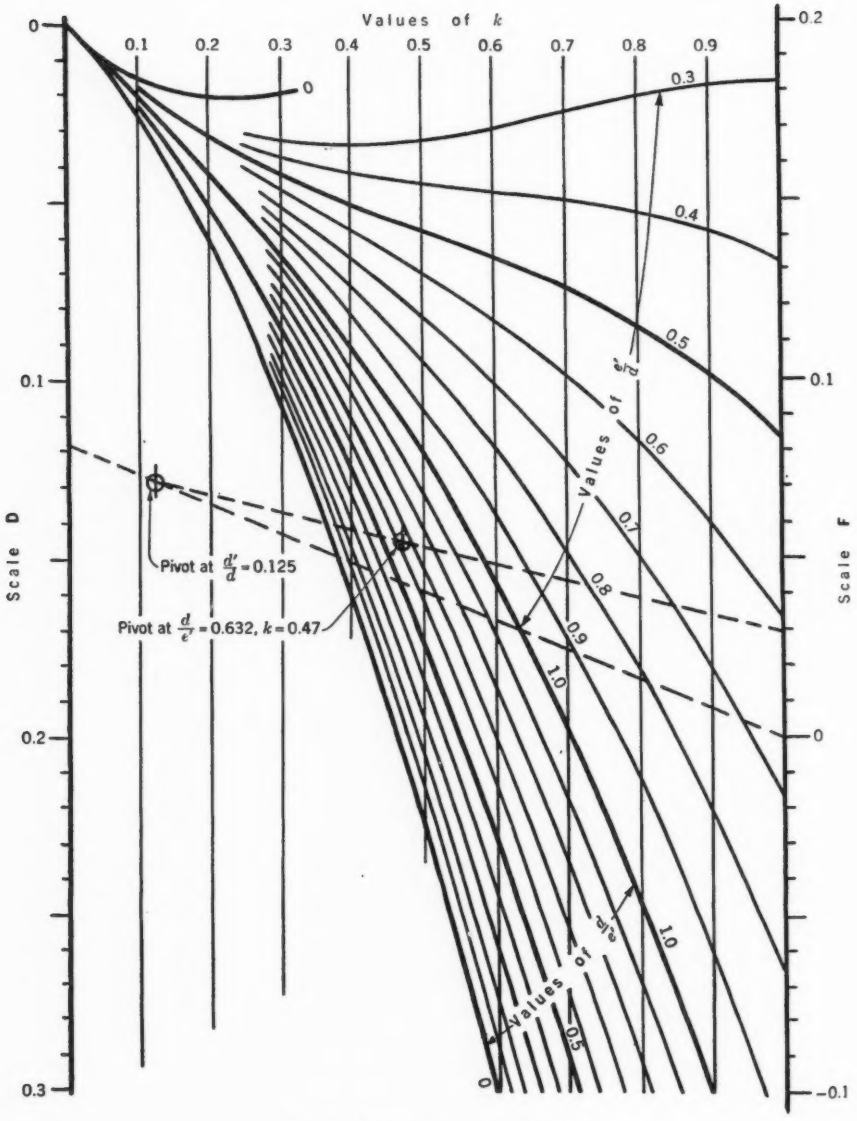

FIG. 6.—INVESTIGATION TO FIND $\bar{k}$

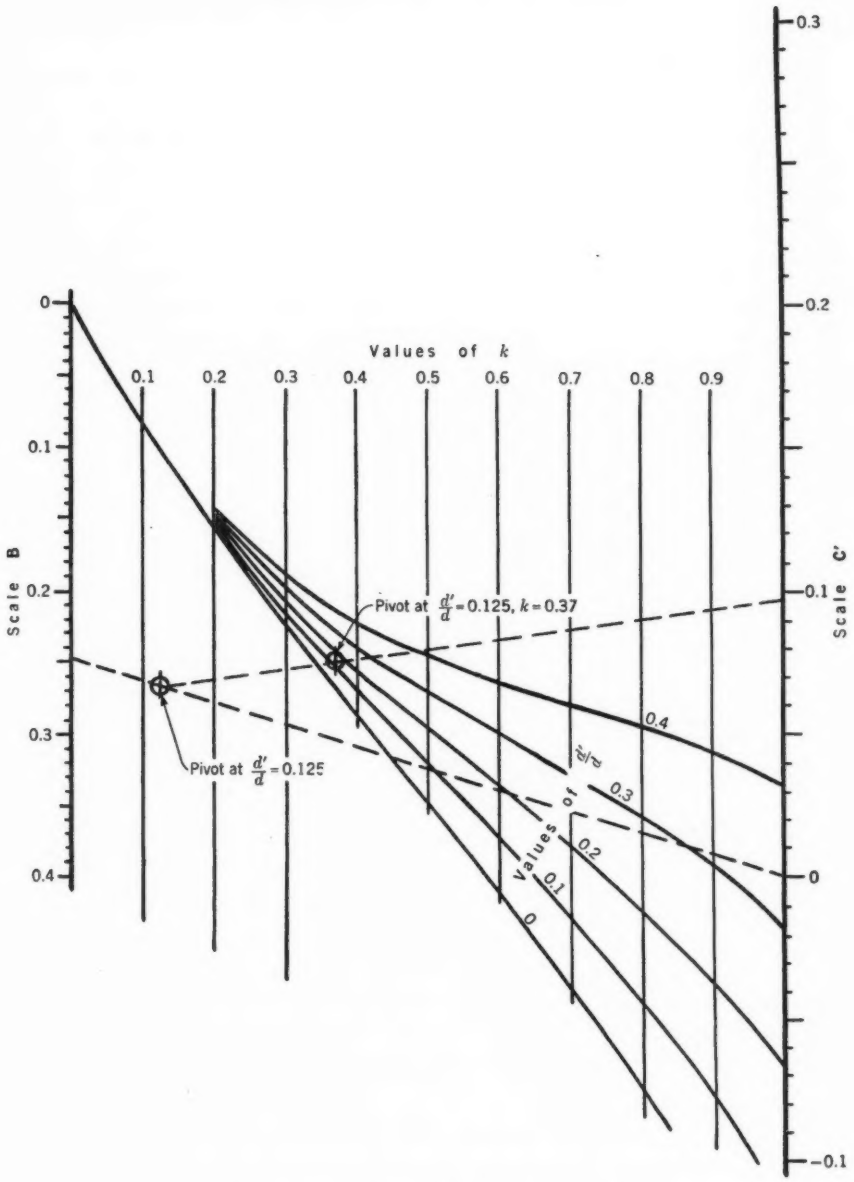

FIG. 7.—DESIGN TO FIND k WHEN $n\,p' = 2\,n\,p$

that calculated by former methods. To comply with this provision, let

$$n\,p'\,b\,d = 2\,n\,A'_s \quad \ldots\ldots (26)$$

for Figs. 2 and 6. The minimum total steel area for this condition would require the revision of Eq. 24 and the redrawing of Fig. 1.

Figs. 2 and 3 present quick solutions of Eqs. 12 and 13, respectively. Some values of k and d'/d produce rather vague readings of $n\,p'$ or $n\,p$, merely indicating that the particular value of $n\,p'$ or $n\,p$ contributes very little to the resistance of the section. It might be mentioned that, if the value of k obtained from Fig. 1 produces a negative value of $n\,p'$ in Fig. 2, k should be recomputed for $n\,p' = 0$ (Fig. 2); and this new value of k should be used to find $n\,p$ in Fig. 3. When k is less than the value for balanced design (usually about 0.33), the value of $n\,p$ obtained in Fig. 3 will cause overstress in the tensile steel. For this condition the approximate formula,

$$A_s = \frac{N}{f_s}\left(\frac{e'}{0.9\,d} - 1.0\right) \quad \ldots\ldots (27)$$

usually will suffice.

Fig. 6 represents the expressions:

$$\frac{e'\,n\,p}{d}(1-k) - n\,p'\left(\frac{e'-d+d'}{d}\right)\left(k - \frac{d'}{d}\right) = \frac{k^2(3\,e' - 3\,d + k\,d)}{6\,d} \quad \ldots (28a)$$

and

$$n\,p\,(1-k) - n\,p'\left(\frac{e'-d+d'}{e'}\right)\left(k - \frac{d'}{d}\right) = \frac{k^2(3\,e' - 3\,d + k\,d)}{6\,e'} \quad \ldots (28b)$$

thus providing a solution for k for investigating or checking a known section. As the factor $\frac{e'}{d}$ may vary from $\left(\frac{1}{2} - \frac{d'}{d}\right)$ to infinity, the value of $\frac{e'}{d}$ is used when it is less than unity and the inverse factor $\frac{d}{e'}$ is used when $e' > d$. When $e' < d$, then, $D = \frac{e'}{d}$ and

$$F = \frac{e'-d+d'}{d}\,\frac{d-d'}{d} \quad \ldots\ldots (29a)$$

and, when $e' > d$, then $D = 1.0$, and

$$F = \frac{e'-d+d'}{e'}\,\frac{d-d'}{d} \quad \ldots\ldots (29b)$$

To investigate a section, compute d'/d, $D\,n\,p$, and $F\,n\,p'$. In Fig. 6, place a straightedge between $D\,n\,p$ on scale **D** and 0 on scale **F**. At the point where the straightedge intersects $k = \frac{d'}{d}$, pivot the straightedge to $F\,n\,p$ on scale **F**. At e'/d, or d/e', Fig. 6, read k. Use this value of k in Fig. 2 to find **B** and then compute f_c and f_s. Referring to Example 1, $\frac{d'}{d} = 0.125$; $\frac{d}{e'} = \frac{16.00}{25.33} = 0.632$;

$D\,n\,p = 1.0 \times 0.118$; and $F\,n\,p' = \dfrac{25.33 - 16 + 2}{25.33}\,\dfrac{16 - 2}{16}\,0.075 = 0.029$. Join 0.118 on scale **D** with 0 on scale **F**. Pivot at $k = \dfrac{d'}{d} = 0.125$ to 0.029 on scale **F**, and at $d/e' = 0.632$ read $k = 0.47$.

There are occasions when it is desirable to use equal steel in the tension and compression faces. Fig. 7 is a nomogram for the equation:

$$(1 - k)\left[\mathbf{B} - \frac{k\,(3 - k)}{6}\right] = \left(k - \frac{d'}{d}\right)\left[2\,\mathbf{C} - \frac{2\,k\,(3\,d' - k\,d)}{6\,d}\right] \quad \ldots\ldots (30)$$

by which to compute the values of k for this condition. Fig. 7 is drawn for the aforementioned new provision that $n\,p'\,b\,d = 2\,n\,A'_s$. Compute **B** and **C** from Eqs. 1*b* and 1*c*, respectively. Compute $\dfrac{d'}{d}$ and $\mathbf{C}' = \mathbf{C}\,\dfrac{d - d'}{d}$. Join **B** on scale **B** with 0 on scale **C**′. At $k = d'/d$, pivot to **C**′ on scale **C**′; and, at d'/d in Fig. 7, read k. With k known, compute $n\,p'$ from Fig. 2 or $n\,p$ from Fig. 3. As a check, $n\,p' = 2\,n\,p$. Referring again to Example 1, $d'/d = 0.125$, $\mathbf{B} = 0.247$, and $\mathbf{C}' = 0.111\,\dfrac{16 - 2}{16} = 0.097$. Join 0.247, scale **B** with 0 scale **C**′. Pivot at $k = \dfrac{d'}{d} = 0.125$ to 0.097, scale **C**′. At $d'/d = 0.125$, read $k = 0.37$. From Fig. 2, $n\,p' = 0.147$; and, from Fig. 3, $n\,p = 0.074$. It is possible to plot a nomogram so that the intersection of a line joining a **B**-value on scale **B** with a **C**-value on scale **C** will show k at d'/d on the chart, but in certain positions there is confusion between the k-lines and the $\dfrac{d'}{d}$-lines of the chart, making it difficult to obtain a satisfactory reading. For this reason, the somewhat longer method illustrated was preferred.

Figs. 6 and 7 are reproduced at a comparatively small scale. Drawn to a larger size with additional subdivisions of the scales, quite satisfactory readings may be obtained from either nomogram.

In conclusion, the writer believes that the author has made a presentation valuable to all designers who are interested in obtaining economical compression members of reinforced concrete.

ROBERT V. HAUER.[14]—The nomograms presented by the author for the design and the analysis of rectangular reinforced concrete sections subjected to combined direct stress and bending have the interesting feature of being based upon the quantities **B** and **C**, which are the two moments of the axial force about the center lines of the tensile and of the compressive reinforcements, divided by $b\,d^2\,f_c$.

The advantage of this particular choice is readily explained by the facts that the tensile reinforcement is independent of **B** and that the compressive reinforcing is independent of **C**. This makes it possible to split the representation of the functional relations between the six quantities **B**, **C**, $n\,p$, $n\,p'$, k, and d'/d into two separate parts, as the author has done in nomograms 2 and 3.

[14] Structural Engr., Albert Kahn, Inc., Detroit, Mich.

The same simplification was utilized before by E. Moersch[15] in his textbook on reinforced concrete. Although Mr. Moersch uses different charts for different ratios of d'/d, his representation, in the writer's opinion, is superior in several other respects. The two parts showing the tensile and the compressive reinforcements, respectively, are placed opposite each other in such a manner that not only can each of these reinforcements be read at a glance but so also can their sum. In addition, this arrangement eliminates the need for a special chart for determining the value of k at which the total reinforcement becomes a minimum, and also solves the problem of finding the reinforcement in the case of a predetermined ratio. Such a case (for instance, if symmetrical reinforcement is required) could be solved with the author's nomograms only by a somewhat cumbersome cut-and-try process.

It may be of interest to mention that nomograms 2 and 3 can be used not only if N is a compressive force, but also if N is a tensile force, provided that **B** and **C** denote the moments of the axial force about the center lines of the tensile and compressive reinforcements, respectively, divided by $b\,d^2\,f_c$. For a tensile force N, the variable **B** becomes smaller than **C**, because tensile and compressive reinforcements are interchanged.

On the other hand, it is regrettable that the author did not extend the nomograms far enough to cover the range of small eccentricities, which, at least in building design, is the case most often encountered. Another shortcoming is that the value of f_c must be known, in order to use the nomograms in design; but most building codes now make the allowable concrete stress a function not only of the relative eccentricity (which is known from the data), but also of the percentage of reinforcement—that is, of the very quantities at which the design computation is aimed.

In conclusion, the writer wants to question some remarks made in the "Synopsis"—namely, that methods in current use for the design of reinforcement are useful mainly in analyzing designs previously made, and that in designing it is customary to assume the amount of reinforcement and then to check the assumption. There are many more formulas, tables, and charts available for design than for analysis. The obvious reason is that the design problem is not only much more important practically, but also, because of its very nature, much simpler than the problem of analysis. If the design is based on predetermined stresses, the location of the neutral axis follows from a very simple formula, whereas in the case of analysis k must be computed from a third-degree equation.

[15] "Der Eisenbetonbau," by E. Moersch, Konrad Wittwer, Stuttgart, Germany, 5th Ed., 1920–1922.

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DISCUSSIONS

MULTIPLE-PURPOSE RESERVOIRS
A SYMPOSIUM

Discussion

BY HERMAN A. HILTON, WESLEY E. GILBERTSON, ROGER E. AMIDON, L. C. BISHOP, AND A. H. DAVISON

HERMAN A. HILTON,[20] M. ASCE.—In those parts of the world where data are available (referring to such items as precipitation, evaporation, size of basin, and runoff factors), the planning and design of multiple-purpose reservoirs can be based on rather exact engineering principles and methods. Where insufficient data exist (for example, where periods are too short to permit reliable conclusions), the problem becomes much more complicated. If other very erratic conditions prevail, such as frequent change of watercourses, extensive disintegration of rock and soil, and enormous sedimentation, and if general difficulties must be considered, such as the lack of skilled and ordinary labor, equipment, and funds, then the problem becomes really grave and even experiences elsewhere, and detailed reports, will be of only limited assistance. This is exactly the situation in many South American Andean countries, where the questions of flood control and protection, irrigation, power, and drinking water supply are of the utmost importance. The writer has described these conditions briefly elsewhere.[21,22]

In such cases, the well-known, and well-proved, engineering methods of planning and design are not sufficient, as the necessary basic information is not at hand, or only partly so. The problems must be solved, however, and the engineers concerned with them found ways, apparently with success as far as the results show. The writer knows of a case in which pluviometric data were compiled by a procedure based on the theory of similitude. One of the stations had precipitation records covering 50 years. The precipitations were plotted for 1-year, 5-year, and 10-year periods, the year being taken as starting with the end of the dry season in October. All these curves had practically the same parabolic form; they differed only in level, according to dry and wet

NOTE.—This Symposium was published in March, 1949, *Proceedings*.

[20] Civ. Engr., Michigan City, Ind.

[21] *Ingenieria* (Mexico), September and October, 1945, p. 273.

[22] *Boletin de la Administracion Nacional del Agua* (Argentina), March, 1945, p. 222.

years. The regularity of the curves is explained by the local meteorological conditions. This phenomenon raised a possibility that, if the precipitations of the first months were known, it would be possible to predict, at the end of December or January, the probable precipitation for the remainder of the year, including the period of highest precipitation, from January to April. When precipitation data were needed at another site some distance away (about 350 miles), the existing records of a few years were examined, and it was found that these curves also had the parabolic form; but, compared with the first place, precipitations for the same years were reduced in a certain proportion. This proportion offered a method of computing the average values of probable precipitations and then determining runoffs. This experience occurred in the spring of 1947, and the writer has no information as to the results of a check by subsequent observations.

In cases where other than meteorological data were lacking, the following procedure has been used. One unknown fact (or even two) was set aside first. The design was made in the regular form; then the influence of the omitted fact or facts was investigated, for the worst conceivable case. If the resulting necessary or desirable changes were not too considerable (above all in time and money), they were included (or at least the possibility of a later change was provided for as far as possible). If the desirable changes were too great, a compromise or a negative decision had to be made, perhaps the entire project would have to be postponed.

If the lack of some data threatens to make a design impossible, it may sometimes be feasible, without too excessive cost, to modify the influence of such data or to limit that influence to certain values and thus to obtain a basis for planning the project.

After such a project has been completed and is at the very beginning of its operation experience, the routine steps in the operation must be based on the conditions assumed in the design. Thereafter, by progressive modification, a definite form and a standard method of operation must be evolved by experience. Sometimes there exist possibilities of providing for later adaptations in design and construction.

Under such conditions it is not sufficient for the engineer to have a theoretical knowledge of hydraulics and experience in hydraulic and general structures. He needs more than knowledge—that is, he needs an instinctive feeling and understanding for all the problems involved. In order to give young engineers this understanding, many technical universities or colleges in South America have a special 1-year course, obligatory for all civil engineering students, entitled "Stream and Sea Hydraulics," of which three fifths or three fourths is concerned with stream hydraulics—both theory and laboratory practice. The course is given in addition to courses in fluid mechanics, sanitary engineering, and other subjects. The writer does not know if any colleges in the United States include a similar special course or if the necessity for such a course exists.

The writer read the Symposium papers with these unique and special conditions in mind, and fails to find any direct reference to this point. It seems to him that all the reservoirs and systems considered by the authors were based

on more or less complete data and knowledge of the prevailing conditions. It would certainly be interesting to read something more concerning experiences with reservoirs designed and originally operated on the basis of insufficient data and under special conditions as outlined in this discussion. The writer felt obliged to raise this point for the benefit of the many engineers in countries other than the United States who read the ASCE *Proceedings*.

WESLEY E. GILBERTSON,[23] ASSOC. M. ASCE.—The importance of multiple-purpose reservoirs is readily acknowledged, and the divergent points of view from which the subject is discussed in this Symposium are quite comprehensive. A serious omission from the Symposium is a paper on the health aspects of reservoirs and water impoundments. It should scarcely be necessary to recall the importance of this consideration. The location, design, and operation of reservoirs may have several important effects, both beneficial and detrimental, on the health of human populations residing above, adjacent to, or below the actual site of the impoundment.

Much work has been done on the control of malaria mosquitoes relative to water impoundments. In fact, this consideration has been a major factor in the location, design, and operation of water impoundments. Some of the results have been to increase costs, others have caused environmental changes that have actually produced savings to people living in the locality. Furthermore, it is now known that water impoundments as related to irrigation have a definite effect on the incidence of eastern and western encephalitis, a mosquito-borne disease. Thus, the geographic area in which the mosquito potential of water impoundment must be considered is enlarged.

The continuous stream flow to provide dilution water for the disposal of sewage or the effluents from sewage treatment plants is an extremely important economic and public health consideration and, alone, has a serious impact on the design of water and sewage treatment plants.

Both state and federal health agencies have recognized the public health significance of reservoirs and have conducted a number of field studies. Careful analysis has been made of the relationship of various health factors to other uses of reservoirs, such as fish and wildlife, recreation, power, and irrigation. The interrelationships of these programs is of great significance; often, changes in one may seriously affect the others.

ROGER E. AMIDON,[24] ASSOC. M. ASCE.—The Symposium on Multiple-Purpose Reservoirs poses a number of important questions to the engineering profession and to all technicians involved in water management, including foresters, economists, and conservationists. An important question not adequately treated in the Symposium is the problem of sedimentation.

People occupying the United States in future centuries will hold a dim view of the reclamation and flood control projects of the twentieth century if needed water control and storage space in these great reservoirs is filled with silt and debris. The question arises, of course, of how distant a view should be taken

[23] San. Engr. Director, Communicable Disease Center, U. S. Public Health Service, Federal Security Agency, Atlanta, Ga.

[24] Civ. Engr., California Forest and Range Experiment Station, Berkeley, Calif.

by planners of reclamation and flood control projects. With exhaustible natural resources, such as large reservoir sites, the probable length of need by man should certainly be given consideration.

Hydro power will probably always be needed. Transportation on navigable rivers will always be economically desirable. Water for domestic and irrigation use will be necessary as long as civilization exists. Sea water may be cheaply converted for use in seaboard areas but its transportation to interior states will always involve costs exceeding the costs of distribution from near-by rivers. There seems little likelihood that precise control of precipitation will obviate the need for control of floodwaters. Fresh-water reservoirs will always be in demand for recreational use.

These considerations point to the permanent need of reservoir space for domestic, industrial, and irrigation waters; the control of floods; recreation; power development; and river navigation.

Silt control and debris control are as much parts of reservoir programs as water control. This principle is recognized under the flood control act and therefore no further legislation is needed to deal with the problem. Without silt and debris control water control becomes a temporary expedient—in many cases an extremely short-lived expedient. In fact, silt control or sediment control in Southern California is a prerequisite to the reduction of flood damage. Although especially important in Southern California, this condition is true elsewhere; if reservoirs are to be applicable to flood control, adequate reservoir capacity not only must be obtained, but also must be maintained. The total and correct cost of water control for multiple use must include the cost of silt and sediment control.

The allotment of space for silt and sediment detention in reservoirs is a recognition that a problem exists; but it is not a solution.

No discussion or apparent thought has been given to the "life of the project" in the Symposium. The size of reservoirs needed to meet the objectives of the planner is one of the major problems of design; but the size for any given set of conditions is directly dependent on the length of time the reservoir is to be used. Encroachment of other uses on flood control space is generally recognized by the authors as reducing the effectiveness of the reservoir. Encroachment of debris and silt on the design space also must reduce the effectiveness of the reservoir for its designed function. Eventually this encroachment will make the reservoir worthless for each and all of its uses. Certainly the engineers conceiving the huge projects being planned and constructed throughout the world are not thinking only in terms of the "economic life" (from 50 to 100 years) of the structure. Studies indicate that the useful life of 64% of all reservoirs in the United States is less than 100 years and that only about 15% will serve for more than 200 years.[25]

Three problems relative to sedimentation should be considered by the planners of multiple-purpose reservoirs: (1) Rate of sedimentation; (2) sediment removal from reservoirs; and (3) reduction of the sediment rate by upstream stabilization measures.

[25] "Sedimentation in Reservoirs," by Berard J. Witzig, *Transactions*, ASCE, Vol. 109, 1944, p. 1047.

(*1*) *Rate of Sedimentation.*—Sufficient data are usually not available to make accurate estimates of sedimentation. Since 1929, however, data have been collected throughout the United States which, when properly analyzed, will provide a basis for estimating with much greater accuracy than was formerly possible. Sedimentation research, too, will undoubtedly improve procedures and methods for making more precise estimates.

(*2*) *Sediment Removal from Reservoirs.*—Eventually sediment deposited in many reservoirs will probably be removed by sluicing and mechanical excavation. Where sluicing is possible, it will undoubtedly be most economical; but the effects of sluicing on downstream installations should be studied. Where mechanical excavation appears necessary, consideration should be given to the availability and accessibility of dumping space. Space should be procured and reserved at the time of construction to prevent intensive development of adjacent areas. The length of haul might dissipate the economy of an otherwise good dam and reservoir site. The cost of sediment removal should be included in the total cost of the project.

(*3*) *Reduction of Sediment Rate by Upstream Stabilization Measures.*—In considering this item the sources of sediment production must be determined. A program must be laid out to reduce the production from these sources. The degree of reduction must be estimated. The cost of the program must be included in the total cost of the reservoir.

Measures involved in upstream stabilization will vary from area to area, and within watersheds. They will generally include intensive fire control in forested areas, vegetation of actively eroding areas, road slope stabilization by vegetation supported by the necessary structures, road drainage for the harmless disposal of water crossing and concentrating on roads, agricultural land treatment, and stabilization of channels. The stabilization of channels will consist mainly of engineering works. In many cases, the stabilization of upstream gullies and severely eroding channels with raveling side slopes will reduce, permanently, the volume of material available for transport to reservoirs. The construction of debris dams, upstream from reservoirs, may be a necessity to provide immediate and positive storage control and to anchor in place large volumes of channel bed material. Reduction of the channel slope by the construction of debris dams and barriers will reduce the transporting power of the flow. Upstream sediment storage may be extremely desirable and necessary but should not be considered a permanent solution.

Various combinations of many measures will ordinarily be needed to reduce reservoir sedimentation. The measures will usually be costly, but in many cases they will be less costly than the continual excavation of material from the reservoirs.

Summary.—In determining the economic justification of a multiple-purpose project, evaluation of the sediment problem over a long period is a requisite to correct analysis. The cost of erosion control measures on the watershed, and the cost of maintaining the reservoir space in whatever quantitative combination is most practical and economical, must be included in the total project cost. The benefits (whomsoever they may help and whensoever they may accrue) should equal or exceed the cost.

L. C. BISHOP,[26] M. ASCE.—One feature of multiple-purpose reservoirs that has been somewhat neglected in the past is silt control. This important consideration has been omitted from the Symposium; yet, unless some plan is devised for holding back most of the silt by better farming practices and erosion control, and then providing a method for washing the remainder through the reservoirs, the day will come when all the fall of the streams is taken up by dams and the basins created behind them will be filled with silt. It will then no longer be economical to operate such reservoirs.

Mr. McBride, in the twelfth Symposium paper, is correct in blaming the states, to some extent, for lack of cooperation with federal agencies. For the most part, the Bureau of Reclamation has made investigations of irrigation projects on interstate streams without regard to the fact that the equities of the states in the right to the use of such waters have not been determined. After considerable agitation by the states in the Upper Colorado River Basin, the United States Department of the Interior did require a compact for the division of the waters, and assisted materially in negotiating it. They should do this in every instance.

A. H. DAVISON,[27] M. ASCE.—Admittedly, there is confusion as to the philosophies and practices discussed in this Symposium that needs to be resolved. Establishment of rational, uniform, and sound economic principles is needed to control the expenditure of public funds. Naturally, since fully 20% of the entire national income is required to finance the federal budget, a tough public will be more than usually critical of the behavior of federal agencies and their management of public funds.

Currently reported in an engineering periodical is the developing controversy as to which government agency shall have control over streams on which public works are projected. This is reminiscent of the "squabble" between military agencies of the Department of National Defense. The public interest is not helped thereby and the responsible agencies earn a poor reputation.

As a fundamental principle, it seems appropriate to emphasize that public agencies are public servants—not public dictators to exploit pet philosophies at public expense. Public resources are not unlimited, and carelessness in their investment will be attended by public distrust, resentment, and loss of good will toward the responsible agency.

Honest engineering in the public interest suggests that departmental philosophies and rivalries should be discouraged. They are more likely to compete with the public interest than to support it.

A justifiable approach to multiple-purpose reservoirs would emphasize objectives that aid people to preserve and improve their means of livelihood. According to the testimony advanced in the Symposium, such limited objectives are most easily coordinated. During a period when the tax burden caused by past emergencies is heavy, every dollar of investment cost should earn at least a dollar's worth of tangible benefit and as much more as is consistent with legitimate objectives. Flood control, as the primary justification for invest-

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[27] Civ. Engr., Glens Falls, N. Y.

ment (because public safety is the legitimate objective), should not be exploited to include speculative intangible benefits. In some parts of the United States, the provision of sanctuaries for fish, wild life, and recreation at appreciable public expense would be considered wasteful and frivolous by many people who are seriously concerned about their own livelihood. On many streams that are highly developed industrially, navigation no longer exists above tidewater, and any representation that navigation should be considered an important factor in that area is "humbug." It is extravagant to design for the development of inherent power possibilities that provide for two or three times as much installed capacity as the stream can possibly operate, at a load factor consistent with the prevailing load factor of the electric system in that area. Experience has proved that such design simply results in public veto of the entire project including its worthwhile flood control feature.

In the northeastern part of the United States, a multiple-purpose reservoir project usually emphasizes the flood control feature as justification for investment of public funds. Provisions for power storage and power development at the site are added under favorable circumstances. The added functions are not allowed to impair the primary function of flood control operation to protect damage centers downstream. An example is a state-owned-and-operated reservoir in the Adirondack section of New York State, which (since about 1930) has been operated to control floods and to increase low stream flow to power plants and industries downstream. The capacity of the reservoir is about 690,000 acre-ft or about 650 acre-ft per sq mile of controlled drainage area.

It has been stated that the primary operating purpose of a reservoir should be clearly defined, and the problem of design is to provide facilities to realize that purpose.

To secure the expected tangible benefits, intelligent operation must be guaranteed, particularly with respect to flood control. Intelligent flood control operation of a reservoir that controls not more than one fourth or one third of the total drainage area above important damage centers downstream depends considerably on shrewd appraisal and understanding of the runoff from that part of the drainage area that is uncontrolled. Prompt closing of reservoir gates at the beginning of a storm, consistent with the maintenance of minimum flow required by industries and other facilities, followed by gradual release of stored water after the uncontrolled runoff has passed the damage centers, is generally indicated. Reservoir operation is not a matter of finicky rule curves devised by theorists.

It seems axiomatic that: (1) A multiple-purpose reservoir should never be caught full to spillway crest at the beginning of a storm; and (2) the storage space reserved for floods should be sufficiently empty at the beginning of a storm to provide adequate flood control. All business is a calculated risk. The degree of "emptiness" of flood storage space might vary with the season of the year when major floods may be expected in that locality. Certainly, the flood storage space should be practically empty at the beginning of the "design storm."

Effective control of a reservoir considers the three elements of inflow, storage, and outflow. Maximum inflow is reflected by the "design storm,"

the magnitude of which is determined by consideration of the records, hydrology, and meteorology applicable to the locality. The designed capacity of the reservoir represents the storage factor. The discharge capacity of spillway and gates represents maximum outflow, which for satisfactory results should not threaten the safety of damage centers downstream. Cost is budgeted to provide for effective controls and their adequate and continuous maintenance, including the salary of a full-time attendant.

It is the function of design to make those controls effective and dependable at all times. A design assumption that considers gates closed or otherwise inoperative and the reservoir full to spillway crest at the beginning of a storm may be permissible in the case of a power storage reservoir because that condition is occasionally an incidental aspect of power storage operation. Such a design assumption does not make sense in the case of a flood control reservoir, especially when applied to the flood storage space of a multiple-purpose reservoir where the chances are excellent that the power storage space is also somewhat depleted at the beginning of a storm.

Extravagant and illogical assumptions result in extravagant design and construction of spillways. An actual flood occurrence that would test the capacity of such a spillway would destroy the damage centers downstream that had trusted in "flood control." If such actual occurrence is unthinkable, why provide the expensive and excessive spillway capacity?

If spillway design applicable to power storage reservoirs is applied to flood control or multiple-purpose reservoirs, the public may suspect that the avowed flood control feature may later disappear and the entire reservoir may ultimately be acquired as power storage by a special interest at public expense. Experience proves that, if design procedure is inconsistent with avowed operation procedure, the public will become suspicious of motives and will probably ascribe the least favorable motive to the public agency involved.

It seems evident that the public interest must be treated fairly by federal agencies or the public may prefer to take its chances with natural forces that appear more trustworthy in behavior and less expensive in results. Such an outcome might be more unfortunate for federal agencies than the reorganization advocated by the report of the Hoover Commission on its recent investigation into the activities of government departments with respect to development of natural resources.

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DISCUSSIONS

EFFECT OF DRIVING PILES INTO SOFT CLAY

Discussion]

BY PHILIP C. RUTLEDGE, A. W. SKEMPTON, JAMES R. SCHUYLER, AND B. A. KANTEY

PHILIP C. RUTLEDGE,[17] ASSOC. M. ASCE.—Effort, money, and time have been spent to obtain the factual data in this paper, on a subject that has long been controversial. The writer commends the authors for securing and presenting the data completely to the engineering profession in their paper, but he cannot agree with the conclusions drawn from these data. A fortunate circumstance of this investigation was the existence of an extremely uniform deposit of soft blue clay below El. 550 from which the authors have plotted extensive test data between El. 550 and El. 525. This discussion will be concerned primarily with this soil layer and these data.

The authors describe this layer as a homogeneous soft blue clay, with the exception that it includes "an appreciable number of small shale pebbles up to ¼ in. in diameter," and as not particularly sensitive to remolding. The authors have included statements of average water contents before and after pile driving from the data shown in Fig. 9(*a*). The writer has used these averages and has abstracted compressive strength data from Fig. 8 and water content data for borings C-1 and C-11 from Fig. 9(*b*) and computed similar averages. These average test results, all taken directly from the authors' data, are summarized in Table 1. Because of the uniformity of the clay layer in question, it is assumed that averages of test data over a 25-ft depth equalize the minor natural variations from sample to sample and represent acceptably the average conditions of the entire clay layer.

The data in Table 1 do not tell the entire story on what happened to the homogeneous clay during the pile driving because considerable periods of time elapsed between the pile driving and the sampling and testing, and both the water content and the compressive strength changed progressively with time. The significance of the data is shown in Fig. 19, in which a part of a consolidation test compression curve for a Detroit clay, with approximately the same

NOTE.—This paper by A. E. Cummings, G. O. Kerkhoff, and R. B. Peck was published in December, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1949, by Leonardo Zeevaert, Jacob Feld, Joseph C. Fegan, and Gregory P. Tschebotarioff.

[17] Prof., Civ. Eng., Technological Inst., Northwestern Univ., Evanston, Ill.

initial water content and plasticity characteristics, is superposed on a plot of points representing the authors' average water content and compressive strength data. Previous work[18,19] has shown that on a semilogarithmic plot of test data for saturated natural clay, the compressive strength-water content

TABLE 1.—AVERAGE SOIL TEST RESULTS (HOMOGENEOUS SOFT BLUE CLAY, EL. 550 TO EL. 525; LIQUID LIMIT, 30.5%; PLASTIC LIMIT, 17.5%; AND PLASTICITY INDEX, 13.0%)

Description	Boring A	Boring P-1	Boring P-11	Boring C-1	Boring C-11	Remolded
Elapsed time after driving, in months.....	Before	1	11	1	11	Before
Water content (per cent of dry weight)...	24.7	22.8	22.0	23.8	24.4	24.7
Compressive strength (tons per sq ft).....	0.61	0.56	0.75	0.56	....[a]	0.28

[a] Not shown.

relation is a curve parallel to the virgin (or essentially straight-line) part of the consolidation test curve. In this case it seems more than a coincidence that the points for the average strengths of completely remolded clay and of clay immediately adjacent to the pile 1 month and 11 months after driving should fall on a single straight line approximately parallel to a consolidation test curve that is reasonable for this clay.

From Fig. 19 the writer's interpretation of the authors' data follows. It is first assumed that the degree of sample disturbance due to sampling operations is approximately the same for all borings because all samples were taken by the same procedure. With this limitation, point A in Fig. 19 for boring A represents the condition of the clay before pile driving. During pile driving the condition of the clay immediately adjacent to the piles moved from point A to point B or to a condition of complete remolding. In the first month after pile driving, this clay consolidated from point B to point C with a consequent increase in strength. In the following 10 months, the consolidation and increase in strength continued to point D, where the strength is greater than the original natural strength of the clay. The one test point, E, on the center of a pile group (about two and one-half diameters from the surfaces of the piles) indicates that the clay in this location was partly remolded and after 1 month had consolidated about one half as much as clay immediately adjacent to the piles.

The data in Fig. 19 show why, when piles are pulled out of clay several months after driving, a cigar-shaped mass of clay sometimes adheres to the pile. The strength of the clay immediately adjacent to the pile (point D, Fig. 19) has become greater than the strength of the clay some distance from the pile (points A or E). On pulling, the clay shears through its weakest zone, which is either at the interface between pile and clay or at some distance away from the pile surface.

[18] "Review of the Cooperative Triaxial Research Program," by P. C. Rutledge, in "Triaxial Shear Research and Pressure Distribution Studies on Soils," Soil Mechanics Fact-Finding Survey, U. S. Waterways Experiment Station, Vicksburg, Miss., April 1947, pp. 20–26 and 79–100.

[19] "Strength of Natural Clays," by P. C. Rutledge, paper presented at ASCE Meeting, Boston, Mass., October, 1948, *Civil Engineering*, November, 1948, p. 23 (abstract).

It is unfortunate for the authors' arguments concerning vertical consolidation settlements of the clay that the piles in this investigation were driven to point bearing in hardpan. With the piles penetrating through the clay mass as fixed vertical reinforcement 4.5 ft on centers, little vertical consolidation should be expected even if all the clay between the piles is completely remolded and tending to consolidate under its own weight. If an area is considered bounded by the center lines of a four-pile group, there is a mass 4.5 ft by 4.5 ft.

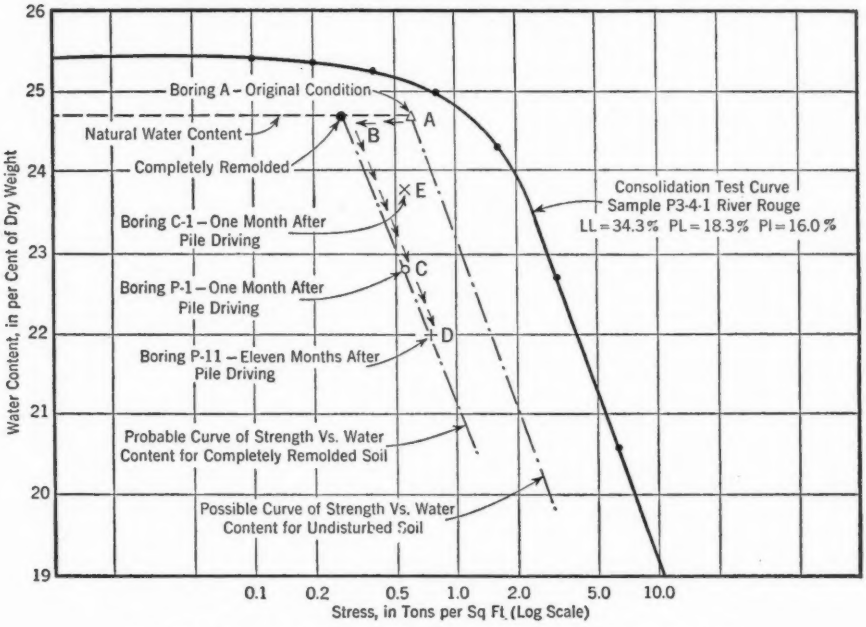

FIG. 19.—PLOT OF AVERAGE DATA STRENGTH VERSUS WATER CONTENT

by 60 ft deep in which 5% of the total volume is perfectly rigid, consisting of four quadrants of a pile at the four corners of the mass. According to the authors' observations, the top of the clay heaved about 2 ft during driving, representing a total volume increase of about 3% as compared with 5% of the total volume occupied by the piles. From these rough computations some decrease in volume of the soil during driving is indicated.

The 95% volume of clay, assumed to be completely remolded, is no longer in equilibrium with the stresses resulting from the weight of overlying material and tends to consolidate and to move downward. However, 22% of the surface area of the clay mass is bounded by nonmoving pile surfaces on which downward motion will develop shear stresses, while the remaining 78% of the surface area is in contact with clay in the same condition as the mass under consideration. Ignoring the possibility of arching effects, a shear stress of 0.26 ton per sq ft at the interface between piles and clay is sufficient to carry completely the weight of the clay mass. From the authors' tests, the shear strength at this point immediately after driving was 0.14 ton per sq ft, and it

increased to 0.28 ton per sq ft in the first month after driving. In other words, downward motion at the interface between pile and clay could develop a shear stress sufficient to transfer half the total weight of the clay to the piles immediately after driving. A stress transfer of such magnitude in this clay would certainly greatly decrease the tendency of the remolded clay to consolidate vertically. After 1 month the consolidation of the clay adjacent to the piles has produced a strength sufficient to transfer the entire weight of the clay to the piles if downward motion of the clay should continue. Thus a relatively small consolidation and a downward motion of the clay are sufficient to transfer a large part of its weight to the rigid piles and to eliminate the stresses that otherwise would cause additional consolidation.

If the piles were friction piles with their lower 20 ft penetrating a nonrigid clay layer which carried the pile load primarily by skin friction, an entirely different situation would exist. The initial downward motion of the remolded clay would transfer a load of approximately 30 tons to each pile. If this load increase did not cause downward motion of the pile, further consolidation could transfer the entire weight of the clay mass, or about 58 tons, to each pile. It seems probable that this magnitude of excess load on a friction pile would cause excessive skin friction on the bottom section of the pile and settlement of the pile.

From these considerations the writer believes that the authors' contentions concerning the absence of vertical consolidation are not valid because:

(1) The conditions of the field tests were such that only small amounts of vertical consolidation should be expected;

(2) The test data do indicate a small over-all decrease in water content and volume of the clay; and

(3) The conditions of this investigation cannot be extended to the entirely different conditions of friction piles.

In general, the authors' test data seem to provide a remarkable confirmation of the predictions by A. Casagrande,[4] M. ASCE, of the remolding effect of pile driving on clay which were made in 1932 on the basis of hypothetical evidence. Finally, the writer again expresses his appreciation to the authors for their efforts in obtaining these factual test data and publishing them in full so that all engineers can analyze and draw conclusions from the observations.

A. W. SKEMPTON.[20]—A comparatively small number of case records are available describing field investigations conducted with the object of obtaining data on the behavior of clays in relation to piling. This paper is an interesting and valuable addition to that literature. In a problem such as that of the effect of driving piles into a stratum of clay an answer can be obtained in any given case only by reference to past records relating to a similar clay, since the difficulties of predicting the influence of pile driving on the clay properties, from laboratory tests alone, are very great.

[4] "The Structure of Clay and Its Importance in Foundation Engineering," by A. Casagrande, *Journal*, Boston Soc. of Civ. Engrs., April, 1932.

[20] Univ. Reader in Soil Mechanics and Asst. Prof., Imperial College, Univ. of London, London, England.

As knowledge on this problem, therefore, is dependent upon the collection of reliable field data, and as the amount of the data is at present very limited, it is all the more important to examine in detail the results given in the present paper. Consequently, the writer feels justified in raising several points on which he does not find himself in full agreement with the authors, and he will also give a brief description of a job in which the driving of piles in a bed of clay of moderately high sensitivity was instrumental in leading to disastrous consequences.

As a basis for examining the results of the authors' investigation, the writer has evaluated the average strength in both the upper and lower clay strata, as found in borings A, P-1, P-11, and C-1 (Figs. 8 and 9). These average values

TABLE 2.—AVERAGE COMPRESSIVE STRENGTHS OF SAMPLES REPORTED BY AUTHORS

Strata	Elevation (ft)	Compressive Strengths (kg per sq cm)[a]				Adjacent to Pile						Center of Pile Group, 3 Weeks After Driving		
						3 Weeks After Driving			11 Months After Driving					
		Undisturbed	Fully remolded	Sensitivity[b]	Remolding loss[c]	Strength (kg per sq cm)	Loss[d] (%)		Strength (kg per sq cm)	Loss[d] (%)		Strength (kg per sq cm)	Loss[d] (%)	
							Undisturbed	Remolding		Undisturbed	Remolding		Undisturbed	Remolded
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Upper	570 to 550	0.48	0.15	3.2	0.33	0.25	48	70	0.39	19	27	0.36	25	36
Lower	550 to 525	0.61	0.27	2.3	0.34	0.56	8	15	0.72	−18	−32	0.56	8	15

[a] 1 kg per sq cm = 14.2 lb per sq in. [b] Sensitivity (dimensionless) = $\frac{\text{undisturbed strength}}{\text{remolded strength}}$. [c] Loss in strength on full remolding. [d] Loss in strength as percentages, respectively, of undisturbed strength and full remolding loss.

are given in Table 2, from which it would seem that the following five conclusions may be drawn:

(1) The upper clay has a moderate sensitivity, the average value for the sensitivity of the clay between El. 570 and El. 550 being 3.2. In the light of this result the writer does not agree with the statement (under the heading, "General Soil Conditions") that:

> "In their undisturbed state, the samples had an unconfined compressive strength approximately eight times that of the same samples when remolded in the laboratory at their natural water content."

This appears to be true only for the two samples at El. 571 and El. 575, and it is clear from Fig. 8 that these two samples are not characteristic of the upper clay.

(2) The upper clay adjacent to the pile, 3 weeks after driving, suffered a loss in strength of 48%, which amounts to 70% of the full remolding loss. At the center of the pile group, at a distance of about two and a quarter diameters from the surface of the pile, the corresponding losses were 25% and 36%.

Thus, in this clay of moderate to low sensitivity the strength 3 weeks after driving varied from one half to three quarters of the original strength and, moreover, this loss amounted to between one third and two thirds of the maximum possible reduction. These results are not so drastic as those suggested by A. Casagrande[4] M. ASCE (and he was certain to err on the side of safety in making a generalization); but they appear, nevertheless, to be rather substantial losses and, at least as far as the upper clay is concerned, the writer cannot altogether agree with the conclusion, drawn by the authors (under the heading, "Conclusions") that the effect of pile driving, even for friction foundation, would quite probably be beneficial.

(3) In the same connection it may be noted from Table 2 that the strength of the upper clay, adjacent to the pile, 11 months after driving is still only about 80% of its original value.

(4) Corresponding to a loss in strength of between 25% and 48% there would presumably be an appreciable increase in compressibility and a decrease in modulus of compression. These effects, again, cannot be considered as beneficial from the point of view of foundation behavior.

(5) Turning now to the lower clay between El. 550 and El. 520 the writer is in substantial agreement with the authors in their conclusion that the effect of pile driving is comparatively unimportant. It has to be remembered, however, that the lower clay is one of low sensitivity—namely, 2.3 (average), and that many geologically recent estuarine and marine clays have sensitivities of between twice and five times this value.

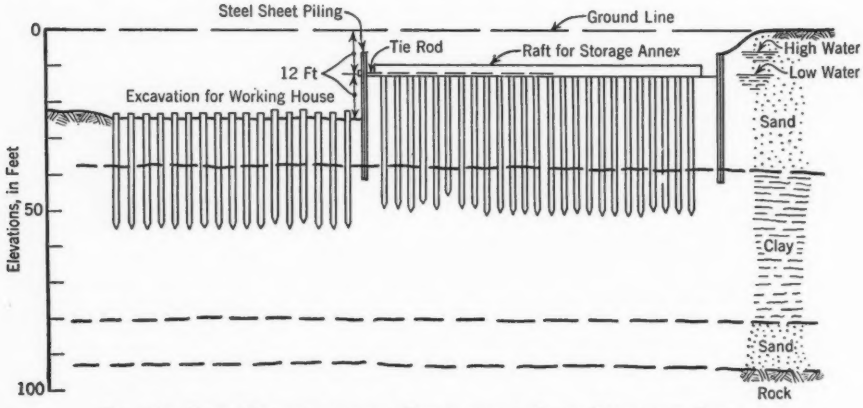

FIG. 20.—CROSS SECTION THROUGH FOUNDATIONS; GRAIN ELEVATORS, DURBAN, UNION OF SOUTH AFRICA, 1922 (CONCRETE PILES, 15 IN. SQUARE)

A case in which the effect of pile driving appears to be one of the main causes of an engineering disaster may now be described. In 1922 an attempt was made to provide a friction pile foundation for a grain silo in Durban Harbour, Union of South Africa.[21] A cross section of the foundation and of the underlying deposits is shown in Fig. 20, from which it will be seen that the

[21] "Report of the Durban Grain Elevator Commission," Govt. Printers, Cape Town, Union of South Africa, 1924.

piles were driven through a layer of sand into a stratum of clay about 42 ft thick overlying (at a depth of 80 ft below surface) a second layer of sand. Rock, consisting of thinly bedded limestones and shales of Cretaceous age, was found at a depth of 92 ft. Borings showed that the clay was "tough" and it was decided to use it as a foundation material for the piles. Seventeen widely spaced test piles were then driven, from which it was concluded that a working load of about 45 tons per pile could be obtained. These tests were not altogether satisfactory since the piles were driven in the storage annex site (see Fig. 20) when the excavation had reached about only one half of its final depth, and the piles were therefore more deeply embedded in the sand than was to be the case in the actual foundation.

Pile driving was commenced in the annex site in August, 1921, by which time the cofferdam of sheet piling had been completed, and it appears that the sets became progressively worse. The specified set had been $2\frac{1}{4}$ in. for ten blows but the final average set for the annex piles was 8.6 in. This would seem to be a result of remolding in the clay, but a contributory cause may also have been the excavation for the working house which, by December, 1921 (when the annex piling was completed), was well advanced.

At that time the sheet piling separating the two excavations was seen to be bulging between the tie bars. This indicates lateral displacement of the ground, which is not surprising since the piles were driven at about 4-ft centers over the entire area of the annex (200 ft by 100 ft in plan) and since photographs taken at the time show that the waling used in the separation cofferdam was inadequate.

In spite of the ominous progressive decrease in driving resistance the annex raft was cast, and additional ties were provided to strengthen the cofferdam. Meanwhile, excavation in the working house site continued and in February, 1922, pile driving was begun in this area. From the start the sets were worse than the specified value, and when the working house piling was completed in April, 1922, the average set for ten blows being obtained was of the order of 25 in. It may be noted from Fig. 20 that these piles are embedded for more then half their length in the clay, and the most obvious reason for the extremely bad set is that the clay was suffering a progressive loss in strength due to remolding, especially since the working house pile driving was the only operation under way during the period from February to April.

During the next 6 weeks twenty-eight extra piles were driven, and construction of the reinforced concrete structure on the annex raft was commenced. By the middle of June, however, the raft had settled about 1 in., and voids were discovered beneath it. After discussion it was concluded that the working house piles were entirely inadequate and it was decided to sink cylinders to rock for this foundation and to grout beneath the annex raft.

Unfortunately the cylinders were not driven with sufficient care to avoid an excessive loss of ground, and the raft settled many inches, with consequent severe cracking. Eventually the entire foundation was abandoned, and entirely new foundations were made by sinking (with careful control) a series of compressed air cylinders, 9 ft and 12 ft in diameter, to the rock. The loss involved on the foundations was estimated at rather more than £200,000.

The writer can give the results of some laboratory tests on the clay since, in 1946, he was requested to act as consultant for the foundations of a large structure which was to be built at a distance of about one quarter of a mile from the grain silo.

Borings showed a succession of strata similar to those beneath the grain silo, and the clay at the new site was not only placed at approximately the same elevation but corresponded with the visual description when the 1921 borings were being made. The relevant laboratory tests are summarized in Table 3, from

TABLE 3.—RECORD OF BORINGS AT DURBAN HARBOR, UNION OF SOUTH AFRICA, MADE IN 1946

Depth (ft)	Consistency				Particle Size (Millimeters)			Specific gravity	Density (lb per cu ft)	Shear Strength		Consolidation			
	Water content (w)	Liquid limit (LL)	Plastic limit (PL)	$\frac{w-PL}{LL-PL}$	Sand 2 to 0.06	Silt 0.06 to 0.002	Clay <0.002			One half compressive strength[a]	Sensitivity	Overburden pressure[b]	Preconsolidation load[c]	Compressibility (sq ft per ton)	Coefficient of consolidation[d]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Boring A															
45	58	111	28	0.36	....	....	....	....	100	1,210	3.9	1.5	....	....	....
48	63	112	33	0.38	....	....	....	2.66	99	1,270	....	1.55	3.2	0.023	0.021
Boring B															
46	56	....	....	....	....	....	....	....	102	1,180	4.5	1.5	....	....	....
50	64	115	32	0.38	....	....	....	....	101	....	....	1.65	....	....	....
53	64	113	35	0.37	11	47	42	2.64	99	1,270	....	1.7	3.4	0.018	0.037
Boring C															
49	65	109	31	0.43	....	....	....	....	99	1,110	....	1.6	3.3	0.025	0.020
Boring D															
48	46	88	27	0.31	19	48	33	....	107	1,330	3.6	1.55	....	....	....
Average Borings, A to D															
....	60	108	32	0.37				2.65	101	1,230	4.0	1.6	3.3	0.022	0.026

[a] Pounds per square foot. [b] Tons per square foot. [c] Compressibility. [d] Square centimeters per minute.

which it will be seen that the clay has a shear strength of about 1,200 lb per sq ft in its undisturbed strength and a sensitivity of about 4.0 (Cols. 11 and 12, Table 3). Typical stress-strain curves are shown in Fig. 21 and they are characteristic of a clay of moderately high sensitivity. The laboratory tests indicate that the clay is lightly overconsolidated. This is probably caused by partial drying subsequent to its formation but prior to the deposition of the

overlying sand, since in one boring the clay was seen to be oxidized to a brown color in the top 2 ft of the stratum.

In conclusion, the writer wishes to emphasize again the great value of observations such as those recorded by the authors, but he feels that experience

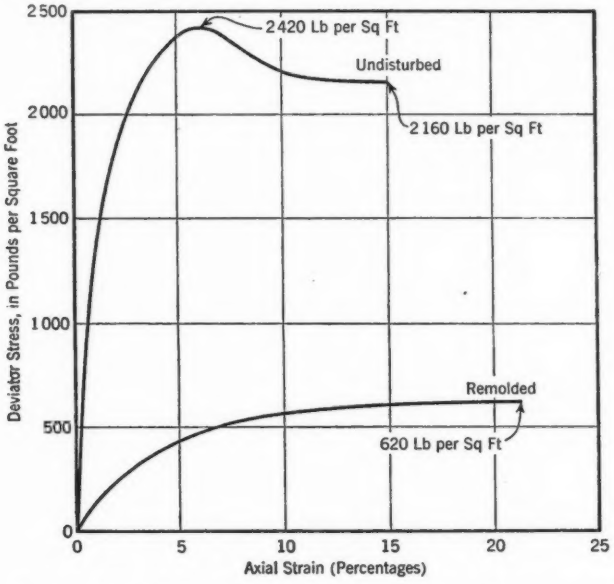

FIG. 21.—TYPICAL STRESS-STRAIN CURVES (ANGLE OF SHEARING RESISTANCE ϕ EQUALS ZERO IN UNDRAINED TRIAXIAL TESTS)

such as that at Durban and, to a more limited degree, the behavior of the upper clay at Detroit, Mich., indicate that in some clays the effect of pile driving can be detrimental. The lower clay of Detroit, on the other hand, shows that this is not always the case. More field research is required before engineers can be reasonably certain as to the border line separating the harmful from the beneficial effects of piling in clays.

JAMES R. SCHUYLER.[22]—Valuable comparative soil strength data related to the effect to driving piles into soft clay are recorded in this paper. Qualitative information derived by the writer from model tests at the Soil Mechanics Laboratory of Princeton University at Princeton, N. J., confirms the authors' conclusion that the original forecast of A. Casagrande, M. ASCE, in 1932, concerning the detrimental effect of pile driving into clays, was extreme, especially when other possible sources of disturbance are analyzed. However, the thesis advanced by the authors that a soft clay stressed by pile driving will not reconsolidate and settle under its own weight is unduly magnified in view of the following information, unless settlement observations between piles can be presented to support that thesis.

[22] Prin. Engr. (Soils), New Jersey State Highway Dept., Trenton, N. J.

As a result of a combined field and laboratory study, Gregory P. Tschebotarioff, M. ASCE, advanced a hypothesis to the effect that the remolding caused by pile driving was smaller in magnitude and more limited in extent than was the remolding accompanying the distortions induced by a heave in the

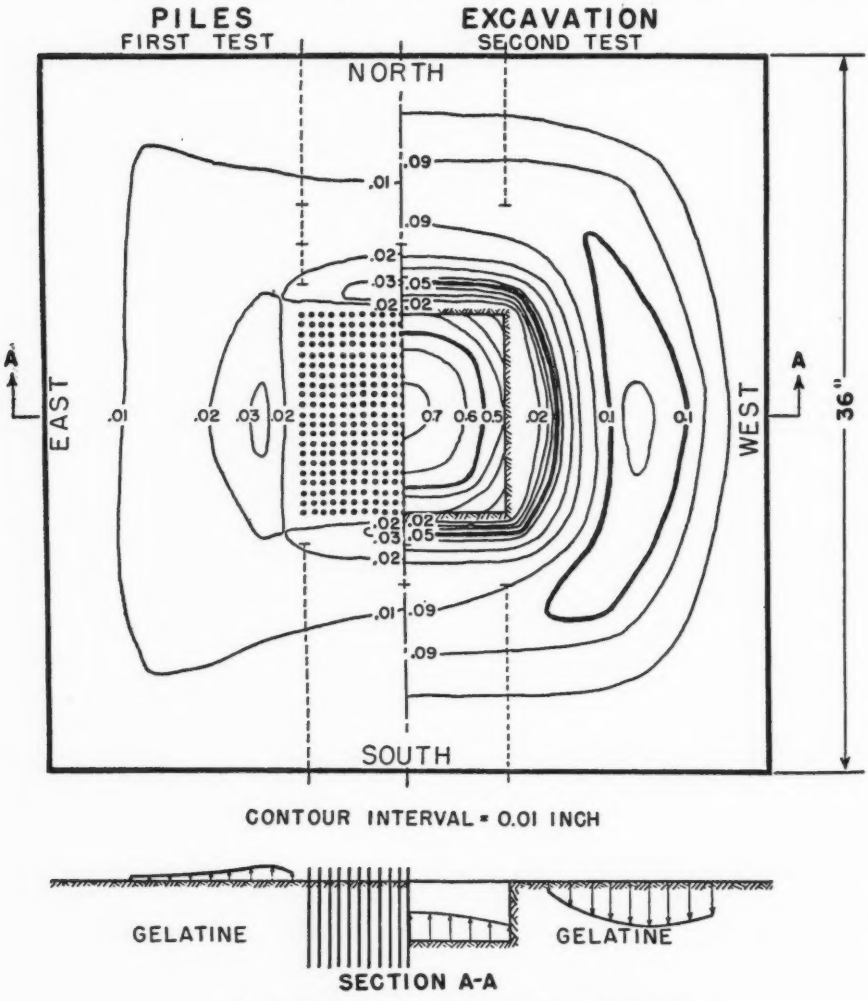

FIG. 22.—COMPARISON OF SURFACE DISPLACEMENTS

bottom of an excavation.[13] Model studies in gelatine were performed to check the disturbance produced by driving piles into plastic clay as compared to the disturbance caused by an unbalanced excavation. The model studies confirmed this hypothesis in a qualitative manner.

[13] "Comparison of the Extent of Disturbance Produced by Driving Piles Into Plastic Clay to the Disturbance Caused by an Unbalanced Excavation," by Gregory P. Tschebotarioff and James R. Schuyler, *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, 1948, Vol. II, pp. 199–205.

A 3-ft by 3-ft metal tank, 1 ft deep, was constructed and filled with gelatine for each test after devices were inserted in the tank to measure displacements within the mass of gelatine. The comparative study was performed: (1) By measuring the deformations of the gelatine surface and the displacements within the mass of gelatine when 441 piles, 4.25 in. long and 0.125 in. thick, were driven into the gelatine, outward from the center of the gelatine surface in a spiral sequence; and (2) by taking measurements at the same locations when an excavation was simulated in the same area to which the piles were confined. All displacements were measured to accuracies of 0.001 in. in relation to a rigid frame. Mechanical extensometers were adjusted by a micrometer screw until contact with ultrathin metallic plates on the surface was signaled by the glow of a neon light. Within the mass of gelatine thin metal disks were connected to light metal rods, which extended through openings in the walls and bottom of

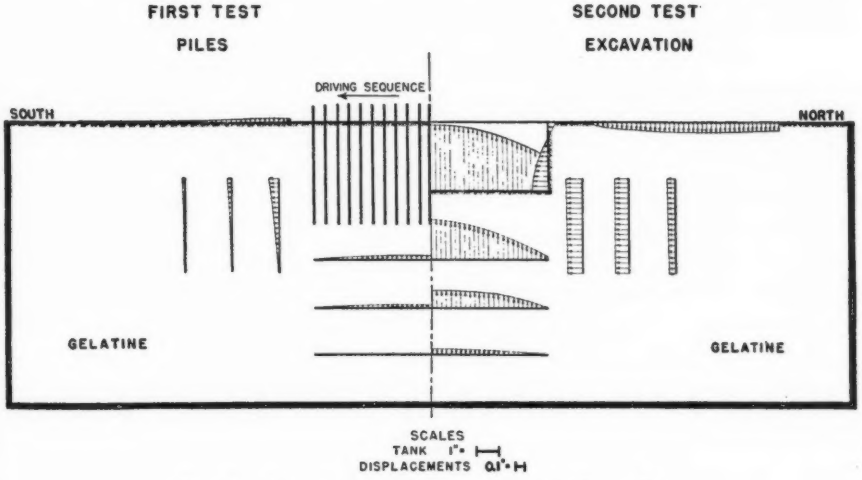

FIG. 23.—COMPARISON OF DISPLACEMENTS BY MODEL TESTS

the tank. These rods were incased in vaseline-filled glass tubes which were extracted after the gelatine had set. Horizontal displacements were measured only along the north-south axis of the tank (Fig. 22), and a comparison with the displacements along the east-west axis of the tank indicated that these devices did not seem to restrain to any appreciable extent the freedom of gelatine displacements in the vertical direction. The strength characteristics of the gelatine were the same for both tests.

The results of both tests are compared graphically in Figs. 22 and 23. The heave between piles reported by the authors is synonymous with the results of this model test. Furthermore, Fig. 23 shows that, if a large number of piles is driven, the gelatine is stressed for an appreciable distance surrounding the volume occupied by the pile group.

Since the authors report that the pile driving reduced the compressive strength of the clay slightly more than half of the maximum possible reduction, the writer contends that this weakening of the soil structure will be prevalent

throughout the zone of clay stressed by pile driving, as indicated by Fig. 23, and that this clay will consolidate under its own weight. Such consolidation would not be detrimental to the structure described by the authors; but, in cases where the soil between piles is utilized for the support of a floor and curtain walls, the authors' hypothesis concerning the process of consolidation initiated by the degree of disturbance reported in the paper to cause no settlement to the ground surface should be accepted with reservations until more certain evidence is presented to substantiate their speculative assumption. Mr. Tschebotarioff has reported consolidation of a glacial clay deposit which was injurious to a floor and partitions in the basement of a building which was founded on long steel piles driven to a firm stratum.

For the present a moderate theory concerning the increase in compressibility which some clays undergo when structurally weakened should fit the category midway between the extremes of Mr. Casagrande's original assumption and the hypothesis advanced by the authors.

B. A. KANTEY,[23] Assoc. M. ASCE.—The subject of this paper is of vital interest to the practicing engineer and far too little is known about it.

A composite vertical section through the piles and subsoil is shown in Fig. 2. This section is based on the results of six borings made at various locations on the site, which indicated that the soil was uniform in its properties in horizontal directions over the entire area investigated. In view of the very small variations in moisture content upon which the conclusions of the authors were based, the writer feels that the details of the six borings might well have been included to indicate the variation in natural moisture content in a horizontal direction throughout the area.

After pile driving had been completed additional borings were put down. These additional borings consisted of one boring next to a pile and one boring in the center of a pile group about 3 weeks after driving. This cycle was repeated about 11 months after driving. In effect then, the original boring A, and these four additional borings, combine to form one set of results which show the variation in moisture content with time and with distance from a pile. The writer feels that the conclusions drawn in this paper are so important that additional sets of results would have been of great value. Could these additional sets of results have been augmented with the extra borings suggested subsequently in this discussion, the data presented might have had even greater value.

Basically the paper undertakes to show that pile driving in the material investigated does not cause such complete and deleterious remolding as has been assumed since the publication of a paper by A. Casagrande,[4] M. ASCE, in 1932. Consequently, the disturbing effect of pile driving in this material would not be so detrimental and therefore would not cause harmful settlement of the surface.

The authors point out that, in spite of Mr. Casagrande's warning that his tentative conclusions should not be accepted without large-scale field research to back them up, these tentative conclusions have been widely accepted. The

[23] Research Officer, Soil Mechanics Div., National Bldg. Research Inst., Pretoria, Union of South Africa.

conclusion with which the authors mainly disagree is that the clay immediately surrounding a pile would be completely remolded to a distance of one half the diameter of the pile, and that it would be disturbed sufficiently to cause a large increase in compressibility to a distance of one and a half diameters.

From the results produced by the authors it seems clear that this is not the case for the material discussed. In fact, the evidence produced shows only partial remolding adjacent to the pile and practically no effect in the center of the pile group. Since this result was also found by W. S. Housel, M. ASCE, and J. R. Burkey,[24] working in the same area, it seems logical to accept the conclusion that driving piles into the material under discussion would not have the detrimental effects that could be forecast on the basis of the work reported by Mr. Casagrande.

It seems clear, then, that there is a great need for more research into the effect of remolding. What appears to be particularly needed is some tie-up between the sensitivity ratio as expressed by the ratio between the unconfined compression test on undisturbed and on remolded samples, the Atterburg limits, the natural moisture content, and the change in void ratio with pressure for undisturbed and remolded conditions.

Tentative examination of results available in various papers seems to indicate the possibility of some kind of correlation between sensitivity to remolding and the Atterburg limits where the natural moisture content is below the liquid limit. A clay with a low liquid limit and a low plasticity index generally appears to be less sensitive than one with higher limits, and this assumption does not appear to be unreasonable when considering Mr. Casagrande's suggested explanation of the cause of remolding.[4]

Mr. Casagrande also suggests that a soil with a simple honeycomb structure will show a smaller difference in compressibility between the undisturbed and remolded states than one with the complex structure described in his paper, and that the larger the plasticity index, the more compressible the soil. Frank A. Marston, M. ASCE, has also pointed out that certain clays appear to have a form of structure permitting a high void ratio yet offering considerable resistance to change in volume.[25] When the structure is broken down, the water in the larger voids is released and the once stiff clay becomes soft and plastic. Accepting Mr. Casagrande's explanation of the cause of this behavior leads to the hypothesis that it is essentially a colloidal phenomenon. This may then indicate that a measure of the colloidal clay content might be a measure of a clay's sensitivity to remolding. It also appears possible that low colloidal clay content would lead to a low liquid limit and, hence, a low plastic limit which could then be correlated with sensitivity.

All the examples found have dealt with clays having a natural moisture content close to, but below, the liquid limit. In Durban, Union of South Africa, there is a clay that has a liquid limit of between 50 and 60 and a natural moisture content generally slightly above the liquid limit. This clay has a rather low sensitivity ratio based on the unconfined compression test but it is very

[24] "Investigation to Determine the Driving Characteristics of Piles in Soft Clay," by W. S. Housel and J. R. Burkey, *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, Vol. V, p. 146.

[25] *Journal*, Boston Soc. of Civ. Engrs., April, 1932, p. 210.

much more compressible when remolded than when it is in the natural state. There is a possibliity, therefore, that the criteria for sensitivity may have to be correlated with the natural moisture content and its relation to the liquid limit.

As regards the Durban clay, some years ago a serious failure occurred in a grain elevator erected on this clay. Although a possible cause for this failure could be the remolding effect of pile driving, this has not yet been definitely established. In the near future, however, it is the intention to conduct a complete investigation and it is hoped that this will lead to more information on the relation between disturbed and undisturbed samples.

The writer feels that there is not sufficient evidence to warrant the conclusion of the authors that the water must have migrated horizontally from the disturbed zone near the pile to the much larger mass of relatively undisturbed clay between the piles. In Fig. 9(*a*) curves are given for the original moisture content, the moisture content about 3 weeks after driving, and about 11 months after driving.

Using the average values quoted, it can be seen that during the first 3 weeks the moisture content dropped by 1.9% whereas 11 months later it had only dropped a further 0.8%. Although this does show a progressive decrease in moisture content with time, it would have been of interest to have obtained several intermediate readings to see when most of the change took place, to have obtained a curve of change in moisture content with time, and to have found out how much of the change in moisture content was a result of the actual pile driving and how much was a result of the consolidation caused by driving. This information might well have been supplemented with void ratio versus load curves for the undisturbed and remolded states.

The writer cannot agree entirely with the finding that the water did not flow upward through the clay mass. Examination of the curve for boring P-11 in Fig. 9(*a*) can be interpreted to indicate a perceptible decrease in moisture content (of 22.2% to 21.4%) from the lower layers upward. The variations in moisture contents are too small, however, to warrant any definite conclusions and the writer hesitates to suggest that the evidence indicates a definite upward flow of moisture.

The authors suggest that the water must then have migrated horizontally; but this too is not generally borne out by the results. A slight indication of this trend is seen in Fig. 9(*c*) where, below El. 530, boring C-11 appears to have a slightly higher moisture content than boring C-1; but this does not appear to be a general characteristic. Even though the slight heaving of the ground during driving may have caused a very slight swelling of the undisturbed clay, which would have been sufficient to take up the reduction in moisture content of the clay next to the pile, borings might have disclosed this fact and a series of borings from the center of a pile group toward the piles soon after driving might have given valuable additional data.

In summary, then, the writer feels that, although the evidence presented in the paper supports the conclusions drawn, the evidence is not sufficiently clear cut to obviate any argument. Additional information along the lines indicated herein may well have served to clarify the position and it is regretted that more borings were not made.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DIFFUSION OF SUBMERGED JETS

Discussion

BY JAMES S. HOLDHUSEN, DUILIO CITRINI, AND STANLEY CORRSIN

JAMES S. HOLDHUSEN,[13] JUN. ASCE.—The experimental program on which this paper is based was planned and executed very carefully, and it seems to justify the assumptions made in deriving the formulas. To the reader of the paper, however, the final form of the equations defining the characteristics of the diffusion process appears to be dependent on the authors' third assumption —that the velocity distribution in the diffusion region follows the normal error curve. Since no semblance of an analytical proof is offered for this assumption, the reader may have some reservations concerning the generality of the subsequent analysis. The following discussion will show that only the first two assumptions are necessary in order to carry through the analysis and arrive at the same form of the final equations.

The first assumption made by the authors—that the pressure is hydrostatically distributed throughout the flow—is entirely justifiable. Theoretical considerations[5] indicate that the pressure differences arising as the consequence of the flow in the y-direction are negligibly small, being of the order of magnitude of a fraction of 1% of the velocity head. The second assumption—that the diffusion process is dynamically similar at all sections of the flow—is apparently justifiable as a consequence of the "free" nature of the diffusion process. This second assumption can be expressed mathematically as

$$v_x = v_{\max} f(\eta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

in which v_x is the velocity at any point in the cross section of the flow that has $v_{\max}$ as a maximum velocity. The variable η is defined by $\eta = y/x$. In this discussion, y is defined as the distance, transverse to the flow direction, from the plane of symmetry in the two-dimensional case or from the axis of symmetry in the three-dimensional case in the zone of established flow. In the zone of flow establishment of both jets, y will be measured from the inner edge of the zone of diffusion, for the sake of convenience. The distance x is measured

NOTE.—This paper by M. L. Albertson, Y. B. Dai, R. A. Jensen, and Hunter Rouse was published in December, 1948, *Proceedings*.

[13] Research Fellow, St. Anthony Falls Hydr. Laboratory, Univ. of Minnesota, Minneapolis, Minn.

[5] "Berechnung turbulenter Ausbreitungsvorgänge," by W. Tollmien, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 6, 1926, p. 468.

from the plane of the slot or orifice. The second assumption states that a single function of η defines the velocity distribution in the diffusion region for any value of x in a particular zone of flow. In the following development, all symbols and terminology will indicate the same quantities that they do in the paper, unless specifically noted otherwise.

In the zone of flow establishment of the two-dimensional jet (Fig. 3) evaluation of the momentum flux integral (Eq. 4) shows directly that

$$\frac{M}{M_0} = 1 = \frac{\int_{0.5B}^{0} (v_0)^2\, dy + \int_0^{\infty} (v_0)^2 f^2(\eta)\, dy}{0.5\, B_0\, (v_0)^2} = \frac{0.5\, B - Y_m\, x}{0.5\, B_0} \quad \ldots\ldots (55)$$

In the discussion concerning both zones of the two-dimensional jet, the following designations will be used, in order to simplify typography:

$$\left.\begin{aligned} Y_m &= \int_0^{\infty} f^2(\eta)\, d\eta \\ Y_q &= \int_0^{\infty} f(\eta)\, d\eta \\ Y_e &= \int_0^{\infty} f^3(\eta)\, d\eta \end{aligned}\right\} \quad \ldots\ldots (56)$$

It will be noted that each of these integrals has a constant, dimensionless, value for an assigned value of $f(\eta)$—that is, for a given velocity distribution. Thus Eq. 55 shows immediately that

$$\frac{B}{2} = \frac{B_0}{2} - Y_m\, x \quad \ldots\ldots (57)$$

The inner boundary of the diffusion region is thus shown to be a plane and

$$x_0 = \frac{B_0}{2\, Y_m} \quad \ldots\ldots (58)$$

Similar evaluation of the volume and energy flux ratios (Eqs. 3 and 5) shows that

$$\frac{Q}{Q_0} = 1 + \frac{2\, x}{B_0}\,(Y_q - Y_m) \quad \ldots\ldots (59a)$$

and

$$\frac{E}{E_0} = 1 + \frac{2\, x}{B_0}\,(Y_e - Y_m) \quad \ldots\ldots (59b)$$

Differentiation of Eq. 59*a* will yield the following equations for the lateral velocity at a considerable distance from the jet:

$$\operatorname*{Limit}_{y \to \infty} \frac{v_y}{v_0} = -\,(Y_q - Y_m) \quad \ldots\ldots (60)$$

Evaluation of the momentum flux ratio in the zone of established flow of the two-dimensional jet shows that

$$\frac{\int_0^\infty (v_{max})^2 f^2(\eta)\, dy}{0.5\, B_0\, (v_0)^2} = 1 = \frac{(v_{max})^2}{(v_0)^2} \frac{Y_m\, x}{0.5\, B_0} \quad \ldots\ldots (61)$$

Thus,

$$\frac{v_{max}}{v_0} = \sqrt{\frac{B_0}{2\, x\, Y_m}} = \sqrt{\frac{x_0}{x}} \quad \ldots\ldots (62)$$

which shows directly that the maximum velocity varies inversely as the square root of the distance from the orifice in the zone of established flow. The three remaining ratios are likewise determined as

$$\frac{Q}{Q_0} = \sqrt{\frac{2\, x\, (Y_q)^2}{B_0\, Y_m}} \quad \ldots\ldots (63a)$$

$$\frac{E}{E_0} = \sqrt{\frac{B_0\, (Y_e)^2}{2\, x\, (Y_m)^3}} \quad \ldots\ldots (63b)$$

and

$$\operatorname*{Limit}_{y \to \infty} \frac{v_y}{v_0} = -\sqrt{\frac{B_0\, (Y_q)^2}{8\, x\, Y_m}} \quad \ldots\ldots (63c)$$

In the zone of establishment of the circular jet, evaluation of the momentum flux ratio will define the inner boundary of the diffusion zone as an ellipsoid of revolution, as was the case in the paper. This indicates that Eq. 54 is not satisfied, and consequently no analysis of the flow characteristics will be made here for this zone. As will be shown, theoretical considerations indicate that similarity of velocity distributions should not be expected in this zone.

In the zone of established flow of the three-dimensional (circular) jet, evaluation of the momentum flux ratio will yield

$$\frac{M}{M_0} = 1 = \frac{\int_0^\infty (v_{max})^2 f^2(\eta)\, dy\, 2\, \pi\, y}{0.25\, \pi\, (D_0)^2\, (v_0)^2} = \frac{(v_{max})^2}{(v_0)^2} \frac{2\, Y_m\, x^2}{0.25\, (D_0)^2} \quad \ldots\ldots (64a)$$

Thus,

$$\frac{v_{max}}{v_0} = \frac{D_0}{x\sqrt{8\, Y_m}} = \frac{x_0}{x} \quad \ldots\ldots (64b)$$

which shows directly that the maximum velocity varies inversely as the distance from the orifice. In the discussion concerning the circular jet, the following designations will be used:

$$\left.\begin{aligned} Y_m &= \int_0^\infty f^2(\eta)\eta\, d\eta \\ Y_q &= \int_0^\infty f(\eta)\eta\, d\eta \\ Y_e &= \int_0^\infty f^3(\eta)\eta\, d\eta \end{aligned}\right\} \quad \ldots\ldots (65)$$

in which, as before, the integrals have a constant dimensionless value for a given value of $f(\eta)$. The three remaining ratios are then determined as

$$\frac{Q}{Q_0} = \frac{x}{D_0}\sqrt{\frac{8\,(Y_q)^2}{Y_m}} \quad \ldots\ldots(66a)$$

$$\frac{E}{E_0} = \frac{D_0}{x}\sqrt{\frac{(Y_e)^2}{8\,(Y_m)^3}} \quad \ldots\ldots(66b)$$

$$\operatorname*{Limit}_{y\to\infty} \frac{v_y\, y}{v_0\, D_0} = -\sqrt{\frac{Y_q{}^2}{8\,Y_m}} \quad \ldots\ldots(66c)$$

The same functional relationships as derived by the authors have been derived without the necessity of assuming any specific distribution of velocity in the diffusion region. The only limitation imposed on this velocity distribution thus far is that the defined integrals exist. One is thus free to choose one of the calculated theoretical distributions, the sources of which have been noted in the paper, and, by a simple graphical integration, the constants in the equation can be determined.

The authors' use of the normal error curve can be justified on the basis of its providing a convenient arithmetic determination of the constants in the final equations. However, in view of the simplicity of making the graphical integrations highly accurate, one is permitted to use an analytically derived form of the velocity distribution, which will permit the extension of the theory to boundary conditions other than those assumed in the present derivation. Also, the use of the error curve has the inherent weaknesses of specifying that the turbulence generated by the mixing extends to infinity, and of indicating a rather erratic variation in the mixing length (Figs. 14 and 20). This discrepancy was pointed out by H. Reichardt[14] when he first postulated the use of the error curve to define the velocity distribution.

The writer has evaluated the constants in Eqs. 55 to 66 using the velocity distribution that has been derived on the basis of the momentum transfer theory, assuming that the mixing length is constant over any cross section.[5] The constants were evaluated for this distribution (Fig. 22) because, in general, it seems to follow experimentally determined distributions as well as, or better than, any other of the analytically determined curves. In addition, this distribution is such that there exists a finite breadth outside which there is no velocity in the x-direction, thus simplifying the evaluation of the integrals. The velocity distribution was analyzed completely by W. Tollmien with the exception of the determination of a single numerical constant in each case which determined the ratio between the mixing length and the distance x. This constant was ascertained by Mr. Tollmien in two cases from rather meager experimental data. However, the excellent data presented by the authors can be used to determine these constants, and the values of Y_m, Y_q, and Y_e have been determined from the values of $x_0/B_0 = 5.2$ and $x_0/D_0 = 6.2$ for the two-dimensional and three-dimensional cases, respectively. In Mr. Tollmein's

[14] "Über eine neue Theorie der Freien Turbulenz," by H. Reichardt, *Zeitschrift für angewandte Mathematik und Mechanik*, October, 1941, pp. 257–264.

paper,[5] it was pointed out that the distance x should not be measured from the plane of the orifice but rather from a plane about two diameters distant from the orifice in order that the profiles may be identical when plotted dimensionlessly. However, at large values of x the values of η would be practically

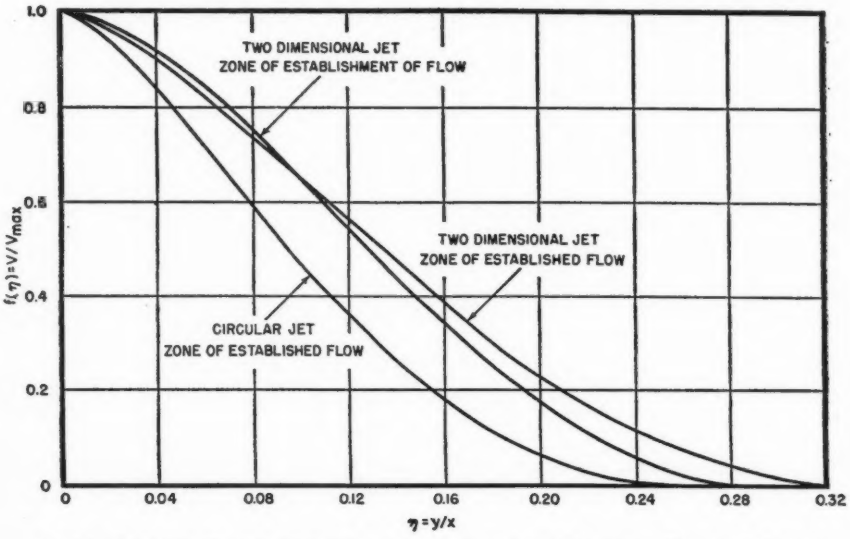

FIG. 22.—VELOCITY DISTRIBUTION IN ZONE OF DIFFUSION BY MOMENTUM TRANSFER THEORY

identical for either definition so that it was assumed that $\eta = y/x$. The curves $f(\eta)$ were first drawn to a convenient scale of η and graphically integrated. The correct scale for Y_m was then determined from Eqs. 57 and 64, and the corresponding values of Y_q and Y_e computed. In Table 1, the general form of the equations are listed with their numerical values computed from the velocity distributions plotted in Fig. 22 and from the formula:

$$f(\eta) = e^{-J} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67)$$

in which, to simplify typography,

$$J = \frac{\eta^2}{2\,(C_1)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (68)$$

Eq. 67 is obtained if it is assumed that the velocity distribution follows the normal error curve. It will be noticed that the differences between the two sets of equations (Cols. 3 and 4, Table 1) are not large and might well fall within normal experimental error.

To the writer, one of the most interesting aspects of free turbulence is the experimental verification, in a wide variety of problems, of the second assumption. However, it has been shown analytically by A. M. Keuthe[7] that the assumption is not valid in the zone of flow establishment of the circular jet.

[7] "Investigations of the Turbulent Mixing Regions Formed by Jets," by A. M. Keuthe, *Journal of Applied Mechanics*, September, 1935, p. A-87.

TABLE 1.—Comparison of Formulas

Zone	Quantity	General form	Error curve	Momentum transfer theory
Zone of establishment; two-dimensional jet	x_0	$\frac{B_0}{2\,Y_m}$	$5.2\,B_0$	$5.2\,B_0$
	$\frac{v_{max}}{v_0}$	1	1	1
	$\underset{y\to\infty}{\text{Limit}}\,\frac{v_y}{v_0}$	$-(Y_q - Y_m)$	-0.040	-0.040
	$\frac{Q}{Q_0}$	$1 + (Y_q - Y_m)\,\frac{x}{B_0/2}$	$1 + 0.080\,\frac{x}{B_0}$	$1 + 0.080\,\frac{x}{B_0}$
	$\frac{E}{E_0}$	$1 + (Y_e - Y_m)\,\frac{x}{B_0/2}$	$1 - 0.035\,\frac{x}{B_0}$	$1 - 0.036\,\frac{x}{B_0}$
	Y_m	$\int_0^\infty f^2(\eta)\,d\eta$	$\frac{\sqrt{\pi}}{2}\,C_1$	0.096
	Y_q	$\int_0^\infty f(\eta)\,d\eta$	$\sqrt{\frac{\pi}{2}}\,C_1$	0.134
	Y_e	$\int_0^\infty f^2(\eta)\,d\eta$	$\sqrt{\frac{\pi}{6}}\,C_1$	0.078
Zone of established flow; two-dimensional jet	x_0	$\frac{B_0}{2\,Y_m}$	$5.2\,B_0$	$5.2\,B_0$
	$\frac{v_{max}}{v_0}$	$\frac{B_0}{2\,Y_m}$	$2.28\sqrt{\frac{B_0}{x}}$	$2.28\sqrt{\frac{B_0}{x}}$
	$\underset{y\to\infty}{\text{Limit}}\,\frac{v_y}{v_0}$	$-\sqrt{\frac{Y_q^2\,B_0}{8\,Y_m\,x}}$	$-0.155\sqrt{\frac{B_0}{x}}$	$-0.160\sqrt{\frac{B_0}{x}}$
	$\frac{Q}{Q_0}$	$\sqrt{\frac{2\,Y_q^2\,x}{B_0\,Y_m}}$	$0.62\sqrt{\frac{x}{B_0}}$	$0.64\sqrt{\frac{x}{B_0}}$
	$\frac{E}{E_0}$	$\sqrt{\frac{Y_e^2\,B_0}{2\,Y_m^3\,x}}$	$1.86\sqrt{\frac{B_0}{x}}$	$1.80\sqrt{\frac{B_0}{x}}$
	Y_m	$\int_0^\infty f^2(\eta)\,d\eta$	$\frac{\sqrt{\pi}}{2}\,C_1$	0.096
	Y_q	$\int_0^\infty f(\eta)\,d\eta$	$\sqrt{\frac{\pi}{2}}\,C_1$	0.140
	Y_e	$\int_0^\infty f^2(\eta)\,d\eta$	$\sqrt{\frac{\pi}{6}}\,C_1$	0.076
Zone of established flow; three-dimensional jet	x_0	$\frac{D_0}{\sqrt{8\,Y_m}}$	$6.2\,D_0$	$6.2\,D_0$
	$\frac{v_{max}}{v_0}$	$\frac{D_0}{x\sqrt{8\,Y_m}}$	$6.2\,\frac{D_0}{x}$	$6.2\,\frac{D_0}{x}$
	$\underset{y\to\infty}{\text{Limit}}\,\frac{v_y\,y}{v_0\,D_0}$	$\frac{-\,Y_q}{\sqrt{8\,Y_m}}$	-0.040	-0.042
	$\frac{Q}{Q_0}$	$\frac{x\sqrt{8\,Y_q^2}}{D_0\sqrt{Y_m}}$	$0.32\,\frac{x}{D_0}$	$0.34\,\frac{x}{D_0}$
	$\frac{E}{E_0}$	$\frac{D_0\,Y_e}{x\sqrt{8\,Y_m^3}}$	$4.1\,\frac{D_0}{x}$	$3.8\,\frac{D_0}{x}$
	Y_m	$\int_0^\infty f^2(\eta)\,\eta\,d\eta$	$\frac{C_2^2}{2}$	0.00325
	Y_q	$\int_0^\infty f(\eta)\,\eta\,d\eta$	C_2^2	0.00684
	Y_e	$\int_0^\infty f^2(\eta)\,\eta\,d\eta$	$\frac{C_2^2}{3}$	0.00201

(Mr. Keuthe's excellent paper is a complete, but rather mathematically involved, theory concerning the zone of establishment of the circular jet.) It can be proved that at the edge of the orifice, the velocity distribution must be the same as that in the zone of establishment of a two-dimensional jet. From this distribution, there must be a gradual transition to the distribution of velocity in the zone of established flow from the orifice (Fig. 22). The equation of the mean motion in the circular jet is

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_y}{\partial y} = \frac{1}{\rho\, y} \frac{\partial(\tau\, y)}{\partial y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (69a)$$

The left side of Eq. 69*a* expresses the acceleration and the right side is the force per unit mass. At $x = 0$ and $y = 1$, designating the radius of the orifice as unity, the equation reduces to the equation of the mean motion in the two-dimensional jet:

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_y}{\partial y} = \frac{1}{\rho} \frac{\partial \tau}{\partial y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (69b)$$

If the same mechanism of turbulent shear exists in both cases, the distribution of velocity in both the circular and two-dimensional jets must be the same at the point where mixing begins. This argument also seems valid intuitively since at this point the breadth of the mixing region is infinitely small compared to the radius of curvature of the orifice. The experimental data presented in Fig. 16 also bear out this argument. Close examination of the plotted points will show that there is a definite tendency for the distribution to vary with x/D_0. Points for smaller values of x/D_0 have a less flat distribution and indeed follow closely the authors' equation for velocity distribution in the zone of establishment of the two-dimensional jet whereas those points for larger values of x/D_0 are approaching the curve for the zone of established flow of the circular jet.

The final forms of the equations presented in Table 1 are dependent on the values determined for x_0/B_0 and x_0/D_0 and as such should be used with some discretion in an application where the distribution of velocity at the slot or orifice is not uniform. A distribution of velocity in an orifice corresponding to a normal turbulent profile might cause a very significant shortening of the length x_0. Since the formulas for the zone of established flow from an orifice are linearly dependent on x_0/D_0, a change of 10% in x_0 would cause an equal change in the discharge flux ratio. The variation in the constants with different inlet velocity distributions could be quite easily determined from measurements of velocity along the center line of the jet, using the principles as derived. Further experiments along this line would be of value in determining the importance of this factor.

DUILIO CITRINI.[15]—This research constitutes a remarkable experimental contribution to the knowledge of the diffusion process of both two-dimensional and three-dimensional jets from slots and orifices. Especially important is the fact that the devised equipment enabled the authors to make observations at

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cross sections at considerable distances from the efflux section, where the values of the velocity and kinetic energy of the fluid are only a small fraction of the initial values. In a strict sense, the simple laws relating to variations of the longitudinal velocity component, the volume flux, and the energy flux, can be applied only to the flow from infinitely small slots and orifices. The measurements reported by the authors, however, seem to indicate the possibility that the laws can be extended to jets having a finite initial section. At least, the similarity between the two cases is reached asymptotically with increasing distance from the efflux section, because, in the region of the jets near the finite initial section (extending somewhat beyond the region which the authors call "of flow establishment"), the flow is certainly governed by more complicated laws.

The analytical presentation of the experimental results is interesting as an approach to the problem by synthesis. As the authors acknowledge, their conclusions must be considered to be only approximations, although in the writer's opinion they are reliable for practical purposes. They are based on substantially empirical hypotheses, as, for example, the assumed distribution of the longitudinal component of the velocity, according to the Gauss normal probability curve. H. Reichardt, who first proposed it in 1941,[16] tries to justify this distribution by an assumed analogy between the diffusion of jets and the diffusion of molecular agitation connected with heat transmission in solids, which is governed by laws that can be expressed through curves of the Gaussian type. Mr. Reichardt shows inductively that, as a consequence of the assumed distribution of the mean longitudinal component of velocity v_x, the turbulent shearing stress can be expressed (in terms of the mean elements of the flow) by the following formula for circular jets (for two-dimensional jets a similar formula can be established):

$$\bar{\tau} = -\rho \left[\Lambda \frac{\partial \overline{(v_x)^2}}{\partial r} + v_x v_r \right] : \ldots\ldots\ldots\ldots\ldots\ldots (70)$$

in which Λ is a linear dimension proportional to x. It does not seem easy however to justify such an expression theoretically.

W. Tollmien,[5] finds a different distribution of v_x by deduction from the well-known expression for $\bar{\tau}$ given by L. Prandtl, and also on the assumption that the turbulent mixing length l is a linear function of x only. The authors call attention to the fact that to make one or another assumption about the detailed characteristics of the turbulent flow pattern does not greatly affect the distribution of the mean velocity; and this is true for most turbulence problems, particularly for the one under discussion. This conclusion is also reached by later studies, such as those of H. Görtler,[17] or those of G. Moretti.[18] Mr. Görtler proposes a theory of the two-dimensional jet based on the hypothesis

[16] "Über eine neue Theorie der Freien Turbulenz," by H. Reichardt, *Zeitschrift für angewandte Mathematik und Mechanik*, 1941.

[17] "Berechnung von Aufgaben der freien Turbulenz auf Grund eines neuen Näherungsansatzes," by H. Görtler, *ibid.*, 1942.

[18] "Scie e getti piani turbolenti," by G. Moretti, *Rendiconti*, Accademia dei Lincei, 1946.

that the so-called kinematic eddy viscosity ϵ is constant on every cross section; and Mr. Moretti bases similar theories on different assumptions. On the other hand, even if all the velocity distributions proposed by the aforementioned investigations, and the distribution devised according to the normal probability curve, are equally useful for supplying an approximate interpretation of the experimental results, the fact cannot be disregarded that there exist between them some remarkable differences. It is enough to remember in this connection that, if the two curves (that by Mr. Tollmien and that of normal probability) identified with the same volume flux are compared for a circular jet, the first gives rise to a kinetic energy flux which is only 87% of that given by the second.[19]

Concerning the comparison of the different curves, it is interesting to consider the respective values of the Coriolis coefficient α—that is, the values of the ratio between the kinetic energy flux of the flow having a velocity distribution according to one of these curves, and that of an ideal flow having the same volume flux and cross section, but with a uniform velocity distribution. In Fig. 23 three curves are compared, with regard to a three-dimensional jet: The Tollmien curve, the normal probability curve, and an experimental curve obtained by the writer[20, 21] as an average of nineteen measurements of mean velocity distribution. The latter observations were made on nineteen cross sections of six different water jets, flowing from the same orifice into still water, with different values of efflux velocity. For the first two curves the abscissa scale, at first, is indeterminate, as it depends (according to the theory of jets issuing from an infinitely small source) upon a constant, which must be established by experiment. This indeterminacy has been eliminated in Fig. 23 by the requirement that the three curves must meet at the place (abscissa $r = r_{0.5}$) where the velocity is reduced to half the axial velocity. The values of the Coriolis coefficient α have been calculated for the three curves, taking into account, for each one, only the part of the cross section covered by the experimental curve. This was done for the two theoretical curves, in an attempt to exclude the external region of the flow, which would have caused a great increase in the section area (infinite for the Gauss curve), while its effect on the volume flux and on the kinetic energy flux would have been negligible. The following values of α were obtained:

Curve	α
Tollmien	2.8
Normal probability	3.64
Experimental	3.5

This comparison demonstrates that the normal probability curve is a good empirical representation of the velocity distribution in the diffusion zone of jets. The results presented in the paper lead to the same conclusion. Naturally this does not apply to the external region of the diffusion zone, where the velocity values really decrease rather quickly toward zero. It is the writer's opinion

[19] "Diffusione di una vena fluida effluente in un campo di fluido in quiete," by Duilio Citrini, *L'Energia Elettrica*, April, 1946.

[20] "Ricerca sperimentale sulla diffusione di una vena liquida effluente in un campo di liquido in quiete," by Duilio Citrini, *ibid.*, August, 1946.

[21] "Nuove ricerche sulla diffusione di una vena liquida in un campo di liquido in quiete," by Duilio Citrini, *ibid.*, June, 1947.

that this circumstance is responsible for the appreciable discrepancy between the experimental values of the volume flux plotted in Fig. 19 and the curve representing Eq. 52, which was obtained by the analytical integration of the Gauss curve. As to the kinetic energy flux, the agreement is rather good because that factor depends on the third power of the velocity, so that the external region contributes very little to it.

The following series of numerical values, defining the writer's experimental curve plotted in Fig. 23, is in good agreement with the results of his tests:

$\frac{r}{r_{0.5}}$	$\frac{v_x}{v_m}$
0.2	0.966
0.4	0.880
0.6	0.757
0.8	0.625
1.0	0.500
1.2	0.387
1.4	0.284
1.6	0.191
1.8	0.106
2.0	0.034

It is the writer's opinion (although lack of necessary numerical data prevents a direct comparison) that this curve also provides very good interpretation of tests presented in the paper.

As to the expansion of the diffusion zone and the law of decrease of the axial velocity (the variations of the volume flux and energy flux being a direct consequence of it), the analysis of the authors seems to have been guided by a criterion of practical convenience, rather than by theoretical exigence. Even accepting for the entire zone of established flow the hypotheses that define a diffusion along a dihedron or a cone (respectively, for the two types of jets studied), there does not seem to be any predictable reason why the edge of the dihedron or the vertex of the cone should lie in the efflux section. Possibly, this appreciable discrepancy between the analytical function and the experimental values (shown by Figs. 11 and 17) is due also to such an assumption, whereas each group of tests could be interpreted more logically by a more suitable Gauss function, correlated to a more suitable law of expansion of the diffusion zone. In order to compare the writer's tests on three-dimensional jets, it must be noted that the Gauss function yields

$$r_{0.5} = 1.178\,\sigma \qquad (71a)$$

The value, $C_2 = 0.081$, given by the authors for the empirical coefficient that enters their theory, corresponds therefore to the expansion law:

$$r_{0.5} = 0.0954\,x \qquad (71b)$$

On the other hand, for the region farthest downstream in the writer's tests, the law—

$$r_{0.5} = 0.1\,x - 0.21\,D_0 \qquad (71c)$$

—has been found most suitable. This law does not differ greatly from the previous law (Eq. 71*b*), but it places the vertex of the diffusion cone somewhat farther than the distance 2 D_0 downstream from the efflux section. Upstream from this region (that is, for approximately $x < 15\ D_0$) and also for the entire

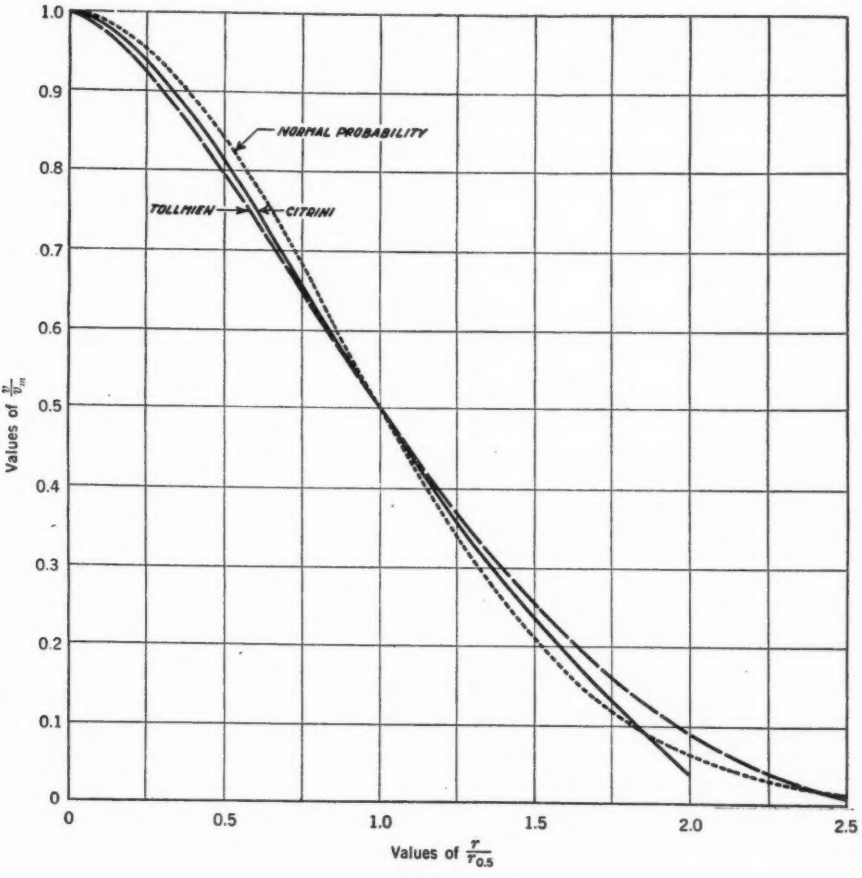

FIG. 23

region which the authors term "flow establishment," the writer has found that the real expansion process is much more exactly represented by a law of parabolic type—that is:

$$\frac{r_{0.5}}{D_0} = 0.55 + \left(0.08\,\frac{x}{D_0}\right)^2 \qquad (71d)$$

Undoubtedly the Iowa experiments lead to much greater values of x than the writer's experiments. This is due partly to the difficulty of measuring low velocities in the water, and partly to the limited dimensions of the stilling basin used by the writer in his tests. In this connection, it is the writer's opinion (differing somewhat from the authors' view) that, if the downstream region has a

greater theoretical meaning, owing to its greater conformity with the assumptions of the theory, the upstream region, which has higher values of velocity and energy flux, is far more interesting for practical purposes. This is the reason that the writer has not derived a law of decrease of the axial velocity of the type of Eq. 47 (which undoubtedly is in much closer agreement with the theoretical assumptions) but has preferred to adopt the form:

$$\frac{v_m}{v_0} = 6.6\,\frac{D_0}{x} - 0.49 \qquad (72)$$

Eq. 72 certainly has an empirical basis; but it seems to follow better the trend of the real process, at least for penetrations of 40 diameters or less.

These differences concern only details, however, for all practical purposes, the authors' and the writer's formulas can be considered as perfectly equivalent. The comparison proves therefore that the results of both groups of tests are reliable, and that all the formulas proposed in the paper can be applied in practical problems.

An interesting property, which should affect the stream surfaces for the circular jet, if all the theoretical assumptions were fulfilled, is that these surfaces should be mutually homothetical, the homothetic center being at the vertex of the diffusion cone.

STANLEY CORRSIN.[22]—This thorough experimental study of the mean flow fields in submerged jets is particularly interesting since the authors have extended the measurements to extremely large distances along the jet axis. In the introductory analytical part of the paper, however, the authors have not explained which assumptions lead to which results in the integral computations of width (σ) and v_{max} variation in the downstream direction. Specifically, the assumption of dynamic similarity alone does not give the linear spread of a jet, and σ/x = constant cannot simply be assumed. (For example, a fully developed two-dimensional laminar jet has dynamic similarity, and spreads as $x^{2/3}$.)

The two assumptions of (*a*) constant static pressure and (*b*) dynamic similarity of both mean and fluctuating motions, together with the equations of motion, yield simultaneously the results for $\sigma(x)$ and $v_{max}(x)$. For example, consider the case of the fully developed round turbulent jet. From the x-component of the Reynolds equation, assuming p constant—

$$v_x \frac{\partial v_x}{\partial x} + v_r \frac{\partial v_x}{\partial r} = \frac{1}{\rho r}\frac{\partial}{\partial r}(r\tau) \qquad (73)$$

For turbulent flow,

$$\tau \doteq -\rho\,\overline{v_x v_r} \qquad (74)$$

By integrating Eq. 73 from the axis to ∞, and by using the equation of continuity, a conservation equation evolves for the rate of flow of momentum,

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analogous to the well known "von Kármán integral relation":

$$\frac{d}{dx}\int_0^\infty (v_x)^2\, r\, dr = 0 \qquad (75a)$$

or

$$2\pi\rho\int_0^\infty (v_x)^2\, r\, dr = \text{constant} = M_0 \qquad (75b)$$

Eq. 75*b* expresses the rate of flow of momentum through all planes perpendicular to the x-axis.

An independent integral relation can be deduced by multiplying Eq. 73 by v_x before integrating. There results the equation for the conservation of rate of flow of mechanical energy—

$$\frac{d}{dx}\int_0^\infty \left[\tfrac{1}{2}\rho\,(v_x)^3\right] r\, dr = -\int_0^\infty \tau\,\frac{\partial v_x}{\partial r}\, r\, dr \qquad (76)$$

The left-hand side of Eq. 76 (within the boundary layer type of approximation) is the x-rate of increase in the rate of flow of mean kinetic energy. The right-hand side is the total rate at which the turbulent shear stress does work on the mean motion over a cross section; thus, it is the rate at which mean flow kinetic energy is transformed to turbulent kinetic energy—a "less available" form of energy.

The assumption of dynamic similarity requires that

$$\frac{v_x}{v_{\max}} = f(\eta) \qquad (77)$$

in which

$$\eta = \frac{r}{\sigma} \qquad (78a)$$

(Of course, any characteristic lateral jet dimension may be used as reference length) and

$$\tau = \rho\,(v_{\max})^2\, g(\eta) \qquad (78b)$$

Substitute the similarity assumptions (Eqs. 77 and 78) into Eqs. 75*b* and 76 to obtain simultaneous equations for the two unknowns, $\sigma(x)$ and $v_{\max}(x)$:

$$(v_{\max})^2\,\sigma^2 = \frac{M_0}{2\pi\rho I_1} \qquad (79a)$$

and

$$\frac{d}{dx}\left[(v_{\max})^3\,\sigma^2\right] = -2\,\frac{I_3}{I_2}\,(v_{\max})^2\,\sigma \qquad (79b)$$

In Eqs. 79, the symbols I are pure numbers depending only upon the assumed dimensionless profile shapes:

$$I_1 \equiv \int_0^\infty f^2\,\eta\, d\eta \qquad (80a)$$

$$I_2 \equiv \int_0^\infty f^3\,\eta\, d\eta \qquad (80b)$$

and

$$I_3 \equiv \int_0^\infty g \frac{df}{d\eta} \eta \, d\eta \quad \text{.....(80c)}$$

Simultaneous solution of Eqs. 79 leads to the required results:

$$v_{max} = \left(\tfrac{1}{2} \frac{I_2}{I_3 \sqrt{I_1}} \sqrt{\frac{M_0}{2\pi\rho}} \right) \frac{1}{x} \quad \text{.....(81a)}$$

and

$$\sigma = 2 \frac{I_3}{I_2} x \quad \text{.....(81b)}$$

The constants of integration have been made equal to zero by (arbitrarily) defining $x = 0$ where $\sigma = 0$ or $v_{max} = \infty$.

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DISCUSSIONS

PROPERTY SURVEYS MUST FIT THEIR TITLES

Discussion

BY RUPERT F. CARROLL

RUPERT F. CARROLL,[7] ASSOC. M. ASCE.—Land surveying is a highly specialized profession. That point is frequently overlooked by men in other branches of engineering who regard surveying as merely the field work necessary to locate structures and provide information from which cost estimates may be prepared. The paper covers this point adequately, and it also lists factors which the engineer must consider in relocating property to conform to its legal and record title. There are special considerations, however, that must be taken into account when applying these factors to survey problems in a given state.

For example, there are no United States public land surveys in Texas. The patents or record titles to most Texas lands are based on original field notes by surveyors from several nations; surveys were made piecemeal and in some instances without enough data on adjoining or conflicting surveys.

Much has been published concerning the special conditions affecting the position of Texas in the current Tidelands controversy due to the state's retention of its public lands. Texas surveyors know that this same retention created problems which are not generally faced by members of their profession practicing in other states. Most persons are vaguely aware that Texas was an independent nation for 10 years, but they do not realize that a colonization race by at least three other nations brought on the war that made Texas a free republic. Also, because of boundary disputes, this colonization did not cease with the independence of Texas in 1836, but was actually accelerated in certain sections, notably the vast south Texas region between the Nueces River and the Rio Grande.

Texas was originally claimed by the United States as a part of the Louisiana Purchase, but this claim was dropped in 1821. It was also claimed by Spain as a part of Mexico and was still considered as such by Mexico after that nation won its independence from Spain. Mexico opened the territory of Texas to foreign colonization in the period from 1821 to 1835 after the boundaries had been closed for more than 200 years.

NOTE.—This paper by William C. Wattles was published in February, 1948, *Proceedings*. Discussion on this paper had appeared in *Proceedings*, as follows: May, 1949, by Robert F. Borg, and S. A. Bauer.

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American colonists moved into east Texas from Louisiana and Arkansas and pushed southward until they came into contact with the Mexican colonists moving northward from the Rio Grande. Disputes arising from this meeting precipitated a war in which the Texans were forced to fight Mexico without organized aid from the United States government. The Texans, therefore, considered that their fight was for independence from Mexico, and having won it, they established the Republic of Texas. Mexico was forced to recognize this new nation but considered the Nueces River as the proper international boundary, whereas the Texans claimed their sovereignty extended to the Rio Grande. During the 10 years of the Texas Republic, both Texas and Mexico colonized the disputed territory extensively. Prior to 1836 it had been populated by a few Mexicans holding large tracts of land by grants from the Spanish crown and the Mexican government.

Texas was annexed to the United States by a treaty in 1845 which specified, among other things, that the new State of Texas would retain title to all public lands within the state. The United States thus became a party to the boundary dispute which precipitated the Mexican War in 1846. This war ended with the Treaty of Guadalupe Hidalgo in 1848. The treaty established the Rio Grande as the international boundary, but recognized the validity of Spanish and Mexican land titles granted prior to November 13, 1835. Incidentally, this treaty also included the cession to the United States of the territory now occupied by New Mexico, Arizona, Nevada, and California. The Texas Legislature passed the Confirmation Act in 1852 which recognized Spanish and Mexican land titles granted prior to November 13, 1835, but invalidated the titles to lands given by Mexico to its citizens as a reward for military service.

Patents to the valid grants were issued by the State of Texas based on the original Spanish or Mexican field notes which contained a minimum of description, and which in many cases were tied to some of the canceled grants. The recognized titles included several huge tracts, the largest of which was the San Juan de Carricitos grant in Kenedy and Willacy counties, covering approximately 650,000 acres. These grants were usually described in the field notes merely by the four corners with sometimes no course and distance calls between. The field notes were in Spanish and the distance calls, if included, were in cordeles, the cordel being 50 varas or approximately 139 ft. The cordel usually employed was of fiber rope and the length was subject to variation by moisture.

There were several ceremonies connected with the Mexican and Spanish procedure of placing a citizen in possession of his land, but the only factor of surveying importance was the requirement in many instances that the landowner set up permanent markers of stone and mortar at the survey corners. These had to be set within a given time or the landowner would forfeit all right to the land. This feature probably accounts for the respect with which their corners were preserved by the Mexican and Spanish colonists. In these early surveys, great importance was paid to natural objects which were used for corners wherever possible. These grants were almost universally excessive since acreage at that time was of small importance.

The General Land Office of the Republic of Texas was established in 1836, and one of the first acts of the first land commissioner was to gather all the available title papers and records from the various colony headquarters and from the Mexican land offices. Many papers were never collected, but the documents obtained are now preserved in the Spanish archives in the Texas General Land Office.

Colonists who had not received land titles from the Mexican government were given lands by the Republic of Texas depending on the marital status of the immigrant and the time of his settlement in Texas. Lands were given for military service and to the heirs of men who had died in the Texas war for independence.

In the early days of the republic, the General Land Office had no accurate maps and the surveyors of the various land districts were required to furnish maps showing the valid surveys in their territory. Today these maps seem at least inaccurate, but when the low fees for surveys at that time are considered, along with the constant danger from marauding Indians and the distances involved, respect for the early surveyor is increased.

When Texas became a state in 1845, the public domain amounted to about 172,700,000 acres, which the United States government could have acquired by the payment of approximately $13,000,000 in debts accumulated by the Republic of Texas. However, Congress allowed the Texans to retain their land and pay their own debt—a very fortunate decision for the people of Texas.

As stated, Texas used the land to pay colonists and soldiers; after statehood, other land was granted to encourage colonization. Millions of acres were given away for the building of railroads, a state capitol building in Austin, the University of Texas at Austin, and many other public improvements. The remainder was finally set aside for the Public Free School Fund. Several thousand acres of unsold land still are retained in this fund.

Many of the land grants were made with the provision that the grantee would survey the land and submit the field notes to the General Land Office. Most of these surveys made in south and west Texas were intentionally excessive, but the fact was not shown in the record field notes which were plotted on maps in the General Land Office. Some of the surveys were carefully made and permanent markers were established. In other cases, it is reasonably certain that the location field notes were written from the land office maps by a surveyor who was never on the ground. Other surveyors who may have been on the ground left no trace of their footsteps.

As the land was settled, it was resurveyed in many instances and these corners were well marked. Where the original survey could be found, the resurvey was called to fit the same corners and natural objects regardless of excess. Sometimes this excess was reported to the land office by the corrected field notes showing the true dimensions.

Thus, a situation is created wherein a citizen has more acreage in his survey than was granted to him by the state, but his title to the entire survey including excess is clear as long as the original survey can be traced. The courts have held that it is not what a surveyor intended to do, it is what he actually did that will govern. In case of mineral lands or other valuable property, it

is generally true that the landowner purchases the excess acreage from the state in order to protect himself should the original corners ever be lost.

The other situation is the so-called "office" survey wherein the original surveyor was not on the ground, or wherein he left no trace of his work. In many cases this survey is built from an older survey, run on the ground and well monumented with excessive dimensions. The office survey calls for adjoinder to the older survey, but the very nature of the calls often shows that the surveyor making them had no knowledge of the survey to which he was building his work. Most of the "office" surveys were resurveyed on the ground with the same excess found in the older survey to which it was called to adjoin. This construction was not contested so long as the land had little value; but court decisions involving the title to mineral lands have held that where a surveyor's work cannot be found, it must be constructed according to the called course and distance from the beginning corner. Even the adjoinder calls were not valid if it could be reasonably proved that the surveyor making such calls did not know where the corners were that he called to adjoin. Under these circumstances, the customary excess cannot be placed in the system of surveys. If it exists on the ground as occupied, it must be considered as vacant or unsurveyed land.

By act of the Texas Legislature in 1939, a landowner may purchase from the state such vacant land found within his enclosure as a "good faith" claimant. If the land is more than 5 miles from a commercial gas or oil well, the State of Texas retains a one-sixteenth royalty interest. If it is less than 5 miles from such a well, the interest retained is one eighth of the oil and gas produced. Other provisions in the law dictate the procedure to be followed in case the vacancy is found and applied for by a so-called "vacancy hunter" before an application to purchase is filed by the "good faith" claimant. These provisions make it very advantageous for a landowner to find and purchase such unsurveyed land before an application is filed by an outsider. Therefore, a surveyor should be very careful in his work of retracing old surveys, and if he cannot definitely prove the position of the title survey, he should advise his client to purchase the doubtful acreage as unsurveyed land, rather than risk a filing by a vacancy hunter.

Surveying of subdivided land in Texas is little different from similar work in other states. However, land holdings in south and west Texas are so large that a property survey usually represents the location of blocks or even of several systems of surveys.

For this reason, a surveyor cannot begin a job of any size without considerable preliminary preparation. He should first consult the General Land Office records, paying particular attention to the following: (1) Patents to the surveys in question; (2) original field notes and all resurvey field notes on the surveys to be located and surrounding surveys; (3) old district and county maps and surveyors' sketches; (4) General Land Office correspondence files; and (5) deed record data from county records or private files.

He should also consult the records of the county surveyor; they sometimes reveal information not available in the General Land Office. If possible, aerial

photographs of the area should be obtained as well as good maps by the United States Geological Survey or the United States Coast and Geodetic Survey.

Careful plotting of the patent field notes in chronological order will often reveal discrepancies before the ground work is begun. These should be p' tted on tracing paper or cloth on the same scale as the aerial pictures if such are available. By superimposing the field notes on the photographs a good idea of the original locations may be obtained.

In the ground survey, it often pays to know something of the characteristics of the original surveyor. It is useless to look for monuments of some surveyors because it has been found from past experience that either they were not on the ground, or (if they were) they left no visible trace of their work. Some surveyors had definite habits concerning "excess" in their work; others were very accurate and careful. Most early surveyors attempted to run on true bearings, but some followed the magnetic course. The field notes usually show the variation or magnetic declination used in the survey which, for work in south Texas and west Texas, usually was the variation in San Antonio at the time of the survey. The extent of the San Antonio or Bexar Land District was so large that this procedure resulted in bearing errors of as much as two degrees in some parts of west Texas.

To summarize, the surveyor should be careful in his report to record only what he found or did not find on the ground, and leave the decisions on points of fact and law to the courts.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DEVELOPMENT OF CBR FLEXIBLE PAVEMENT DESIGN METHOD FOR AIRFIELDS

A SYMPOSIUM

Discussion

BY JACOB FELD

JACOB FELD,[53] M. ASCE.—Within about one hundred pages, the contributors have recorded 20 years of experience in developing, testing, and applying a method for designing soil layers to support moving loads. Each author treats his phase of the subject in detail, making available the story of the empirical deduction from field observations, followed by the confirmation from controlled tests for highway traffic, the extension of method to the design of airport pavements and, the program of field testing to confirm and modify the method so applied to airport pavement design. In all this detailed record, however, there is no clear definition of the California bearing ratio (CBR), or of the soil characteristic which it is supposed to represent.

This discussion is limited to the question of correlation of CBR value with soil identification and classification. In the "Foreword" the CBR of the soil is stated to be an index of strength obtained by a simple shear test. In paper No. 1, Messrs. McFadden and Pringle infer that a shear test is proper in the design of flexible pavements, although they also state that the CBR test may not supply the ultimate answer.

The basis of the design method is actually the following:

(1) There is a unique experimental procedure to be performed according to fixed rules—namely, a soil sample in a rigid mold, 6 in. in diameter by 5 in. high, is compressed by a static load of 2,000 lb per sq in. (144 tons per sq ft); then it is saturated while loaded with weights equal to the pavement and other static loads to be imposed above the soil layer under consideration; followed, finally, by a measurement of the load required to force a flat surfaced piston (area 3 sq in.) into the soil a distance of 0.1 in. The ratio of this load to the load required for the same penetration of the piston into crushed rock road

NOTE.—This Symposium was published in January, 1949, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: May, 1949, by Miles S. Kersten, and F. C. Tyrrell and L. A. Palmer.

[53] Cons. Engr., New York, N. Y.

base material is the "CBR" of the soil. For field tests, the static compression load is replaced by standard impact compaction methods.

(2) The result of this procedure is the major criterion of soil action under traffic loaded flexible pavements.

(3) Identity of test results represents identity of soil action, so that, in effect, for this design problem the CBR test is a complete method of soil identification and classification.

The primary purpose of engineering design is to avoid failures (sufficiency), and the requirement of economy (necessity) is a close second. Mr. Porter's original conclusions resulted from the observation of pavement failures (paper No. 2), which are classified as

(*a*) Lateral displacement of the soil as a result of softening by water absorption (viscous flow);
(*b*) Differential settlement (failure of the solid state); and
(*c*) Excessive deflections resulting from repetitions of load (failure of a plastic material).

The first two types of failure are assumed to be caused by insufficient compaction of the soil during construction, and the third is an indication of insufficient shear strength because of either the nature of the soil or poor compaction. Therefore, the CBR test seems to be limited to the strength determination and classification of such soil as will exhibit plastic characteristics (in the compacted state) under repetitions of load. The design method then becomes the choice of the necessary thickness of soil layers of proper CBR value to prevent the plastic deformation of the soils below such layers.

In a proper design by the CBR method, as is stated by Messrs. Middlebrooks and Bertram in paper No. 3, there is no shear deformation, but there are still the settlement of the subgrade, the compaction of the base and surface, and the elastic deformation. These three factors are not direct functions of the CBR value. In addition, it might be argued that, even if the shearing stress does not exceed the shearing strength, there may also be a measurable strain. In any case, it must be made clear that the CBR method disregards the possibility of soil strains accumulating to failure from other causes, even if the necessary shear strength is provided.

A study of the test data reports brings up some questions along this same line of thought. In the Stockton Field tests in California, Mr. Porter reports pavement settlements of from $\frac{1}{2}$ in. to $\frac{3}{4}$ in. during the traffic tests without causing pavement failure. These settlements are credited to the heavy wheel loads as added compaction over that obtained by the rollers during construction (paper No. 5 under the heading, "Description of Project"). However, the data in Table 1 indicate that subgrade deflections at the bottom of the crushed gravel base were substantial, and at least 70% of the measured values at the top of the pavement. Correspondingly, in the Barksdale Field test at Shreveport, La., Mr. Hansen reports (in Table 5, under the heading, "Summary of Test Results: Compaction Requirements to Prevent Excessive Consolidation," in paper No. 6) that the subgrade was compacted permanently by traffic, to

depths of 6 in. for the 20,000-lb track and 12 in. for the 50,000-lb track, with no detrimental shear deformation.

In the Marietta (Ga.) test, Mr. Griffith (in paper No. 7, under the heading, "Discussion of Tests and Test Results") reports on two unsatisfactory pavement assemblies, in both of which the clay subgrade and the sand-clay base were consolidated to higher dry densities. In Table 7 there is an interesting phenomenon reported, for which Mr. Griffith may have an explanation. Although traffic caused additional consolidation of the subgrade, the pressure cells rose as much as 2 in. during the test. Such displacement of the recording cells must be accompanied by a local soil distortion, and readings of questionable accuracy. It seems pertinent to inquire why there is no dependence of soil changes and shear strength and, further, whether a decrease in CBR value denotes that shear failure, and only shear failure, is responsible.

The greater advance made in soil technology by the CBR method, often overlooked, is the premise that the soils as found are to be used, modified if necessary by physical manipulation, and covered with stronger layers for proper distribution of loading. Every soil can be used, sometimes at great expense, but no soil has zero value.

The weakness of the CBR method lies in the premise that only plastic failure must be guarded against and that other strains in the soil body will not cause failure of the pavement, either directly or by affecting the plastic range of action. It would seem more logical to set up a criterion that the soil characteristics (density, moisture content, and the CBR, or some other measure of shear resistance) are not altered permanently by traffic loading, for adequacy of design.

The respective authors should be complimented for making available this complete record of data and the correlation with the CBR design procedure, with special commendation for the work of Mr. Porter, so apparently visible in each phase of the Symposium report.

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DISCUSSIONS

CADASTRAL SURVEYS AND RESURVEYS

Discussion

BY WILLIAM R. BANDY, J. C. CARPENTER, AND WILLIAM H. RICHARDS

WILLIAM R. BANDY,[5] M. ASCE.—Beginning with a brief description of the manner in which the rectangular system for surveying the public lands was devised and inaugurated, this paper describes the magnitude of the job that has been accomplished since its inception and then goes on to show that the job is far from complete. There still remain approximately 116,060,842 acres of unsurveyed land in the United States, and the demand for resurveys is increasing, as a result, to a large extent, of the need for the resurveying of the vast areas scheduled for intensive development in the various river basins throughout the west. The Forest Service of the United States Department of Agriculture has extensive areas to be resurveyed because forest fires have destroyed the wooden corners and bearing trees in certain parts of the forest reserves.

As Mr. Horton very properly states:

> "All land titles, land transactions, and considerations of geographical position in the area of the public domain rest upon and are in terms of the Bureau's cadastral public land survey * * *."

A fact that is not generally known, except to abstractors and realtors, is that the title to every city lot, and parcel of land in the area of the public domain, is based upon the cadastral survey made by the Bureau of Land Management, or its predecessor, the United States General Land Office.

The demand for the remonumentation of the cadastral survey corners on the various irrigation projects that are being developed by the Bureau of Reclamation is explained by the fact that the original surveys of those areas, for the most part, were made in early days when the prime importance of permanent monumentation of corner positions was not realized as it is today, and also because the settlers and subsequent owners of the land failed to appreciate the importance of protecting and preserving the corner monuments. Even today, it is advisable to conduct an educational program to impress upon property

NOTE.—This paper by A. C. Horton, Jr., was published in January, 1949, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1949, by Scott P. Stewart and Clarence S. Jarvis, and Roger E. Amidon.

[5] Regional Chf. of Eng., Region III, Bureau of Land Management, Helena, Mont.

owners and road builders the many advantages of protecting the re-established public survey corners.

Mr. Horton gives a clear description of the cooperative cadastral resurveys now being executed for the Bureau of Reclamation. He mentions the fact that in many instances these cadastral surveys serve the Bureau of Reclamation as horizontal control for its mapping activities. This use of the cadastral surveys by the Bureau of Reclamation illustrates the great saving being effected by the cooperation of the various governmental agencies participating in the development of the Missouri River Basin. This dual use of the cadastral survey rectangular net not only supplies the Bureau of Reclamation with an official property boundary survey on its projects, monumented with iron corner posts, but also furnishes a system of horizontal control for its topographic and planning maps, both for the price of one. Formerly it cost as much to establish a triangulation control system as it costs to cover the area with a cadastral survey.

To insure proper coordination of the work being performed by the governmental agencies in the Missouri River Basin is the principal function of the Interior Missouri Basin Field Committee and the Missouri Basin Inter-Agency Committee, both of which are headed by W. G. Sloan, Assoc. M. ASCE, coauthor of the Pick-Sloan Plan. These committees are doing a very worthwhile job.

Mr. Horton calls attention to the improvements in instrumental equipment and methods for making cadastral surveys over those in use in the early days. In 1948 a test was completed in this region, comparing the lines run by the cadastral engineers with those established by the United States Coast and Geodetic Survey (U.S.C.G.S.) On the Missouri-Souris Unit in North Dakota the Bureau resurveyed 3,262 sq miles, re-established all the section and quarter section corners, and determined the true bearings and distances between the corners. The same area was likewise covered by a control system established by the U.S.C.G.S., its stations tied to the public survey by the cadastral engineers as the work progressed. The cadastral resurvey of this area was executed by field parties headed by eleven cadastral engineers over a period of three seasons—1946, 1947, and 1948.

The tests recorded were taken at random over a large area to include some work of each of the engineers. These tests are made by comparing the measurements of the cadastral survey lines with those made by the U.S.C.G.S. which are considered the ultimate in measurement. The cadastral survey traverses between the triangulation stations were taken from the township plats, using the bearings and distances along the section lines as reported by the cadastral engineers, with ties made to the triangulation stations from the public survey corners. The errors of closure against the U.S.C.G.S. work varied from 1:3996 to 1:35000. The best closure was made against the International Boundary, but this may be partly accounted for because it is the largest area closed. However, the average closure for the seventeen separate tests made is found to be 1:10624.

One interesting feature discovered in computing these closures is that the distances measured by the cadastral engineers are usually shorter than those measured by the U.S.C.G.S. This may have occurred because the chainmen

for the cadastral engineers used too much tension on the tape since spring balances were not used to gage the tension. On the other hand, it may be due to failure of the cadastral engineers to take into account corrections for temperature. No allowance for differences in temperature were made by the cadastral engineers, and a large part of the lines was run during the hot summer months.

J. C. CARPENTER,[6] M. ASCE.—The subdivision of 2,000,000 sq miles of public lands was one of the most extensive surveying and mapping projects in the history of the world. It was a major undertaking of the nineteenth century. The work had to be done in advance of settlement in an undeveloped wilderness with few inhabitants except wild animals and Indians, sometimes hostile.

Land values at the time of the survey did not encourage the use of a high degree of accuracy. The magnetic needle for direction and the link chain for distance were considered satisfactory. Prior to 1910 the contract method was employed and this called for speed at the sacrifice of quality.

In spite of these conditions, which adversely influenced the character of the surveys, a remarkably satisfactory final result was attained. The sections were laid off with an accuracy in keeping with the raw nature of the land; section and exterior quarter corners were marked with temporary monuments—usually a charred stake, quart of charcoal, or marked stone, around which was heaped a mound of sod from two or four standard pits. Settlers followed the tracks of the surveyors across the prairies or past blazed trees in the forests. The maps were indispensable in guiding the settlement, and the orderly arrangement of sections and townships has provided a simple self-indexing permanent record and a location guide to every tract in this vast domain.

As Mr. Horton points out, the ravages of time and the elements have all but obliterated the evidence of the primitive monuments. The concrete filled metal pipe markers now being used by the Bureau of Land Management will provide practically permanent monuments for the boundary corners. Local surveyors have perpetuated many of the original corners with stone or concrete monuments and have re-established lost corners in the manner prescribed by the government regulation. In eastern South Dakota 3,000 sq miles were resurveyed from 1900 to 1910 and all exterior section corners were marked with granite monuments 8 in. square and 3 ft long.

The local surveyor searching for a lost corner (or re-establishing it, if it has been obliterated) must follow the footsteps of the original surveyor until he has determined the exact point where the original marker was set. Having accomplished this objective he perpetuates the location by setting a durable monument and establishing adequate references.

A method of reference has been developed which will provide a much more secure and trustworthy identification than has ever been available. This reference medium is the system of standard plane coordinates now established for each state. The United States Coast Survey, early in the nineteenth century, found it necessary to connect the east and west coast surveys and so projected an arc of triangulation across the continent. Other arcs were extended east and west and north and south and these surveys were all adjusted by the

[6] Senior Highway Engr., U. S. Public Roads Administration, Washington, D. C.

United States Coast and Geodetic Survey (U.S.C.G.S.) in 1927 so that a rigid control network of geodetic arcs has been established. In 1933 the U.S.C.G.S. laid out the first system of state plane coordinates, in the State of North Carolina. This provided a method of converting spherical coordinates of latitude and longitude to plane rectangular (X and Y) coordinates on a single grid, expressed in feet. Any surveyor can use these coordinates as he does latitudes and departures in plane surveying. Every state in the Union now has its own standard system of plane coordinates. In twenty-four states the legislatures have officially adopted the standard plane coordinate system for defining and stating the position or location of points on the surface of the earth. Nine of these states (California, Louisiana, Minnesota, Ohio, Oregon, South Dakota, Washington, Alabama, and Nevada) are among the twenty nine in the public lands area.

The use of plane coordinates by the Bureau of Land Management to define the exact location of land boundary corners will have a far-reaching and very valuable result.

In the plane coordinate system the numerical coordinates represent one location only and when accurately determined they are applicable to that particular spot and no other. Until the network of triangulation and the determination of accurate geodetic positions has been extended so that land boundary corners may be tied in with the assurance that they will meet the necessary standards of accuracy, this method must be used with extreme caution. However, the Bureau of Land Management, in cooperation with the U.S.C.G.S., can select the areas where the system may be applied with confidence of its successful use, and these areas can gradually be widened as the system demonstrates its value. Adoption of plane coordinate control for land boundaries will justify an extension of the geodetic network and will result in a closer spacing of monuments, which will provide more control for important surveys. This control will be invaluable in all future mapping. In the recent remapping of the State of Massachusetts the U. S. Geological Survey found that the cost of the topographic work was reduced by using the control placed by the Massachusetts Geodetic Survey in an amount greater than the entire cost of the geodetic work performed by the state, and the topographic maps were completed and ready for use considerably earlier than if the geodetic survey had not been in place.

No geodetic monuments were available for ties when the public land surveys were made. At first the magnetic needle was used to determine direction and more recently the solar transit has become standard equipment. Now that the geodetic network has been extended throughout the entire United States newer surveys and resurveys should certainly be tied in and the plane coordinates accurately established. By this method the accuracy of the surveys can be measured and errors that may be important can be located and corrected.

The endorsement of plane coordinate control by the Bureau of Land Management will stimulate its use by local land surveyors and engineers on public and private projects. It will convince many "doubting Thomases" who have been reluctant about using the control, possibly because it has not been universally adapted by the Bureau of Land Management.

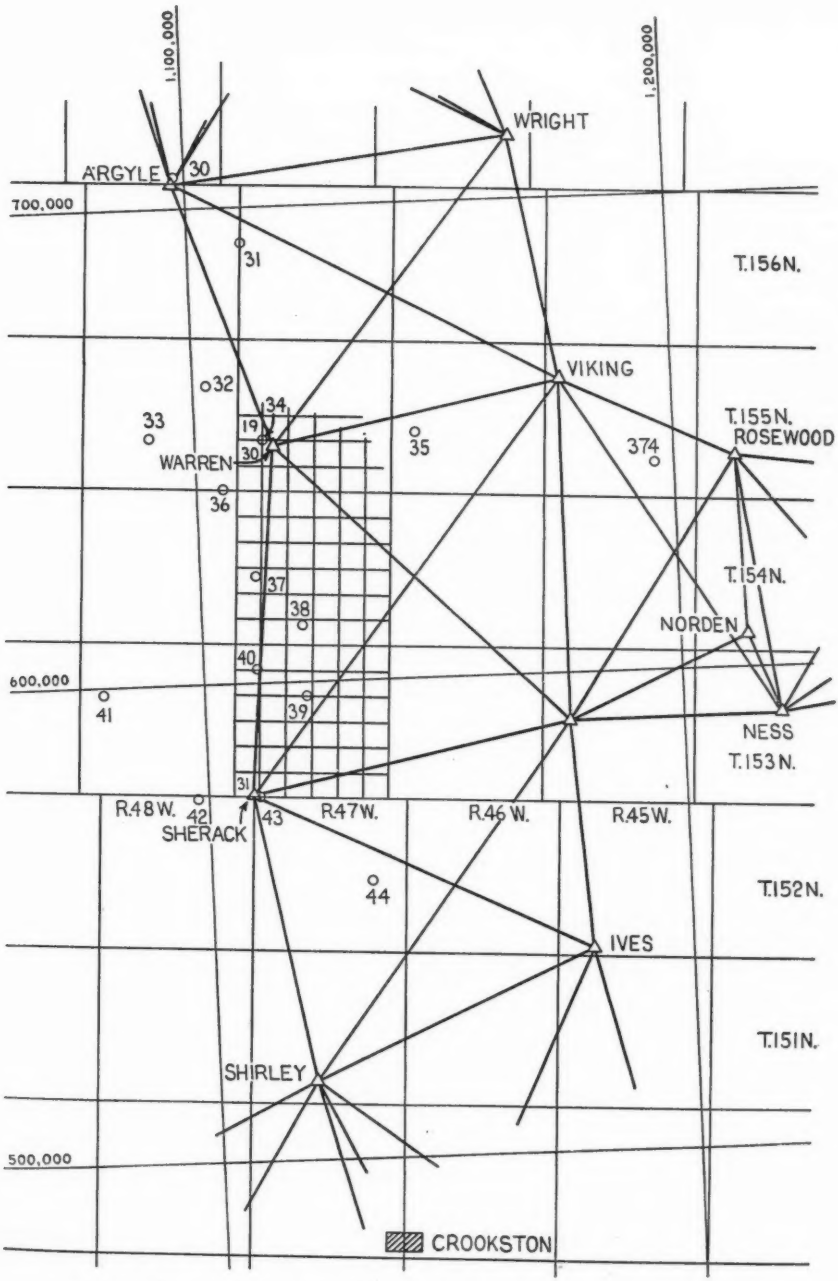

FIG. 2.—PUBLIC LAND SURVEY, MINNESOTA, 98TH MERIDIAN TRIANGULATION AND PLANE COORDINATES (SCALE 1 IN. = 40,000 FT)

Use of plane coordinate control by the Bureau of Land Management will extend the system into virgin areas ahead of development. It will provide an accurate control system for all kinds of surveys—highway, irrigation, power, municipal, aviation—and particularly for topographic mapping. The coordinate system will then be recognized as a truly standard state system.

The location of section corners by plane coordinates will allow accurate determination of their geographic coordinates and will provide for conversion to the military grid and for other uses in accurate military operations.

Some of the original land survey corners were tied to the geodetic network when the triangulation arcs were projected across the United States. To illustrate the theme of this discussion, an example was chosen in Polk County, Minnesota, where two section corners were used as reference marks for principal points on the 98th meridian arc (see Fig. 2).

The public lands survey of this area was made in 1872 and section corners marked with charred stakes, mounds, and pits. In 1906 the U.S.C.G.S. executed its triangulation and established two principal points, "Warren" and "Sherack." Warren was close to the southeast corner of Section 19, T 155 N, R 47 W (Fig. 3), and Sherack, near the southeast corner of Section 31, T 153 N, R 47 W (Fig. 4). The azimuth and distance to these corners were measured. The locations of these two section corners and numerous other supplementary references, such as church spires and elevators, were included in the general readjustment of the first order triangulation in the western part of the United States and these stations are now on the North American datum of 1927. The plane coordinates have been computed for all the points and these coordinates, with a wealth of supplemental data, are readily available.[7]

The markers for the principal points Warren and Sherack were surface and underground monuments and they can be found. For the section corner at the southeast corner of Section 19, T 155 N, R 47 W, the description shows that in 1935 it is reported to have been destroyed by road construction. The published description[7] of the section corner at the southeast corner of Section 31, T 153 N, R 47 W, states that roads have been graded. Both of these corners may be relocated accurately by using the information in the special publication. On this arc the record shows seventeen section corners to have been tied in, and the descriptions indicate that practically all have been covered or destroyed by road construction. The measurements from points on the 98th meridian arc triangulation to these corners will allow their locations to be re-established, and where the roadway is in embankment the original stake may sometimes be discovered by excavating to the level of the prairie sod.

If this were a resurvey, or a new survey of areas not previously subdivided, numerous other corners on the 14-mile length should be tied to the network. Plane coordinates for these additional corners would then bring the system into active use for local surveys. In the U.S.C.G.S. description[7] the plane coordinates for the southeast corner of Section 19, T 155 N, R 47 W (station 34), are shown as $X = 1{,}113{,}173.32$ ft and $Y = 649{,}795.26$ ft. For the southeast corner of Section 31, T 153 N, R 47 W (station 43), the plane coordinates are $X = 1{,}110{,}051.58$ ft and $Y = 575{,}784.83$ ft. This develops a right triangle

[7] *Special Publication No. 203*, U. S. Coast and Geodetic Survey, Washington, D. C., 1935.

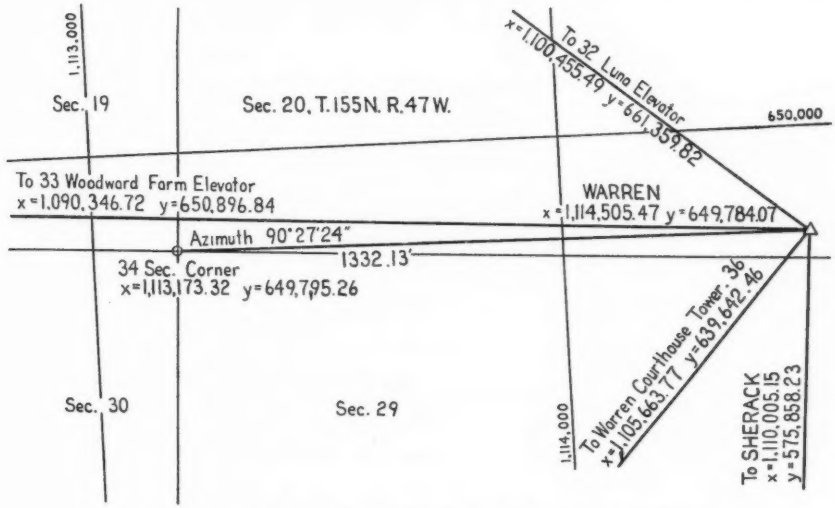

FIG. 3.—LOCATION OF SOUTHEAST CORNER OF SECTION 19 WITH REFERENCE TO "WARREN" AND SUPPLEMENTARY POINTS

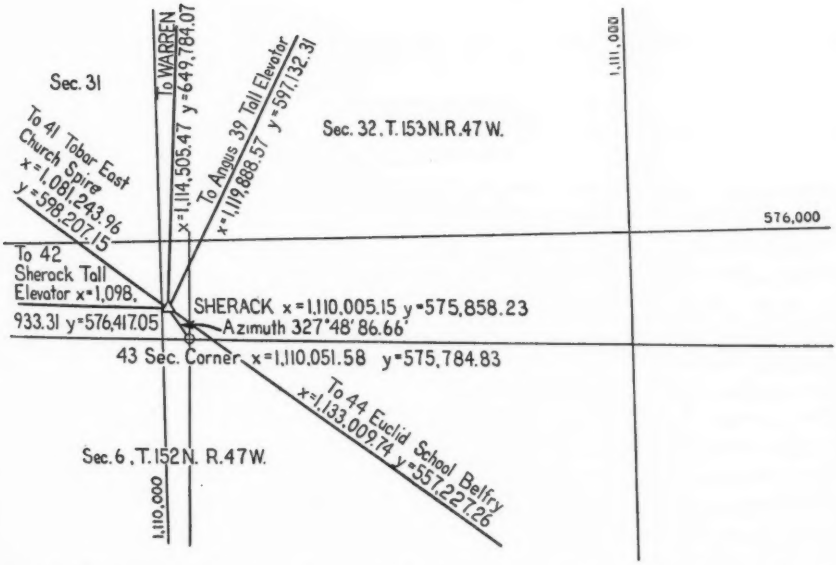

FIG. 4.—LOCATION OF SOUTHEAST CORNER OF SECTION 31, WITH REFERENCE TO "SHERACK" AND SUPPLEMENTARY POINTS, 98TH MERIDIAN TRIANGULATION ARC (SCALE 1 IN. = 400 FT)

with the *X* leg 3,121.74 ft and the *Y* leg 74,010.43 ft. The geodetic distance on the hypotenuse is 74,076.24 ft. The field notes of the 1872 survey in the Washington (D.C.) office of the Bureau of Land Management show compass courses with a needle variation from 13°00′ to 13°30′ on the 14 miles of line between these two corners. Each of the sections is a mile long except the north section in Township 153 N, which shows a length of 80.22 chains or 5,294.52 ft. The chained length on the 1872 survey between these two section corners was 73,934.52 ft. The difference of 141.72 ft gives an average error of 10.12 ft per mile which is an accuracy of 1:522. Considering the circumstances existing at the time of the original survey, this accuracy is reasonable. In the link chain used for linear measurement there is a loop on each end of each link and there are two rings between the links so that there are 588 bearing surfaces to wear and 396 loops and rings that may elongate as the chain is dragged over the ground. The excess length of 10.12 ft per mile means 1.52 in. per chain or a 0.0015-in. average wear or stretch for each bearing surface loop or ring. The chains used were tested as often as was considered necessary, but this element of wear and stretch undoubtedly accounts for part of the almost universal overrun in length of these original measurements.

With the data available it is possible to compute the variation from true north of a line between these two corners. Other valuable checks on linear and angular measurement can be made using the plane coordinates as a method of simplification of the computations. In the surveys now being executed by the Bureau of Land Management the use of plane coordinate control for position determination will provide a very valuable basic system, which can be used for all other subsequent surveys. The coordinates will not supplant the section and quarter corners as a description of the tract, but will supplement this description and serve as an accurate engineering location of the position of the corner. For subdivision inside the section the plane coordinates will provide a positive, accurate description of every point. For any such interior point the distance and azimuth from the section and quarter corners can be computed precisely using plane trigonometry.

The methods now specified by the Bureau of Land Management for its surveys may result in better than third order accuracy. The ties to the geodetic network (wherever economically feasible), and the use of plane coordinates in position computation will provide accurate, simple ground control for all kinds of adjacent surveys and particularly for mapping by modern methods. The markers originally used for the section and quarter corners were intended to be temporary but this marker has been established as the true legal boundary position even though the measurements do not agree with the original field notes. The charred stake, quart of charcoal, and durable stone about 18 in. long, are definite and positive identifications of this original position, provided they can be found and legally recognized. Once found, the position must be perpetuated by granite markers or noncorrodible metal pipes filled with concrete and with reference points accurately measured, or other reliable procedures, all legally recorded. In addition, if the corners are tied to the geodetic network, and plane coordinates are used to reference the location to the equator and the North Pole, the best present available method of identification will be utilized.

The value of universal adoption of plane coordinate control for positive identification is so large that it cannot be estimated. It is certain that this value as a method of standard control for land boundaries, for engineering projects of all kinds, and particularly for mapping, is many times the actual monetary cost of developing this control.

WILLIAM H. RICHARDS,[8] M. ASCE.—In the years following the close of the Revolutionary War the Continental Congress was confronted with the problems involved in the administration and disposal of the lands recently ceded to the federal government by the several states. The ordinance of May 20, 1785, provided in principle for the rectangular system of survey of these public lands. It inaugurated a program of land identification and description that continues to the present time. For magnitude and continuity of execution it is without parallel anywhere in the world.

As the public land states were admitted into the Union, the United States continued to hold title to the unappropriated lands. Under the public land laws the Secretary of the Interior was vested with the jurisdiction over the administration, survey, and disposal of these lands. These activities are now the immediate responsibility of the Bureau of Land Management.

The Department of the Interior and the Bureau must consider and determine what are public lands, what lands have been, or should be, surveyed, and what lands have been, or remain to be, disposed of. It has the authority to extend the surveys of the public lands as may be necessary, including the lands that were omitted from earlier surveys.

These administrative functions, as well as those having to do with resurveys, involve legal as well as technical questions. The Bureau must establish the fact that the lands in question are within its jurisdiction and must plan the survey procedure in such a manner as to protect, adequately, the rights of adjoining privately owned lands. The cadastral engineer who undertakes the field work must be familiar with the fundamental principles of federal and local law. Finally, the record of the survey must be reviewed and accepted by the Bureau.

As stated by Mr. Horton, an original survey of public lands does not ascertain boundaries; it creates or identifies such lines for the first time. A field execution of the survey and the preparation of the plat and field notes are not sufficient in themselves to constitute an official survey. The public lands are not regarded as surveyed until the surveys are accepted by the duly authorized administrative officer of the Bureau.

Resurveys of large areas of the public lands in the western public land states have constituted an important part of the survey program of the Bureau for many years. However, a general recognition by federal mapping agencies of the importance of complete identification and accurate location of the public land survey network as a basis for precise horizontal control for mapping purposes is a relatively recent development.

In the past, engineers engaged in topographic mapping have endeavored to locate all possible section corners. Frequently, only a limited number are

[8] Chf., Branch of Surveys, Bureau of Land Management, U. S. Dept. of the Interior, Washington, D.C.

definitely identified with consequent uncertainty as to the correct position of the remaining section lines. Although the results have been fairly satisfactory for general purpose, small-scale topographic maps, considerably more precise results are required for the larger scale maps now in demand. These results can best be obtained by a complete official resurvey involving recovery of all evidence of the original survey combined with proper restorative procedure for missing corners.

The paper makes reference to the fact that the plats and fields notes of all public land surveys are on file in the Interior Department Building in Washington, D. C. An interesting use of these records has developed since 1947. Active exploration for oil and gas in many of the western states has developed a demand for small-scale planimetric maps showing the land net. A number of private mapping organizations engaged in the preparation of such maps have found that a rapid, convenient and space-saving method of securing copies of the official plats is to photograph them on 35 mm microfilm. In this way copies of the plats covering an entire state can be put on eight or ten rolls of film occupying only a few cubic inches of space.

The paper by Mr. Horton fills a definite need in presenting a brief authoritative statement of the activities of the Bureau of Land Management, in the field of cadastral surveys and resurveys. The author has prepared a comprehensive outline of the origin, purpose, scope, and character of the public land rectangular system of surveys in the United States and Alaska.